OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4768/01 Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4768/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A certain industrial process requires a supply of water. It has been found that, for best results, the mean water pressure in suitable units should be 7.8. The water pressure is monitored by taking measurements at regular intervals. On a particular day, a random sample of the measurements is as follows.

7.50 7.64 7.68 7.51 7.70 7.85 7.34 7.72 7.74

These data are to be used to carry out a hypothesis test concerning the mean water pressure.

(i) Why is a test based on the Normal distribution not appropriate in this case? **[2]**

(ii) What distributional assumption is needed for a test based on the *t* distribution? **[1]**

(iii) Carry out a *t* test, with a 2% level of significance, to see whether it is reasonable to assume that the mean pressure is 7.8. **[9]**

(iv) Explain what is meant by a 95% confidence interval. **[2]**

(v) Find a 95% confidence interval for the actual mean water pressure. **[4]**

2 A particular species of reed that grows up to 2 metres in length is used for thatching. The lengths in metres of the reeds when harvested are modelled by the random variable X which has the following probability density function, $\mathrm{f}(x)$.

$$\mathrm{f}(x) = \begin{cases} \frac{3}{16}\ (4x - x^2) & \text{for } 0 \leqslant x \leqslant 2 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Sketch $\mathrm{f}(x)$. **[3]**

(ii) Show that $\mathrm{E}(X) = \frac{5}{4}$ and find the standard deviation of the lengths of the harvested reeds. **[8]**

(iii) Find the standard error of the mean length for a random sample of 100 reeds. **[2]**

Once the harvested reeds have been collected, any that are shorter than 1 metre are discarded.

(iv) Find the proportion of reeds that should be discarded according to the model. **[2]**

(v) Reeds are harvested from a large area which is divided into several reed beds. A sample of the harvested reeds is required for quality control. How might the method of cluster sampling be used to obtain it? **[3]**

3 In the manufacture of child car seats, a resin made up of three ingredients is used. The ingredients are two polymers and an impact modifier. The resin is prepared in batches. Each ingredient is supplied by a separate feeder and the amount supplied to each batch, in kg, is assumed to be Normally distributed with mean and standard deviation as shown in the table below. The three feeders are also assumed to operate independently of each other.

	Mean	Standard deviation
Polymer 1	2025	44.6
Polymer 2	1565	21.8
Impact modifier	1410	33.8

(i) Find the probability that, in a randomly chosen batch of resin, there is no more than 2100 kg of polymer 1. **[3]**

(ii) Find the probability that, in a randomly chosen batch of resin, the amount of polymer 1 exceeds the amount of polymer 2 by at least 400 kg. **[4]**

(iii) Find the value of b such that the total amount of the ingredients in a randomly chosen batch exceeds b kg 95% of the time. **[4]**

(iv) Polymer 1 costs £1.20 per kg, polymer 2 costs £1.30 per kg and the impact modifier costs £0.80 per kg. Find the mean and variance of the total cost of a batch of resin. **[3]**

(v) Each batch of resin is used to make a large number of car seats from which a random sample of 50 seats is selected in order that the tensile strength (in suitable units) of the resin can be measured. From one such sample, the 99% confidence interval for the true mean tensile strength of the resin in that batch was calculated as (123.72, 127.38). Find the mean and standard deviation of the sample. **[4]**

[Question 4 is printed overleaf.]

4 **(a)** At a college, two examiners are responsible for marking, independently, the students' projects. Each examiner awards a mark out of 100 to each project. There is some concern that the examiners' marks do not agree, on average. Consequently a random sample of 12 projects is selected and the marks awarded to them are compared.

(i) Describe how a random sample of projects should be chosen. **[2]**

(ii) The marks given for the projects in the sample are as follows.

Project	1	2	3	4	5	6	7	8	9	10	11	12
Examiner A	58	37	72	78	67	77	62	41	80	60	65	70
Examiner B	73	47	74	71	78	96	54	27	97	73	60	66

Carry out a test at the 10% level of significance of the hypotheses H_0: $m = 0$, H_1: $m \neq 0$, where m is the population median difference. **[8]**

(b) A calculator has a built-in random number function which can be used to generate a list of random digits. If it functions correctly then each digit is equally likely to be generated. When it was used to generate 100 random digits, the frequencies of the digits were as follows.

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	6	8	11	14	12	9	15	5	14	6

Use a goodness of fit test, with a significance level of 10%, to investigate whether the random number function is generating digits with equal probability. **[8]**