OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4768**

Statistics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Each month the amount of electricity, measured in kilowatt-hours (kWh), used by a particular household is Normally distributed with mean 406 and standard deviation 12.

(i) Find the probability that, in a randomly chosen month, less than 420 kWh is used. **[3]**

The charge for electricity used is 14.6 pence per kWh.

(ii) Write down the distribution of the total charge for the amount of electricity used in any one month. Hence find the probability that, in a randomly chosen month, the total charge is more than £60. **[3]**

(iii) The household receives a bill every three months. Assume that successive months may be regarded as independent of each other.

Find the value of b such that the probability that a randomly chosen bill is less than £b is 0.99. **[4]**

In a different household, the amount of electricity used per month was Normally distributed with mean 432 kWh. This household buys a new washing machine that is claimed to be cheaper to run than the old one. Over the next six months the amounts of electricity used, in kWh, are as follows.

404 433 420 423 413 440

(iv) Treating this as a random sample, carry out an appropriate test, with a 5% significance level, to see if there is any evidence to suggest that the amount of electricity used per month by this household has decreased on average. **[9]**

2 **(a)** **(i)** What is stratified sampling? Why would it be used? **[4]**

(ii) A local authority official wishes to conduct a survey of households in the borough. He decides to select a stratified sample of 2000 households using Council Tax property bands as the strata. At the time of the survey there are 79 368 households in the borough. The table shows the numbers of households in the different tax bands.

Tax band	A – B	C – D	E – F	G – H
Number of households	32 298	33 211	9739	4120

Calculate the number of households that the official should choose from each stratum in order to obtain his sample of 2000 households so that each stratum is represented proportionally. **[2]**

(b) **(i)** What assumption needs to be made when using a Wilcoxon single sample test? **[2]**

(ii) As part of an investigation into trends in local authority spending, one of the categories of expenditure considered was ‘Highways and the Environment’. For a random sample of 10 local authorities, the percentages of their total expenditure spent on Highways and the Environment in 1999 and then in 2009 are shown in the table.

Local authority	A	B	C	D	E	F	G	H	I	J
1999	9.60	8.40	8.67	9.32	9.89	9.35	7.91	8.08	9.61	8.55
2009	8.94	8.42	7.87	8.41	10.17	10.11	8.31	9.76	9.54	9.67

Use a Wilcoxon test, with a significance level of 10%, to determine whether there appears to be any change to the average percentage of total expenditure spent on Highways and the Environment between 1999 and 2009. **[10]**

3 The masses, in kilograms, of a random sample of 100 chickens on sale in a large supermarket were recorded as follows.

Mass (m kg)	$m < 1.6$	$1.6 \leqslant m < 1.8$	$1.8 \leqslant m < 2.0$	$2.0 \leqslant m < 2.2$	$2.2 \leqslant m < 2.4$	$2.4 \leqslant m < 2.6$	$2.6 \leqslant m$
Frequency	2	8	30	42	11	5	2

(i) Assuming that the first and last classes are the same width as the other classes, calculate an estimate of the sample mean and show that the corresponding estimate of the sample standard deviation is 0.2227 kg. **[3]**

A Normal distribution using the mean and standard deviation found in part **(i)** is to be fitted to these data. The expected frequencies for the classes are as follows.

Mass (m kg)	$m < 1.6$	$1.6 \leqslant m < 1.8$	$1.8 \leqslant m < 2.0$	$2.0 \leqslant m < 2.2$	$2.2 \leqslant m < 2.4$	$2.4 \leqslant m < 2.6$	$2.6 \leqslant m$
Expected frequency	2.17	10.92	f	33.85	19.22	5.13	0.68

(ii) Use the Normal distribution to find f. **[3]**

(iii) Carry out a goodness of fit test of this Normal model using a significance level of 5%. **[9]**

(iv) Discuss the outcome of the test with reference to the contributions to the test statistic and to the possibility of other significance levels. **[3]**

4 A timber supplier cuts wooden fence posts from felled trees. The posts are of length $(k + X)$ cm where k is a constant and X is a random variable which has probability density function

$$f(x) = \begin{cases} 1 + x & -1 \leqslant x < 0, \\ 1 - x & 0 \leqslant x \leqslant 1, \\ 0 & \text{elsewhere.} \end{cases}$$

(i) Sketch $f(x)$. **[3]**

(ii) Write down the value of $E(X)$ and find $\text{Var}(X)$. **[5]**

(iii) Write down, in terms of k, the approximate distribution of $\bar{L}$, the mean length of a random sample of 50 fence posts. Justify your choice of distribution. **[4]**

(iv) In a particular sample of 50 posts, the mean length is 90.06 cm. Find a 95% confidence interval for the true mean length of the fence posts. **[4]**

(v) Explain whether it is reasonable to suppose that $k = 90$. **[1]**