

June 2006
6691 Statistics S3
Mark Scheme

Question Number	Scheme	Marks
1 (a)	<p><u>Advantages:</u></p> <ul style="list-style-type: none"> - does not require the existence of a ^{Sampling frame} population list - <u>field work can be done quickly</u> as representative sample can be achieved with a small sample size - costs kept to a minimum (<u>cheaply</u>) - administration relatively <u>easy</u> - non-response not an issue <p><u>Disadvantages:</u></p> <ul style="list-style-type: none"> - not possible to estimate sampling errors - interviewer choice and may not be able to judge easily / <u>may lead to bias</u> - non-response not recorded - non-random process 	<p style="text-align: right;">any one B1</p> <p style="text-align: right;">any one B1</p> <p style="text-align: right;">(2)</p>
(b)	<p><u>Advantages:</u></p> <ul style="list-style-type: none"> - <u>random process</u> so possible to <u>estimate sampling errors</u> - free from <u>bias</u> <p><u>Disadvantages:</u></p> <ul style="list-style-type: none"> - not suitable when sample size is large - <u>sampling frame required</u> which <u>may not exist</u> or may be difficult to construct for a large population. 	<p style="text-align: right;">any one B1</p> <p style="text-align: right;">any one B1 (2)</p> <p style="text-align: right;">TOTAL 4</p>

NO REPETITION / OPPOSITES

Question Number	Scheme	Marks
2 (a)	$\bar{X} \sim N(90, \frac{\Sigma^2}{100}) \text{ i.e. } N_9(90, 0.25)$ <p>Application of <u>central limit theorem</u> as (sample large)</p>	M1A1 B1 (3)
2 (b)	$P(\bar{X} \geq 91) = 1 - P(Z < \frac{91-90}{0.5}) \quad \text{Stand.}$ $= 1 - P(Z < 2)$ $= 1 - 0.9772$ $= 0.0228 \quad \text{aust } 0.0228$	M1A1 A1 (3) TOTAL 6
3 (a)	$H_0: \mu_A = \mu_B, H_1: \mu_A \neq \mu_B \quad \mu_1, \mu_2 \text{ OK both}$ $s_e = \sqrt{\frac{47^2}{70} + \frac{23^2}{90}} (= \sqrt{37.43492...})$ <p>Test statistic is $\pm \frac{198-201}{s_e} = \pm 0.4903 \quad \text{aust } 0.99$ <small>M1A1 probab aust 0.312 B1 probab cv 0.025</small></p> $cv = (\pm) 1.96$ <p>Insufficient evidence to reject H_0, no significant difference between the mean cholesterol content of the two samples. (require correct comparison for FT) <u>correct required.</u></p>	B1 M1A1 M1A1 B1 A1 ✓ (7)
3 (b)	<ul style="list-style-type: none"> - require 1 egg from each of 70 chickens of diet A to ensure <u>independence</u>, similarly for diet B. - no chickens in common between the two samples to ensure <u>independence</u> - not same chickens on diet A and diet B because if it were we need to do a <u>paired analysis</u>. <p style="text-align: right;">Any 1</p> <p>not same chickens on diet A and diet B because if it were we need to do a paired analysis</p>	B1, B1 (2) TOTAL 9

4.

Rank:

Shop	Distance	Price	d	d ²
A	1	9	8	64
B	2	7	5	25
C	3	10	7	49
D	4	6	2	4
E	5	4	1	1
F	6	8	2	4
G	7	2	5	25
H	8	1	7	49
I	9	5	4	16
J	10	3	7	49

ranking

M1

Reverse ranking on price, $\sum d^2 = 44$
Hairs

286 $\sum d^2$

M1, A1

(a)

$$r_s = 1 - \frac{6 \times 286}{10(100-1)} = -0.73 \text{ or } \frac{-11}{15} \text{ or } -0.733$$

or 0.733 for $\sum d^2 = 44$

M1 A1

(5)

(b)

$H_0: \rho = 0$

$H_1: \rho < 0$

($H_1: \rho > 0$ if reverse ranking)

$cv = -0.5636$

(0.5636)

B1

B1

B1

Reject H_0 , evidence there is a significant negative correlation between the price of an ice cream and the distance from a tourist attraction.

(Ice cream gets cheaper further from the tourist attraction)

B1

(4)

(-cv from correct table required) (positive in context)

TOTAL 9

5. $M = \text{wt of male worker}$ $M \sim N(78.5, 12.6^2)$
 $F = \text{wt of female worker}$ $F \sim N(62.0, 9.8^2)$

(a) $W = M_1 + \dots + M_7 + F_1 + \dots + F_8$
 $E(W) = 7 \times 78.5 + 8 \times 62.0 = 1045.50$ awrt 1050 M1A1
 $\text{Var}(W) = 7 \times 12.6^2 + 8 \times 9.8^2 = 1879.64$ 1880 M1A1 (4)

(b) Independent: (used in Variance formula) B1 (1)

(c) $P(W > 1090) = P\left(Z > \frac{1090 - 1045.5}{\sqrt{1879.64}}\right)$ M1
 $= P(Z > 1.03)$ awrt 1.03 A1
 $= 1 - 0.8485$ 1 - H1
 $= \underline{0.1515}$ A1 (4)
 Awrt(0.152)

(9)

<p>6.</p>	<p>H_0 : No association between age and colour (independent)</p> <p>H_1 : Association between age and colour (Not independent)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>O</th> <th>E</th> <th>$\frac{(O-E)^2}{E}$</th> </tr> </thead> <tbody> <tr> <td>12</td> <td>10.08</td> <td>0.3657...</td> </tr> <tr> <td>6</td> <td>7.92</td> <td>0.4654...</td> </tr> <tr> <td>10</td> <td>9.52</td> <td>0.0242...</td> </tr> <tr> <td>7</td> <td>7.48</td> <td>0.0308...</td> </tr> <tr> <td>6</td> <td>8.4</td> <td>0.6857...</td> </tr> <tr> <td>9</td> <td>6.6</td> <td>0.8727...</td> </tr> </tbody> </table> <p>$\sum \frac{(O-E)^2}{E} = 2.4446...$</p> <p>$\nu = (3-1)(2-1) = 2, \chi^2 = 5.991$</p> <p>Insufficient evidence to reject H_0. No association between age and colour (cv for correct h/c for ft)</p>	O	E	$\frac{(O-E)^2}{E}$	12	10.08	0.3657...	6	7.92	0.4654...	10	9.52	0.0242...	7	7.48	0.0308...	6	8.4	0.6857...	9	6.6	0.8727...	<p>BI</p> <p>BI</p> <p>MIAI</p> <p>MIAI</p> <p>MIAI</p> <p>BI BI√</p> <p>A I√ (ii)</p> <p>TOTAL 11</p>
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<p>7.(a)</p>	<p>$\bar{x} = \frac{500}{10} = 50$</p> <p>$s^2 = \frac{1}{9} (25001.74 - \frac{500^2}{10}) = 0.193$</p> <p>limits are 50 ± 1.966 $= (49.02, 50.98)$</p> <p>Confidence interval is $(50 - 2.5758 \times \frac{0.5}{\sqrt{10}}, 50 + 2.5758 \times \frac{0.5}{\sqrt{10}})$ $= (49.59273, 50.40727...)$</p> <p>use of estimate in (a) in (b) AND (c) assume MISREAD.</p>	<p>MIAI</p> <p>MIAIAI (5)</p> <p>MIBI</p> <p>AIAI (4)</p> <p>MIBIAN</p> <p>AIAI (5)</p> <p>TOTAL 14</p>																					

8 (a)	$B_7(5, 0.5)$	MIAI (2)																																											
(b)	<p>H_0: $B(5, 0.5)$ is a suitable model (good fit)</p> <p>H_1: $B(5, 0.5)$ is not a suitable model (not a good fit) ✓ for $\hat{p} = 0.466$.</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: left;">No. of heads</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> <td style="text-align: center;">5</td> <td></td> </tr> <tr> <td style="text-align: left;">Expected</td> <td style="text-align: center;">3.125</td> <td style="text-align: center;">15.625</td> <td style="text-align: center;">31.25</td> <td style="text-align: center;">31.25</td> <td style="text-align: center;">15.625</td> <td style="text-align: center;">3.125</td> <td rowspan="2" style="font-size: small; vertical-align: middle;">100% (100%) For Bin, 1 correct = AI All correct = AI 3st or better</td> </tr> <tr> <td style="text-align: left;">Actual</td> <td style="text-align: center;">6</td> <td style="text-align: center;">18</td> <td style="text-align: center;">29</td> <td style="text-align: center;">34</td> <td style="text-align: center;">10</td> <td style="text-align: center;">3</td> </tr> </table> <table style="width: 100%; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">0</td> <td style="text-align: center;">E</td> <td style="text-align: center;">$\frac{(O-E)^2}{E}$</td> </tr> <tr> <td style="text-align: left;">0 or 1</td> <td style="text-align: center;">24</td> <td style="text-align: center;">18.75</td> <td style="text-align: center;">1.47</td> </tr> <tr> <td style="text-align: left;">2</td> <td style="text-align: center;">29</td> <td style="text-align: center;">31.25</td> <td style="text-align: center;">0.162</td> </tr> <tr> <td style="text-align: left;">3</td> <td style="text-align: center;">34</td> <td style="text-align: center;">31.25</td> <td style="text-align: center;">0.242</td> </tr> <tr> <td style="text-align: left;">4 or 5</td> <td style="text-align: center;">13</td> <td style="text-align: center;">18.75</td> <td style="text-align: center;">1.763</td> </tr> </table> <p style="margin-left: 20px;">$\sum \frac{(O-E)^2}{E} = 3.6373$</p> <p style="margin-left: 20px;">$\chi^2_{(0.10)} = 6.251$</p> <p style="margin-left: 20px;">Insufficient evidence to reject H_0 $B(5, 0.5)$ is a suitable model. No evidence that coins are biased</p>	No. of heads	0	1	2	3	4	5		Expected	3.125	15.625	31.25	31.25	15.625	3.125	100% (100%) For Bin, 1 correct = AI All correct = AI 3st or better	Actual	6	18	29	34	10	3		0	E	$\frac{(O-E)^2}{E}$	0 or 1	24	18.75	1.47	2	29	31.25	0.162	3	34	31.25	0.242	4 or 5	13	18.75	1.763	<p>BIV ✓</p> <p>MIAIAI</p> <p>MIAI</p> <p>MIAI</p> <p>BIBI ✓</p> <p>AI ✓ (11)</p> <p>TOTAL 13</p>
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	<p>Ungrouped gives a wt 5.44, $\chi^2_5 = 9.236$</p> <p>grouped gives a wt 5.44, $\chi^2_5 = 9.236$</p>																																												