OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4768/01**

MATHEMATICS (MEI)

Statistics 3

FRIDAY 12 JANUARY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

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1 The continuous random variable X has probability density function

$$f(x) = k(1 - x) \quad \text{for } 0 \leqslant x \leqslant 1$$

where k is a constant.

(i) Show that $k = 2$. Sketch the graph of the probability density function. [4]

(ii) Find $E(X)$ and show that $\text{Var}(X) = \frac{1}{18}$. [5]

(iii) Derive the cumulative distribution function of X. Hence find the probability that X is greater than the mean. [4]

(iv) Verify that the median of X is $1 - \frac{1}{\sqrt{2}}$. [2]

(v) $\overline{X}$ is the mean of a random sample of 100 observations of X. Write down the approximate distribution of $\overline{X}$. [3]

2 The manager of a large country estate is preparing to plant an area of woodland. He orders a large number of saplings (young trees) from a nursery. He selects a random sample of 12 of the saplings and measures their heights, which are as follows (in metres).

0.63 0.62 0.58 0.56 0.59 0.62 0.64 0.58 0.55 0.61 0.56 0.52

(i) The manager requires that the mean height of saplings at planting is at least 0.6 metres. Carry out the usual t test to examine this, using a 5% significance level. State your hypotheses and conclusion carefully. What assumption is needed for the test to be valid? [11]

(ii) Find a 95% confidence interval for the true mean height of saplings. Explain carefully what is meant by a 95% confidence interval. [5]

(iii) Suppose the assumption needed in part **(i)** cannot be justified. Identify an alternative test that the manager could carry out in order to check that the saplings meet his requirements, and state the null hypothesis for this test. [2]

3 Bill and Ben run their own gardening company. At regular intervals throughout the summer they come to work on my garden, mowing the lawns, hoeing the flower beds and pruning the bushes. From past experience it is known that the times, in minutes, spent on these tasks can be modelled by independent Normally distributed random variables as follows.

	Mean	Standard deviation
Mowing	44	4.8
Hoeing	32	2.6
Pruning	21	3.7

(i) Find the probability that, on a randomly chosen visit, it takes less than 50 minutes to mow the lawns. [3]

(ii) Find the probability that, on a randomly chosen visit, the total time for hoeing and pruning is less than 50 minutes. [3]

(iii) If Bill mows the lawns while Ben does the hoeing and pruning, find the probability that, on a randomly chosen visit, Ben finishes first. [4]

Bill and Ben do my gardening twice a month and send me an invoice at the end of the month.

(iv) Write down the mean and variance of the **total** time (in minutes) they spend on mowing, hoeing and pruning per month. [2]

(v) The company charges for the **total** time spent at 15 pence per minute. There is also a fixed charge of £10 per month. Find the probability that the total charge for a month does not exceed £40. [6]

4 **(a)** An amateur weather forecaster has been keeping records of air pressure, measured in atmospheres. She takes the measurement at the same time every day using a barometer situated in her garden. A random sample of 100 of her observations is summarised in the table below. The corresponding expected frequencies for a Normal distribution, with its two parameters estimated by sample statistics, are also shown in the table.

Pressure (a atmospheres)	Observed frequency	Frequency as given by Normal model
$a \leqslant 0.98$	4	1.45
$0.98 < a \leqslant 0.99$	6	5.23
$0.99 < a \leqslant 1.00$	9	13.98
$1.00 < a \leqslant 1.01$	15	23.91
$1.01 < a \leqslant 1.02$	37	26.15
$1.02 < a \leqslant 1.03$	21	18.29
$1.03 < a$	8	10.99

Carry out a test at the 5% level of significance of the goodness of fit of the Normal model. State your conclusion carefully and comment on your findings. [9]

(b) The forecaster buys a new digital barometer that can be linked to her computer for easier recording of observations. She decides that she wishes to compare the readings of the new barometer with those of the old one. For a random sample of 10 days, the readings (in atmospheres) of the two barometers are shown below.

Day	A	B	C	D	E	F	G	H	I	J
Old	0.992	1.005	1.001	1.011	1.026	0.980	1.020	1.025	1.042	1.009
New	0.985	1.003	1.002	1.014	1.022	0.988	1.030	1.016	1.047	1.025

Use an appropriate Wilcoxon test to examine at the 10% level of significance whether there is any reason to suppose that, on the whole, readings on the old and new barometers do not agree. [9]