



Mark Scheme (Results)

Summer 2013

GCE Statistics 3 (6691/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.
 8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks
1.	Label females 1 – 100 (or 0 – 99) and males 1 – 300 (or 0 – 299) Using <u>random numbers</u> for <u>each group</u> in range 1 – 100 (0 – 99) <u>select 15</u> females and using 1 – 300 (or 0 – 299) select <u>45</u> <u>males</u>	B1 B1 B1 [Total 3]
	Notes	
	1 st B1 for labelling\numbering\listing females <u>and</u> males 2 nd B1 for use of random numbers or “randomly select” in <u>each group</u> (may be implied) 3 rd B1 for selecting the correct number of females <u>and</u> males e.g. randomly select 45 males and 15 females scores 2 nd and 3 rd B marks since randomly selecting and the “each group” is implied, If using systematic sampling within each strata allow 1 st B1 and 3 rd B1 (if earned) but 2 nd B0	

Question Number	Scheme	Marks
2.	$X \sim N(40, 3^2) \quad \bar{X} \sim N\left(40, \frac{9}{n}\right)$ <p style="text-align: right;">(Condone $Y \sim N\left(40, \frac{9}{n}\right)$)</p> $P(\bar{X} > 42) = P\left(Z > \frac{42 - 40}{\sqrt{\frac{9}{n}}}\right)$ $\frac{42 - 40}{\sqrt{\frac{9}{n}}} \geq 1.6449$ $n \geq 6.087$ $n = 7$	B1 M1 B1 dM1 A1 [Total 5]
	<p>1st B1 for stating the correct distribution for \bar{X}. May be implied if correctly used in line 2 and no incorrect version seen elsewhere.</p> <p>1st M1 for an attempt to standardise with 42, 40 and their $\sqrt{\frac{9}{n}}$, must have n. Allow \pm</p> <p>2nd B1 for using $z = \pm 1.6449$ (or better)</p> <p>2nd dM1 Dep on 1st M1 for forming an equation in n or \sqrt{n}. Allow “=” or “<” i.e. setting their standardised expression = their z value ($z > 1.5$)</p> <p>A1 for $n = 7$ only The A1 must follow from correct working so e.g. $n < 6.087$ leading to $n = 7$ is A0</p>	

Question Number	Scheme											Marks
3 (a)	Town	A	B	C	D	E	F	G	H	I	J	M1
	Pop	1	2	3	4	5	6	7	8	9	10	
	Empl	2	1	3	5	4	6	10	8	9	7	
	d	1	1	0	1	1	0	3	0	0	3	
	d^2	1	1	0	1	1	0	9	0	0	9	
	$\sum d^2 = 22$											M1A1
	$r_s = 1 - \frac{6 \times 22}{10 \times 99}$											dM1
	$= \frac{143}{165} = 0.866$											A1 (5)
	awrt 0.867											
(b)	$H_0: \rho = 0 ; H_1: \rho > 0$ CV = 0.6485											B1 B1
	in critical region / significant/ reject H_0											M1
	evidence of <u>positive</u> correlation between population and no. of employees											A1 (4)
(c)	CV = <u>0.6319</u> [not in critical region / not significant/ do not reject H_0] No evidence of <u>positive</u> correlation											B1 B1 (2)
(d)	No evidence to suggest that as pop' increased the no. of employees increased <u>linearly</u> . Villages <u>ranked</u> highly for pop' were also <u>ranked</u> highly for the no. of employees.											B1 B1 (2)
ALT	Alternate for part (d) if different conclusions in part (b) and part (c) Data probably not (bivariate) normal therefore Spearman's coefficient is more suitable than the product moment correlation coefficient.											[Total 13]
Notes												
(a)	1 st M1 for an attempt to rank no of employees against the populations 2 nd M1 for attempting $\sum d^2$ (must be using ranks) ft their ranks 1 st A1 for 22 3 rd dM1 dep on 1 st M1 for use of the correct formula with their $\sum d^2$. If ans. is not correct an expr' is required.											
(b)	1 st B1 for both hypotheses in terms of ρ , H_1 must be one tail and compatible with their ranking M1 for a correct statement relating their r_s ($ r_s < 1$) with their cv but cv must be such that $ cv < 1$ A1 for a correct contextualised comment that is rejecting H_0 Must mention "population" and "no. of employees" and "positive correlation". Follow through their r_s and their cv (provided it is $ cv < 1$ Use of "association" is A0											
(c)	1 st B1 for 0.6319 2 nd B1 does not require context just no <u>positive</u> correlation mentioned											
(d)	1 st B1 for a comment relating to pmcc (i) no <u>linear</u> relationship or (ii) pmcc requires (joint) normal distribution 2 nd B1 for a second comment relating to Spearman's (i) there is a (non-linear) relationship between <u>rank</u> s.. <u>or</u> (ii) data not (joint) normal so Spearman's is better											

Question Number	Scheme	Marks																									
4 (a)	$\frac{282 \times 100}{600}$ (Do not accept $282 - 114.2 - 90.2 - 30.6$ (o.e.))	B1 (1)																									
(b)	9	B1 (1)																									
(c)	2.5 or better (Do not accept 0.025)	B1 (1)																									
(d)	H_0 : hair colour occurs in the ratio 2:6:1:3 H_1 : hair colour does not occur in the ratio 2:6:1:3	B1																									
	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 20%;"></th> <th style="width: 20%;">black</th> <th style="width: 20%;">brown</th> <th style="width: 20%;">red</th> <th style="width: 20%;">blonde</th> </tr> </thead> <tbody> <tr> <td>observed</td> <td>105</td> <td>282</td> <td>48</td> <td>165</td> </tr> <tr> <td>expected</td> <td>100</td> <td>300</td> <td>50</td> <td>150</td> </tr> <tr> <td>$\frac{(O_i - E_i)^2}{E_i}$</td> <td>0.25</td> <td>1.08</td> <td>0.08</td> <td>1.5</td> </tr> <tr> <td>$\frac{O_i^2}{E_i}$</td> <td>110.25</td> <td>265.08</td> <td>46.08</td> <td>181.5</td> </tr> </tbody> </table>		black	brown	red	blonde	observed	105	282	48	165	expected	100	300	50	150	$\frac{(O_i - E_i)^2}{E_i}$	0.25	1.08	0.08	1.5	$\frac{O_i^2}{E_i}$	110.25	265.08	46.08	181.5	B1 expected M1 A1
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	$\sum \frac{(O_i - E_i)^2}{E_i} = 2.91 \quad \text{or} \quad \sum \frac{O_i^2}{E_i} - 600 = 602.91 - 600 = 2.91 \quad (\text{awrt } \mathbf{2.91})$ $\nu = 3$ cv is 7.815 $[2.91 < 7.815]$ so insufficient evidence to reject H_0 <u>or</u> not significant There is evidence to suggest that hair colour does occur in the given ratio.	A1 B1 B1 dM1 A1 (9) [Total 12]																									
	Notes																										
(d)	1^{st} B1 for both hypotheses. Must mention hair colour and ratio e.g. "hair colour in the given ratio" Allow use of ditto 2^{nd} B1 for all 4 correct expected frequencies 1^{st} M1 for at least 2 correct calculations from 3^{rd} or 4^{th} row 1^{st} A1 for all correct calculations to at least 3sf if row 4 If awrt 2.91 is seen with no incorrect working award B1M1A1A1 2^{nd} dM1 Dep on 1^{st} M1 for a correct statement linking their test statistic and their cv ($cv > 3.5$) 3^{rd} A1 for a correct comment in context - must mention "hair colour" and "ratios" <u>or</u> "model" e.g. "There is evidence of to support the given model" No follow through If hypotheses are the wrong way round score A0.																										

Question Number	Scheme	Marks
<p>5 (a)</p> <p>$\bar{x} = \frac{1}{2}(118.8 + 121.2) = 120$</p> <p>“their 1.6449” $\frac{\sigma}{\sqrt{n}} = 121.2 - 120$</p> <p>“their 2.3263” $\frac{\sigma}{\sqrt{n}} = 2.3263 \times \left(\frac{121.2 - 120}{1.6449} \right)$</p> <p>So 98% C.I. = $120 \pm 1.424\dots = (118.3028\dots, 121.699\dots)$</p> <p>(b) awrt $(118\pi, 122\pi)$ or $(371/372, 382/383)$</p> <p>(c) $P(\text{All}) = (0.98)^3$ = 0.941</p>	<p>1.6449 (or better)</p> <p>2.3263 (or better)</p> <p>awrt (118, 122)</p>	<p>B1</p> <p>B1 M1</p> <p>B1 dM1</p> <p>A1</p> <p>B1ft</p> <p>M1 A1</p> <p>(6)</p> <p>(1)</p> <p>(2)</p> <p>[Total 9]</p>
Notes		
<p>(a)</p> <p>(c)</p>	<p>NB in part (a) only lose one of the B1 marks for not using the percentage points table</p> <p>1st B1 for $\bar{x} = 120$</p> <p>2nd B1 for 1.6449 or better in an attempt (could be $1.6449\sigma = k$ or even $1.6449\sigma^2 = k$) Condone strange notation for standard error (E) here if it is <u>used</u> correctly</p> <p>1st M1 for an attempt to find “width” or “half-width” of a 90% CI ft their z value ($z > 1.5$) e.g. for $z E = 121.2 - 120$ (o.e.) N.B. $E = 0.7295\dots$ Condone missing 2 here.</p> <p>3rd B1 for 2.3263 or better in an attempt at CI. If score 2nd B0 for using 1.64 or 1.645 allow 3rd B1 for 2.32 or 2.33 here</p> <p>2nd dM1 for a correct attempt at “width” or “half-width” of a 98% CI ft their z value ($z > 2$) Dependent on 1st M1 and ft their value or expression for s.e.</p> <p>A1 for lower limit in range [118, 118.35) <u>and</u> upper limit in range (121.65, 122]</p> <p>Answer only of awrt (118, 122) with no incorrect working seen scores 6/6/ if 1.6449 and 2.3263 are seen and 5/6 (B1B1M1B0M1A1) otherwise.</p> <p>M1 for a correct expression i.e. $(0.98)^3$</p> <p>A1 for awrt 0.941</p>	

Question Number	Scheme	Marks
<p>6 (a)</p> <p>(b)</p>	$\text{Var}(X) = \frac{(a+5-a+1)^2}{12} [=3]$ $\bar{X} \sim N\left(a+2, \frac{3}{50}\right)$ $17.2 - 1.96 \times \sqrt{\frac{3}{50}} < \mu < 17.2 + 1.96 \times \sqrt{\frac{3}{50}}$ $17.2 - 1.96 \times \sqrt{\frac{3}{50}} < a+2 < 17.2 + 1.96 \times \sqrt{\frac{3}{50}}$ $14.7 < a < 15.7$	<p>M1</p> <p>A1, A1ft</p> <p>(3)</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>(4)</p> <p>[Total 7]</p>
Notes		
<p>(a)</p> <p>(b)</p>	<p>M1 for a correct expression for $\text{Var}(X)$ in terms of a <u>or</u> $\text{Var}(X) = 3$</p> <p>1st A1 for normal and correct mean must be $a + 2$</p> <p>NB $N(17.2, \dots)$ is A0 and $N(17.2, \frac{3}{50})$ is M1A0A1</p> <p>2nd A1ft for correct $\text{Var}(\bar{X})$, i.e. (their "3")/50</p> <p>1st B1 for correct use of $z = 1.96$ in an attempt e.g. $\bar{x} \pm z\sigma$ or $\bar{x} \pm z\sigma^2$</p> <p>M1 for $17.2 \pm z \times \sqrt{\frac{3}{50}}$ where $z > 1.5$ accept just + or just -</p> <p>Answer of (16.7, 17.7) scores B1M1B0A0</p> <p>2nd B1 for either of the inequalities with $a + 2$ and any z ($z > 1.5$) or $a = 15.2 \pm z \times \sqrt{\frac{3}{50}}$</p> <p>A1 for awrt 14.7 and 15.7</p>	

Question Number	Scheme	Marks
<p>7 (a)</p>	<p>$H_0: \mu_a = \mu_b, \quad H_1: \mu_a < \mu_b$</p> $\text{s.e.} = \sqrt{\frac{25^2}{100} + \frac{10^2}{150}}, \quad z = \frac{67 - 60}{\sqrt{\frac{25^2}{100} + \frac{10^2}{150}}} \quad \text{CR} = 1.6449 \times \sqrt{\frac{25^2}{100} + \frac{10^2}{150}}$ <p>$z = \pm 2.6616\dots \quad = \pm 4.326\dots \quad (\text{awrt } 2.66/4.33)$</p> <p>One tailed critical value $z = 1.6449$ (or prob of awrt 0.004 (<0.05)) [Condone 0.996 if compared correctly with 0.95 for the B1] 2.6616 > 1.6449 so] significant evidence to reject H_0 There is evidence that the amount of lead present in the soil has decreased.</p>	<p>B1</p> <p>M1,dM1</p> <p>A1</p> <p>B1</p> <p>dM1</p> <p>A1ft</p> <p>(7)</p>
	Notes	
<p>(a)</p>	<p>1st B1 for both hypotheses in terms of μ not words. Accept μ_1, μ_2 etc if there is some indication of which is which e.g $X \sim N(67, 25^2)$ implies X is “before”.</p> <p>1st M1 for attempt at s.e. - condone one number wrong or mis-matched variances i.e. $\sqrt{\frac{p}{q} + \frac{r}{s}}$ (3 of p, q, r & s correct) or $\sqrt{\frac{10^2}{100} + \frac{25^2}{150}}$</p> <p>2nd dM1 Dep on 1st M1 for using their s.e. in correct formula for test statistic. Num of $\pm (67 - 60)$ <u>or</u> for correct expression for CR</p> <p>3rd dM1 dep. on 2nd M1 for a correct statement based on their normal cv ($cv > 1.5$) and their test statistic</p> <p>2nd A1ft for correct comment in context. Must mention “lead” or “soil” and “factory”. Allow ft If hypotheses are the wrong way round score A0 If hypotheses are not for a difference between 2 means award A0</p>	<p>B1</p> <p>(1)</p>
<p>(b)</p>	<p>B1 must mention <u>mean</u> and <u>normal</u>. In words or symbols e.g. $\bar{X} \sim N(\dots$</p>	<p>B1</p> <p>(1)</p>
		[Total 9]

Question Number	Scheme	Marks
<p>8 (a)</p> <p>Let $W = D_1 - D_2$ $W \sim N(0, 2.88)$ $P(W > 3) = 2 \times P(W > 3)$ $= 2 \times P\left(Z > \frac{3-0}{\sqrt{2.88}}\right)$ $= 2 \times P(Z > 1.76776\dots)$ $= 2 \times (1 - 0.9616)$ $= 0.0768$</p> <p>(b)</p> <p>Let $T = 5C - 4D$ or $4D - 5C$ or $C - \frac{4}{5}D$ or $\frac{4}{5}D - C$ $T \sim N(\pm 4, 39.04)$ or $N(\pm 0.8, 1.5616)$ $P(T < 0) = P\left(Z < \frac{0-4}{\sqrt{39.04}}\right)$ or $P\left(Z < \frac{0-0.8}{\sqrt{1.5616}}\right)$ $= P(Z < -0.64018\dots)$ $= (1 - 0.7389)$ $= 0.2611$</p> <p>(c)</p> <p>Let $P = D + D + D + D + D + D + B$ Let $Q = C + C + C + C + C + C + B$ $P \sim N(352, 13.64)$ and $Q \sim N(292, 8.84)$ [Let $R = P - Q$ $R \sim N(\pm 60, 22.48)$</p> <p>$P(R > 50) = P\left(Z > \frac{50-60}{\sqrt{22.48}}\right)$ $= P(Z > -2.10\dots)$ $= 0.9821$</p>	<p>awrt 0.077</p> <p>awrt 0.261</p> <p>awrt 0.982 ~ 0.983</p>	<p>M1 A1, A1 M1 dM1 A1</p> <p>M1 A1 A1 M1 A1</p> <p>M1 A1, A1 M1 dM1 A1</p> <p>(6)</p> <p>(5)</p> <p>(6)</p> <p>[Total 17]</p>
Notes		
<p style="text-align: center;">Award full marks in each part for a correct answer with no incorrect working seen.</p> <p>(a) 1st M1 for explicitly defining a suitable W and attempt to find the distribution of W. May be implied by sight of $N(0, 2.88)$ 1st A1 for normal and mean of 0, 2nd A1 for variance of 2.88. Award M1A1A1 for $N(0, 2.88)$ seen. 2nd M1 for realising need $2 \times P(W > 3)$ 3rd dM1 Dep on 1st M1 for standardising with 3, 0 and their s.d. Must lead to $P(Z > +ve)$ (o.e.)</p> <p>(b) 1st M1 for explicitly defining a suitable T but may be implied by sight of one of these normals 1st A1 for normal and correct mean, 2nd A1 for correct variance. Accept awrt 3sf i.e. 39.0, 1.56 2nd M1 for standardising with 0 and their mean and their s.d. Must lead to $P(Z < -ve)$ (o.e.)</p> <p>(c) 1st M1 for explicitly defining a correct P or Q. May be implied by a correct distribution for P or Q 1st A1 for a correct distribution for P 2nd A1 for a correct distribution for Q 2nd M1 for attempting R and obtaining its distribution- ft their P and Q means and variances 3rd dM1 for attempting $P(R > 50)$ <u>and</u> standardising with 50 and their $E(R)$ and their $\sqrt{\text{Var}(R)}$ Dependent on 2nd M1. Must lead to a $P(Z > -ve)$ (o.e.)</p>		

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