Q1	$\mathrm{F}(t)=1-\mathrm{e}^{-t/3}$ (t>0)			
(i)	For median *m*, $\frac{1}{2}=1-\mathrm{e}^{-m/3}$ $\therefore\ \mathrm{e}^{-m/3}=\frac{1}{2}\Rightarrow-\frac{m}{3}=\ln\frac{1}{2}=-0.6931$ $\Rightarrow m=2.079$ For 90th percentile *p*, $0.9=1-\mathrm{e}^{-p/3}$ $\therefore \mathrm{e}^{-p/3}=0.1\Rightarrow-\frac{p}{3}=\ln 0.1=-2.3026$ $\Rightarrow\ p=6.908$	M1 M1 A1 M1 A1	attempt to solve, here or for 90th percentile. Depends on previous M mark.	5
(ii)	$\mathrm{f}(t)=\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{F}(t)$ $=\frac{1}{3}\mathrm{e}^{-t/3}$ $\mu=\int_0^\infty\frac{1}{3}t\mathrm{e}^{-t/3}\mathrm{d}t$ $=\frac{1}{3}\left\{\left[\frac{t\mathrm{e}^{-t/3}}{-1/3}\right]_0^\infty+3\int_0^\infty \mathrm{e}^{-t/3}\mathrm{d}t\right\}$ $=\left[0-0\right]+\left[\frac{\mathrm{e}^{-t/3}}{-1/3}\right]_0^\infty=3$	M1 A1 M1 M1 A1	(for *t*>0, but condone absence of this) Quoting standard result gets 0/3 for the mean. attempt to integrate by parts	5
(iii)	$\mathrm{P}(T>\mu)=$ [from cdf] $\mathrm{e}^{-\mu/3}=\mathrm{e}^{-1}$ =0.3679	M1 A1	[or via pdf] ft c's mean (>0)	2
(iv)	$\bar{T}\sim$ (approx) $\mathrm{N}\left(3,\frac{9}{30}=0.3\right)$	B1 B1 B1	N ft c's mean (>0) 0.3	3
(v)	**<u>EITHER</u>** can argue that 4.2 is more than 2 SDs from μ ($3+2\sqrt{0.3}$ = 4.095; <u>must</u> refer to SD $(\bar{\mathrm{T}})$, not SD(T)) i.e. outlier $\Rightarrow$ doubt **<u>OR</u>** formal significance test: $\frac{4.2-3}{3/\sqrt{30}}=2.191$, refer to N(0,1), sig at (eg) 5% $\Rightarrow$ doubt	M1 M1 A1 M1 M1 A1	Depends on first M, but could imply it.	3 18

Q2	$X \sim N(180,\ \sigma = 12)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	P (X <170) = P$(Z < \frac{170-180}{12} = -0.8333)$ = 1–0.7976 = 0.2024	M A1 A1	For standardising. Award once, here or elsewhere.	3
(ii)	$X_1 + X_2 + X_3 + X_4 + X_5 \sim N(900, \sigma^2 = 720[\sigma = 26.8328]$ $P\,(\text{this} < 840) = P(Z < \frac{840-900}{26.8328} = -2.236)$ $= 1 - 0.9873 = 0.0127$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iii)	$Y \sim N(50, \sigma = 6)$ $X + Y \sim N(230, \sigma^2 = 180[\sigma = 13.4164])$ $P\,(\text{this} > 240) = P(Z > \frac{240-230}{13.4164} = 0.7454)$ $= 1 - 0.7720 = 0.2280$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iv)	$\frac{1}{4}X \sim N\left(45, \sigma^2 = \frac{1}{16} \times 144 = 9[\sigma = 3]\right)$ Require t such that $0.9 = P(\text{this} < t) = P\left(Z < \frac{t-45}{3}\right) = P(Z < 1.282)$ $\therefore t - 45 = 3 \times 1.282 \Rightarrow t = 48.85\,(48.846)$	B1 M B1 A1	Variance. Accept sd. FT incorrect mean. Formulation of requirement. 1.282 ft only for incorrect mean	4
(v)	I = 45 + T where T ~ N (120, σ = 10) $\therefore I \sim N(165, \sigma = 10)$ $P(I < 150) = P(Z < \frac{150-165}{10} = -1.5)$ $= 1 - 0.9332 = 0.0668$	B1 A1	for unchanged σ (candidates might work with P (T<105)) c.a.o.	2
(vi)	$J = 30 + \frac{3}{5}T$ where T ~ N (120, σ = 10)		Cands might work with $P(\frac{3}{5}T < 75)$. $\frac{3}{5}T \sim N(72,36)$	

	$\therefore J \sim \mathrm{N}\left(102, \sigma^2 = \frac{9}{25} \times 100 = 36 \left[\sigma = 6\right]\right)$ $\mathrm{P}(J < 105) = \mathrm{P}(Z < \frac{105-102}{6} = 0.5) = 0.6915$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
				18

Q3				
(a)	H$_0$: $\mu_D = 0$ (or $\mu_A = \mu_B$)	B1	Hypotheses in words only must include “population”.	
	H$_1$: $\mu_D > 0$ (or $\mu_B > \mu_A$)	B1	Or “<” for $A - B$.	
	where μ_D is “mean for B – mean for A”	B1	For adequate verbal definition. Allow absence of “population” if correct notation μ is used, but do NOT allow “$\overline{X}_A = \overline{X}_B$” or similar unless $\overline{X}$ is clearly and explicitly stated to be a population mean.	
	Normality of differences is required	B1		
	MUST be PAIRED COMPARISON t test.			
	Differences are: 2.1 1.0 0.8 0.6 0.4 -1.0 -0.3 0.8 0.9 1.1			
	$\bar{d} = 0.64$ $s_{n-1} = 0.8316$	B1	s_n = 0.7889 but do NOT allow this here or in construction of test statistic, but FT from there.	
	Test statistic is $\frac{0.64 - 0}{\frac{0.8316}{\sqrt{10}}}$	M	Allow c’s $\bar{d}$ and/or s_{n-1}. Allow alternative: 0 + (c’s 1.833) × $\frac{0.8316}{\sqrt{10}}$ (= 0.4821) for subsequent comparison with $\bar{d}$. (Or $\bar{d}$ – (c’s 1.833) × $\frac{0.8316}{\sqrt{10}}$ (= 0.1579) for comparison with 0.)	
	=2.43(37).	A1	c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{d}$ scores M1A0, but ft.	
	Refer to t_9.	M	No ft from here if wrong.	
	Single-tailed 5% point is 1.833.	A1	No ft from here if wrong.	
	Significant.	E1	ft only c’s test statistic.	
	Seems mean amount delivered by B is greater that that by A	E1	ft only c’s test statistic. Special case: (t_{10} and 1.812) can score 1 of these last 2 marks if either form of conclusion is given.	11
(b)	We now require Normality for the amounts delivered by machine A.	B1		

	For machine A, $\bar{x} = 250.19$ $s_{n-1} = 3.8527$	B1	s_n = 3.6549(83) but do NOT allow this here or in construction of CI.	
	CI is given by $250.19 \pm 2.262 \frac{3.8527}{\sqrt{10}}$	M B1 M	ft c's $\bar{x}$ ±. 2.262 ft c's s_{n-1}.	
	= 250.19 ± 2.75(6) = (247.43(4), 252.94(6))	A1	c.a.o. Must be expressed as an interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_9 is OK.	
	250 is in the CI, so would accept $H_0 : \mu = 250$, so no evidence that machine is not working correctly in this respect.	E1		7
				18

Q4				
(i)				

	$\underbrace{1 \quad 30}_{31}$	62	70	$\underbrace{34 \quad 3}_{37}$
e_i	$\underbrace{1.49 \quad 37.85}_{39.34}$	55.62	58.32	$\underbrace{44.62 \quad 2.10}_{46.72}$

		M	for grouping	
	$X^2 = 1.7681 + 0.7318 + 2.3392 + 2.0222$	M	Allow the M1 for correct method from wrongly grouped or ungrouped table.	
	$= 6.86$	A1		
	Refer to χ_1^2.	M	Allow correct df (= cells – 3) from wrongly grouped or ungrouped table, and FT. Otherwise, no FT if wrong.	
	Upper 5% point is 3.84	A1	No ft from here if wrong.	
	Significant	E1	ft only c's test statistic.	
	Suggests Normal model does not fit	E1	ft only c's test statistic.	7
(ii) (A)	t test unwise …	E1		
	… because underlying population appears non-Normal	E1	FT from result of candidate's work in (i)	2

<table>
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<td>(B)</td>
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<tr><th>Data</th><th>Median 301</th><th>Difference</th><th>Rank of |diff|</th></tr>
<tr><td>301.3</td><td></td><td>0.3</td><td>3</td></tr>
<tr><td>301.4</td><td></td><td>0.4</td><td>4</td></tr>
<tr><td>299.6</td><td></td><td>- 1.4</td><td>8</td></tr>
<tr><td>302.2</td><td></td><td>1.2</td><td>7</td></tr>
<tr><td>300.3</td><td></td><td>- 0.7</td><td>5</td></tr>
<tr><td>303.2</td><td></td><td>2.2</td><td>10</td></tr>
<tr><td>302.6</td><td></td><td>1.6</td><td>9</td></tr>
<tr><td>301.8</td><td></td><td>0.8</td><td>6</td></tr>
<tr><td>300.9</td><td></td><td>- 0.1</td><td>1</td></tr>
<tr><td>300.8</td><td></td><td>- 0.2</td><td>2</td></tr>
</table>

T = 1 +2 + 5 + 8 = 16 (or 3+4+6+7+9+10 = 39)
Refer to tables of Wilcoxon single sample (/paired) statistic
Lower (or upper if 39 used) 5% tail is needed
Value for n = 10 is 10 (or 45 if 39 used)
Result is not significant
No evidence against median being 301
</td>
<td>M
M
A1
B1
M
M
A1
E1
E1</td>
<td>for differences. ZERO in this section if differences not used.
for ranks. FT if ranks wrong.</td>
<td>9</td>
</tr>
<tr><td></td><td></td><td></td><td></td><td>18</td></tr>
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