



**ADVANCED GCE  
MATHEMATICS (MEI)**

**4768/01**

Statistics 3

**TUESDAY 15 JANUARY 2008**

Morning  
Time: 1 hour 30 minutes

**Additional materials:** Answer Booklet (8 pages)  
Graph paper  
MEI Examination Formulae and Tables (MF2)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

- 1 (a) The time (in milliseconds) taken by my computer to perform a particular task is modelled by the random variable  $T$ . The probability that it takes more than  $t$  milliseconds to perform this task is given by the expression  $P(T > t) = \frac{k}{t^2}$  for  $t \geq 1$ , where  $k$  is a constant.
- (i) Write down the cumulative distribution function of  $T$  and hence show that  $k = 1$ . [3]
- (ii) Find the probability density function of  $T$ . [2]
- (iii) Find the mean time for the task. [3]
- (b) For a different task, the times (in milliseconds) taken by my computer on 10 randomly chosen occasions were as follows.

6.4    5.9    5.0    6.2    6.8    6.0    5.2    6.5    5.7    5.3

From past experience it is thought that the median time for this task is 5.4 milliseconds. Carry out a test at the 5% level of significance to investigate this, stating your hypotheses carefully.

[10]

- 2 In the vegetable section of a local supermarket, leeks are on sale either loose (and unprepared) or prepared in packs of 4.

The weights of unprepared leeks are modelled by the random variable  $X$  which has the Normal distribution with mean 260 grams and standard deviation 24 grams. The prepared leeks have had 40% of their weight removed, so that their weights,  $Y$ , are modelled by  $Y = 0.6X$ .

- (i) Find the probability that a randomly chosen unprepared leek weighs less than 300 grams. [3]
- (ii) Find the probability that a randomly chosen prepared leek weighs more than 175 grams. [3]
- (iii) Find the probability that the total weight of 4 randomly chosen prepared leeks in a pack is less than 600 grams. [3]
- (iv) What total weight of prepared leeks in a randomly chosen pack of 4 is exceeded with probability 0.975? [3]
- (v) Sandie is making soup. She uses 3 unprepared leeks and 2 onions. The weights of onions are modelled by the Normal distribution with mean 150 grams and standard deviation 18 grams. Find the probability that the total weight of her ingredients is more than 1000 grams. [3]
- (vi) A large consignment of unprepared leeks is delivered to the supermarket. A random sample of 100 of them is taken. Their weights have sample mean 252.4 grams and sample standard deviation 24.6 grams. Find a 99% confidence interval for the true mean weight of the leeks in this consignment. [3]

## 3

- 3 Engineers in charge of a chemical plant need to monitor the temperature inside a reaction chamber. Past experience has shown that when functioning correctly the temperature inside the chamber can be modelled by a Normal distribution with mean  $380^{\circ}\text{C}$ . The engineers are concerned that the mean operating temperature may have fallen. They decide to test the mean using the following random sample of 12 recent temperature readings.

374.0	378.1	363.0	357.0	377.9	388.4
379.6	372.4	362.4	377.3	385.2	370.6

- (i) Give three reasons why a  $t$  test would be appropriate. [3]
- (ii) Carry out the test using a 5% significance level. State your hypotheses and conclusion carefully. [9]
- (iii) Find a 95% confidence interval for the true mean temperature in the reaction chamber. [4]
- (iv) Describe briefly one advantage and one disadvantage of having a 99% confidence interval instead of a 95% confidence interval. [2]
- 4 (a) In Germany, towards the end of the nineteenth century, a study was undertaken into the distribution of the sexes in families of various sizes. The table shows some data about the numbers of girls in 500 families, each with 5 children. It is thought that the binomial distribution  $B(5, p)$  should model these data.

Number of girls	Number of families
0	32
1	110
2	154
3	125
4	63
5	16

- (i) Use this information to calculate an estimate for the mean number of girls per family of 5 children. Hence show that 0.45 can be taken as an estimate of  $p$ . [3]
- (ii) Investigate at a 5% significance level whether the binomial model with  $p$  estimated as 0.45 fits the data. Comment on your findings and also on the extent to which the conditions for a binomial model are likely to be met. [12]
- (b) A researcher wishes to select 50 families from the 500 in part (a) for further study. Suggest what sort of sample she might choose and describe how she should go about choosing it. [3]