Q1	$f(x) = 12x^3 - 24x^2 + 12x, \qquad 0 \le x \le 1$			
(i)	$E(X) = \int_0^1 x f(x)\,dx$	M1	Integral for E(*X*) including limits (which may appear later).	
	$= 12\left[\frac{x^5}{5} - 2\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$	A1	Successfully integrated.	
	$= 12\left[\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right] = 12 \times \frac{1}{30} = \frac{2}{5}$	A1	Correct use of limits leading to final answer. C.a.o.	
	For mode, $f'(x) = 0$	M1		
	$f'(x) = 12(3x^2 - 4x + 1) = 12(3x-1)(x-1)$ $\therefore f'(x) = 0$ for $x = 1$ and $x = \frac{1}{3}$	A1		
	Any convincing argument (e.g. $f''(x)$) that $\frac{1}{3}$ (and not 1) is the mode.	A1		6
(ii)	Cdf $F(x) = \int_0^x f(t)\,dt$	M1	Definition of cdf, including limits (or use of "+c" and attempt to evaluate it), possibly implied later. Some valid method must be seen.	
	$= 12\left(\frac{x^4}{4} - 2\frac{x^3}{3} + \frac{x^2}{2}\right)$ $= 3x^4 - 8x^3 + 6x^2$	A1	Or equivalent expression; condone absence of domain [0,1].	
	$F\left(\frac{1}{4}\right) = \frac{3}{256} - \frac{8}{64} + \frac{6}{16} = \frac{3-32+96}{256} = \frac{67}{256}$ $F\left(\frac{1}{2}\right) = \frac{3}{16} - \frac{8}{8} + \frac{6}{4} = \frac{3-16+24}{16} = \frac{11}{16}$ $F\left(\frac{3}{4}\right) = \frac{3\times81}{256} - \frac{8\times27}{64} + \frac{6\times9}{16} = \frac{243}{256}$	B1	For all three; answers given; must show convincing working (such as common denominator)! Use of decimals is not acceptable.	3
(iii)	<table><tr><td>o_i</td><td>126</td><td>209</td><td>131</td><td>46</td></tr><tr><td>e_i</td><td>134</td><td>352 – 134 = 218</td><td>486 – 352 = 134</td><td>26</td></tr></table>	B2	For e_i. B1 if any 2 correct, provided Σ = 512.	
	X^2 = 0·4776 + 0·3716 + 0·0672 + 15·3846 = 16·30(1)	M1 A1		
	Refer to χ_3^2.	M1	Must be some clear evidence of reference to χ_3^2, probably implicit by reference to a critical point (5% : 7·815; 1% : 11·34). No ft (to the A marks) if incorrect χ^2 used, but E marks are still available.	
	Very highly significant. Very strong evidence that the model does not fit.	A1 A1	There must be at least one reference to "very …", i.e. the extremeness of the test statistic.	
	The main feature is that we observe many		Or e.g. "big/small" contributions	

	more loads at the "top end" than expected. The other observations are below expectation, but discrepancies are comparatively small.	E1 E1	to X^2 gets E1, … … and directions of discrepancies gets E1.	9
				18

Q2	A to B : $X \sim N(26,\ \sigma = 3)$ B to C : $Y \sim N(15,\ \sigma = 2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$P(X < 24) = P\left(Z < \frac{24-26}{3} = -0\cdot6667\right)$ $= 1 - 0{\cdot}7476 = 0{\cdot}2524$	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	3
(ii)	$X + Y \sim N(41,$ $\sigma^2 = 9 + 4 = 13\ [\sigma = 3\cdot6056])$ P(this < 42) = $P\left(Z < \frac{42-41}{3\cdot6056} = 0\cdot2774\right) = 0\cdot6093$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iii)	$0\cdot85X \sim N(22\cdot1,$ $\sigma^2 = (0\cdot85)^2 \times 9 = 6\cdot5025\ [\sigma = 2\cdot55])$ P(this < 24) = $P\left(Z < \frac{24-22\cdot1}{2\cdot55} = 0\cdot7451\right)$ $= 0{\cdot}7719$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iv)	$0\cdot9X + 0\cdot8Y \sim N(23\cdot4 + 12 = 35\cdot4,$ $\sigma^2 = (0\cdot9)^2 \times 9 + (0\cdot8)^2 \times 4 = 9\cdot85\ [\sigma = 3\cdot1385])$ Require t such that 0·75 = P(this < t) $= P\left(Z < \frac{t-35\cdot4}{3\cdot1385}\right) = P(Z < 0\cdot6745)$ $\therefore t - 35\cdot4 = 3\cdot1385 \times 0\cdot6745 = 2\cdot1169$ $\Rightarrow t = 37\cdot52$ Must therefore take scheduled time as 38	B1 B1 M1 B1 A1 M1	Mean. Variance. Accept sd. Formulation of requirement (using c's parameters). Any use of a continuity correction scores M0 (and hence A0). 0·6745 c.a.o. Round to next integer above c's value for t.	6
(v)	CI is given by $13\cdot4 \pm 1\cdot96 \frac{2}{\sqrt{15}}$ $= 13{\cdot}4 \pm 1{\cdot}0121 = (12{\cdot}38(79),\ 14{\cdot}41(21))$	M1 B1 A1	If <u>both</u> 13·4 and $2/\sqrt{15}$ are correct. (N.B. 13·4 is given as $\bar{x}$ in the question.) (If $3/\sqrt{15}$ used, treat as mis-read and award this M1, but not the final A1.) For 1·96 c.a.o. Must be expressed as an interval.	3
				18

Q3				
(i)	Simple random sample might not be representative - e.g. it might contain only managers.	E1 E1	 Or other sensible comment.	 2
(ii)	Presumably there is a list of staff, so systematic sampling would be possible. List is likely to be alphabetical, in which case systematic sampling might not be representative. But if the list is in categories, systematic sampling could work well.	E1 E1 E1	 Or other sensible comments.	 3
(iii)	Would cover the entire population. Can get information for each category.	E1 E1		 2
(iv)	5, 11, 24	B1	(4·8, 11·2, 24)	1
(v)	$\bar{x}$ = 345818, s_{n-1} = 69241 Underlying Normality H_0: μ = 300 000, H_1: μ > 300 000 Test statistic is $\dfrac{345818 - 300000}{\frac{69241}{\sqrt{11}}}$ =2·19(47). Refer to t_{10}. Upper 5% point is 1·812. Significant. Evidence that mean wealth is greater than 300 000. CI is given by 345818 ± 2·228 $\times \dfrac{69241}{\sqrt{11}}$ = 345818 ± 46513·84 = (299304(·2),	 M1 A1 M1 A1 A1 A1 M1 B1 M1 A1	 All given in the question. Allow alternatives: 300000 + (c's 1·812) × $\frac{69241}{\sqrt{11}}$ (= 337829) for subsequent comparison with 345818. or 345818 – (c's 1·812) × $\frac{69241}{\sqrt{11}}$ (= 307988) for comparison with 300000. c.a.o. but ft from here in any case if wrong. Use of $\mu - \bar{d}$ scores M1A0, but ft. No ft from here if wrong. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. Special case: (t_{11} and 1·796) can score 1 of these last 2 marks if either form of conclusion is given. c.a.o. Must be expressed as an	 10

PMT

	392331(·8))		interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{10} is OK.	
				18

Q4				
(i)	<table><tr><th>Differences</th><th>Rank of |diff|</th></tr><tr><td>– 2</td><td>2</td></tr><tr><td>– 1</td><td>1</td></tr><tr><td>– 6</td><td>5</td></tr><tr><td>– 3</td><td>3</td></tr><tr><td>4</td><td>4</td></tr><tr><td>– 12</td><td>9</td></tr><tr><td>7</td><td>6</td></tr><tr><td>– 8</td><td>7</td></tr><tr><td>– 10</td><td>8</td></tr></table>	M1 M1 A1	For differences. ZERO in this section if differences not used. For ranks. FT from here if ranks wrong	
	T = 4 +6 = 10 (or 1+2+3+5+7+8+9 = 35)	B1		
	Refer to tables of Wilcoxon paired (/single sample) statistic. Lower (or upper if 35 used) 5% tail is needed. Value for n = 9 is 8 (or 37 if 35 used). Result is not significant. No evidence to suggest a real change.	M1 M1 A1 A1 A1	No ft from here if wrong. i.e. a 1-tail test. No ft from here if wrong. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic.	9
(ii)	Normality of <u>differences</u> is required.	B1		
	CI MUST be based on DIFFERENCES. Differences are 53, 15, 32, 13, 61, 82, 70		ZERO/6 for the CI if differences not used. Accept negatives throughout.	
	$\bar{d} = 46 \cdot 5714$ $s_{n-1} = 27 \cdot 0485$	B1	Accept $s_{n-1}^2 = 731{\cdot}62$ … [$s_n = 25{\cdot}0420$, but do <u>NOT</u> allow this here or in construction of CI.]	
	CI is given by 46·5714 ± 3·707 $\times \frac{27 \cdot 0485}{\sqrt{7}}$	M1 B1 B1 M1	Allow c's $\bar{d}$ ± … **If t_6 used.** 99% 2-tail point for c's t distribution. (Independent of previous mark.) Allow c's s_{n-1}.	
	= 46·5714 ± 37·8980 = (8·67(34), 84·47)	A1	c.a.o. Must be expressed as an interval. [Upper boundary is 84·4694]	
	Cannot base CI on Normal distribution because sample is small population s.d. is not known	 E1 E1	 Insist on "population", but allow "σ".	 9
				18