

Mark Scheme (Results)

June 2011

GCE Statistics S3 (6691) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol \checkmark will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

June 2011
Statistics S3 6691
Mark Scheme

Question Number	Scheme	Marks
1.	X_1, X_2, \dots, X_n is a random sample of size n , for large n , drawn from a population of any distribution with mean μ and variance σ^2 then \bar{X} is (approximately) $N\left(\mu, \frac{\sigma^2}{n}\right)$	B1 B1 B1 (3) 3
	1 st B for large sample or equivalent 2 nd B for ‘population of any distribution’ or ‘any population’ 3 rd B require mean or symbol and normal (parameters not required)	

Question Number	Scheme	Marks																																								
2. (a)	<table border="1" data-bbox="296 376 1161 591"> <thead> <tr> <th>Town</th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> </tr> </thead> <tbody> <tr> <td><i>h</i> rank</td> <td>1</td> <td>5</td> <td>2</td> <td>3</td> <td>7</td> <td>4</td> <td>6</td> </tr> <tr> <td><i>c</i> rank</td> <td>4</td> <td>3</td> <td>2</td> <td>1</td> <td>6</td> <td>7</td> <td>5</td> </tr> <tr> <td>d</td> <td>3</td> <td>2</td> <td>0</td> <td>2</td> <td>1</td> <td>3</td> <td>1</td> </tr> <tr> <td>d^2</td> <td>9</td> <td>4</td> <td>0</td> <td>4</td> <td>1</td> <td>9</td> <td>1</td> </tr> </tbody> </table> $\sum d^2 = 28$ $r_s = 1 - \frac{6 \times 28}{7 \times 48}$ $= 0.5$	Town	A	B	C	D	E	F	G	<i>h</i> rank	1	5	2	3	7	4	6	<i>c</i> rank	4	3	2	1	6	7	5	$ d $	3	2	0	2	1	3	1	d^2	9	4	0	4	1	9	1	M1 M1 M1A1 M1 A1 (6)
Town	A	B	C	D	E	F	G																																			
<i>h</i> rank	1	5	2	3	7	4	6																																			
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$ d $	3	2	0	2	1	3	1																																			
d^2	9	4	0	4	1	9	1																																			
(b)	$H_0 : \rho = 0, H_1 : \rho \neq 0$ Critical values are $r_s = \pm 0.7857$ $0.5 < 0.7857$ insufficient evidence to reject H_0 Councillor's claim is supported.	B1 B1ft M1 A1ft (4) 10																																								

Question Number	Scheme	Marks
<p>Notes (a)</p> <p>(b)</p>	<p>1st M1 for an attempt to rank the hardship against calls</p> <p>2nd M1 for attempting d for their ranks. Must be using ranks.</p> <p>3rd M1 for attempting $\sum d^2$ (must be using ranks)</p> <p>1st A1 for sum of 28 (or 84)</p> <p>4th M1 for use of the correct formula with their $\sum d^2$. If answer is not correct an expression is required.</p> <p>2nd A1 for awrt 0.5 (or -0.5)</p> <p>1st B1 for both hypotheses in terms of ρ, H_1 must be two tail.</p> <p>2nd B1 for cv of ± 0.7857 (or 0.7143 to ft from 1-tailed H_1)</p> <p>M1 for a correct statement relating their r_s with their cv but cv must be such that $cv < 1$</p> <p>A1ft for a correct contextualised comment. Must mention “Councillor” and “claim” <u>or</u> “hardship” and “number of calls (to the emergency services)”</p> <p>Follow through their r_s and their cv (provided it is $cv < 1$)</p> <p>Condone use of “association” in conclusion for A1</p> <p>Condone ‘positive’ in conclusion.</p>	

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3.	<table border="1" data-bbox="292 333 1131 562"> <thead> <tr> <th>Defect Type Shift</th> <th>D₁</th> <th>D₂</th> <th></th> </tr> </thead> <tbody> <tr> <td>First Shift</td> <td>47.25</td> <td>15.75</td> <td>63</td> </tr> <tr> <td>Second Shift</td> <td>56.25</td> <td>18.75</td> <td>75</td> </tr> <tr> <td>Third Shift</td> <td>46.5</td> <td>15.5</td> <td>62</td> </tr> <tr> <td></td> <td>150</td> <td>50</td> <td>200</td> </tr> </tbody> </table> <p data-bbox="276 640 1043 674">H₀ : Type of defect is independent of Shift (no association)</p> <p data-bbox="276 689 1043 723">H₁ : Type of defect is not independent of Shift (association)</p> <table border="1" data-bbox="292 804 1121 1133"> <thead> <tr> <th><i>O</i></th> <th><i>E</i></th> <th>$\frac{(O-E)^2}{E}$</th> <th>$\frac{O_i^2}{E_i}$</th> </tr> </thead> <tbody> <tr> <td>45</td> <td>47.25</td> <td>0.1071...</td> <td>42.857...</td> </tr> <tr> <td>18</td> <td>15.75</td> <td>0.3214...</td> <td>20.571..</td> </tr> <tr> <td>55</td> <td>56.25</td> <td>0.02777...</td> <td>53.777...</td> </tr> <tr> <td>20</td> <td>18.75</td> <td>0.0833...</td> <td>21.333...</td> </tr> <tr> <td>50</td> <td>46.5</td> <td>0.2634...</td> <td>53.763...</td> </tr> <tr> <td>12</td> <td>15.5</td> <td>0.7903...</td> <td>9.290...</td> </tr> </tbody> </table> <p data-bbox="276 1178 973 1261">$\frac{(O-E)^2}{E} = 1.5934..$ or $\frac{O_i^2}{E_i} - 200 = 201.5934 - 200 = 1.5934..$</p> <p data-bbox="276 1272 528 1305">$\nu = (3-1)(2-1) = 2$</p> <p data-bbox="276 1317 496 1350">$\chi^2_2(0.10) = 4.605$</p> <p data-bbox="276 1361 895 1395">1.59 < 4.605 so insufficient evidence to reject H₀</p> <p data-bbox="276 1406 1002 1440">Insufficient evidence to support manager's belief /claim.</p>	Defect Type Shift	D ₁	D ₂		First Shift	47.25	15.75	63	Second Shift	56.25	18.75	75	Third Shift	46.5	15.5	62		150	50	200	<i>O</i>	<i>E</i>	$\frac{(O-E)^2}{E}$	$\frac{O_i^2}{E_i}$	45	47.25	0.1071...	42.857...	18	15.75	0.3214...	20.571..	55	56.25	0.02777...	53.777...	20	18.75	0.0833...	21.333...	50	46.5	0.2634...	53.763...	12	15.5	0.7903...	9.290...	<p data-bbox="1318 528 1406 562">M1A1</p> <p data-bbox="1318 674 1358 707">B1</p> <p data-bbox="1318 1099 1406 1133">M1A1</p> <p data-bbox="1318 1178 1358 1211">A1</p> <p data-bbox="1318 1272 1358 1305">B1</p> <p data-bbox="1318 1317 1382 1350">B1ft</p> <p data-bbox="1318 1361 1358 1395">M1</p> <p data-bbox="1318 1406 1358 1440">A1</p> <p data-bbox="1433 1451 1473 1485">10</p>
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Notes	<p>1st M1 for some use of $\frac{\text{Row Total} \times \text{Col.Total}}{\text{Grand Total}}$ May be implied by correct E_i</p> <p>1st A1 for all expected frequencies correct</p> <p>B1 for both hypotheses. Must mention “defect” and “shift” at least once</p> <p>Use of “relationship” or “correlation” or “connection” is B0</p> <p>2nd M1 for at least two correct terms (as in 3rd or 4th column) or correct expressions with their E_i</p> <p>2nd A1 for all correct terms. May be implied by a correct answer.(2 dp or better-allow eg 0.10...)</p> <p>3rd M1 for a correct statement linking their test statistic and their cv . Must be χ^2 not normal.</p> <p>4th A1 for a correct comment in context - must mention “manager’s belief” or “shift” and “defect type” - condone “relationship” or “connection” here but not “correlation”. No follow through e.g. “There is evidence of a relationship between shift and type of defect”</p>	

Question Number	Scheme	Marks
4. (a)	$\bar{x} = \frac{5320}{80} = 66.5$ $s^2 = \frac{392000 - 80 \times (66.5)^2}{79}$ $= 483.797\dots$ <p style="text-align: right;">awrt 484</p>	M1,A1 M1A1ft A1 (5)
(b)	$H_0: \mu_m = \mu_{nm}, \quad H_1: \mu_m > \mu_{nm}$ <p>(accept μ_1, μ_2 with definition)</p> $z = \frac{69.0 - 66.5}{\sqrt{\frac{483.797}{80} + \frac{446.44}{60}}}$ $= 0.6807$ <p>One tailed cv 1.6449 0.752)</p> <p>0.6807 < 1.6449 (or 0.248 > 0.05) insufficient evidence to reject H_0 Mean money spent is not greater with music playing.</p> <p style="text-align: right;">awrt 0.681 (Probability is awrt</p>	B1B1 M1dM1 A1 B1 dM1 A1ft (8) 13

Question Number	Scheme	Marks
(b)	<p style="text-align: center;">Notes</p> <p>No definition award B1B0. 1st M1 for attempt at s.e. - condone one number wrong or switched 60 & 80 . 2nd dM1 for using their s.e. in correct formula for test statistic. 3rd dM1 dep. on 2nd M1 for a correct statement based on their normal cv and their test statistic 2nd A1 for correct comment in context. Must mention “money spent” and “music playing”. Allow ft.</p> <p>Critical Region for (b) Standard error x z value for 2nd M1 Standard error x 1.6449= awrt 6.04 for 1st A1 2.5 < 6.04</p>	

Question Number	Scheme									Marks
5.										
(a)	Hurricanes: occur singly / are independent or occur at random / are a rare event / at a constant rate									B1B1 (2)
(b)	From data $\frac{1 \times 2 + 2 \times 5 + 3 \times 17 + \dots + 7 \times 12}{80} = 4.4875$									M1A1 (2)
(c)	No of hurricanes, h	0	1	2	3	4	5	6	7+	M1A1A1 (3)
	$80P(X = h)$	0.9	4.038	$r=9.06\dots$	13.55	$s=15.205$	13.647...	10.206...	13.388...	
	Combine to give expected frequencies >5	13.9991		13.55	15.205...	13.647...	10.206...	13.388...		
(d)	Observed	7		17	20	12	12	12		M1
	$\frac{(O - E)^2}{E}$	3.499...		0.876...	1.511...	0.198...	0.315...	0.143...		
	$\frac{O_i^2}{E_i}$	3.500...		21.322...	26.306...	10.551...	14.108...	10.755..		
	H_0 : Poisson distribution is a good fit o.e. H_1 : Poisson distribution is not a good fit o.e. $\sum \frac{(O_i - E_i)^2}{E_i} = 6.545\dots$ or $\frac{O_i^2}{E_i} = 86.545 - 80 = 6.545\dots$ (awrt 6.55 or 6.54) $\nu = 6 - 2 = 4$ cv is 9.488 (ft their ν i.e. $\chi^2_{\nu}(0.05)$ $6.545 < 9.488$ so insufficient evidence to reject H_0 (Hurricanes) can be modelled by a Poisson distribution									B1 A1 B1 B1ft A1
										(6) 13

Question Number	Scheme	Marks
<p>(b)</p> <p>(c)</p> <p>(d)</p>	<p style="text-align: center;">Notes</p> <p>M for at least 2 terms on numerator. 359/80 only award M0A0</p> <p>M for 80xPoisson probability with 4.4875 and either 2 or 4.</p> <p>1st A1 for awrt 9.06 and 2nd A1 for awrt 15.20 or 15.21</p> <p>1st M1 for some pooling and attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$, at least 3 correct expressions or values.</p> <p>1st B1 no value for parameter permitted</p> <p>2nd A1 for a correct comment suggesting that Poisson model is suitable.</p> <p>No ft</p>	

Question Number	Scheme	Marks
6. (a)	$L = A_1 + A_2 + \dots + A_6$ Mean is $E(L) = 6 \times 20 = 120$ Standard deviation is $\sqrt{\text{Var}(W)} = \sqrt{6 \times 5^2} = 5\sqrt{6} = 12.247\dots$ 12.2 awrt	B1 B1 (2)
(b)	$P(L > 110) = P\left(Z > \left(\frac{110 - 120}{12.247\dots}\right)\right)$ $= P(Z < 0.8164\dots)$ $= 0.7939 \text{ (or 0.7929 using interpolation or 0.79289 by calc)}$	M1 A1 (2)
(c)	Let $X = 4B - \sum_1^6 A_i$ $E(X) = 140 - 120 = 20$ $\text{Var}(X) = 16 \times 8^2 + 6 \times 5^2 = 1174$ $P(X < 0) = P\left(Z < \frac{-20}{\sqrt{1174}}\right) = P(Z < -0.583\dots)$ $= 0.2797 \text{ (or 0.2810 if no interpolation) or 0.27971 by calc.}$	B1 M1M1A1 M1 A1 (6) 10

Question Number	Scheme	Marks
	Notes	
(b)	M1 for identifying a correct probability (they must have the 110) and attempting to standardise with their mean and sd. This can be implied by the correct answer. A1 for awrt 0.794 or 0.793	
(c)	Accept ± 20 for B mark. Only award for probability statement if 2 terms in var 1 st M1 for 1024, 2 nd M1 for 150 3 rd M for standardising with their mean and 2 term sd and finding probability < 0.5 2 nd A1 for awrt 0.280 or 0.281	

Question Number	Scheme	Marks
7. (a)	$H_0: \mu = 250, H_1: \mu < 250,$ $z = \frac{248 - 250}{\frac{5.4}{\sqrt{90}}}$ $= -3.513\dots$ awrt - 3.51 Critical value -1.6449 -3.513.. < -1.6449 so sufficient evidence to reject H_0 Manager's claim is justified.	B1 M1 A1 B1 A1 (5)
(b)	98% CI for μ is $248 \pm 2.3263 \times \frac{5.4}{\sqrt{90}}$ = awrt (247,249) 2.33 dependent upon z value awrt	M1B1 A1A1 (4)
(c)	Hypothesis test is significant or CI does not contain stated weight. (Manager should ask the company to investigate if their) stated weight is too high o.e.	B1 B1 (2)
(d)	$P(\bar{x} - \mu < 1) = 0.98$ $\frac{1}{3} = 2.3263$ $\frac{1}{\sqrt{n}}$ $n = (3 \times 2.3263)^2 = 48.7\dots$ Sample size 49 required.	M1 A1 dM1A1 A1 (5) 16

Question Number	Scheme	Marks
	<p style="text-align: center;">Notes</p> <p>(a) 1st B1 for H_0 and for H_1 (must be <250) They must use μ not x, p, λ or \bar{x} etc 1st M1 for attempt at standardising using 248, 250 and sd. Can accept \pm. Critical region: $250 - 0.936 = 249.064$ for M1A1 (and compare with 248.) 3rd B1 for ± 1.6449 seen (or probability of 0.0002 or better) 2nd A1 for a correct contextualised comment. Must mention “Manager” and “claim” <u>or</u> “weight” and “stated weight”. No follow through.</p> <p>(b) 2.3263 or better for B mark. Any z value replacing 2.3263 award M.</p> <p>(d) 1st M for LHS = z value >1 1st A for RHS awrt 2.33 2nd A1 for answers in the range 48.7-48.9 3rd A1 don't condone \geq</p>	

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