publication

June 2005
6685 Statistics S3
Mark Scheme

FINAL

Question Number	Scheme	Marks
1.	(a) Population divides into mutually exclusive; groups distinct strata	B1; B1 (2)
	(b) Advantages – enables fieldwork to be done quickly – costs kept to a minimum – administration is relatively easy Any ONE	B1
	Disadvantages – non-random so not possible to estimate sampling errors – subject to possible interviewer bias Any ONE – non-response not recorded	B1 (2)
2.	$X \sim N(10, 3^2)$ $\therefore$ $\bar{X} \sim N(10, 9/5)$ Can be implied 10; 9/5	B1; B1
	$P(7 \le \bar{X} \le 10) = P\left(\frac{7-10}{\sqrt{9/5}} < Z < 0\right)$ Standardising with 10 & their σ	M1 A1
	$= P(-2.236 < Z < 0)$	
	$= \Phi(0) - \{1 - \Phi(2.24)\}$	M1 ($p<0.5$)
	$= \underline{0.4875}$	A1 (6)

Question Number	Scheme	Marks
3.		

	No action	Remove diseased branches	Spray with Chemicals	Totals
Tree died within 1 year	10 (7)	5 (7)	6 (7)	21
Survived 1-4 years	5 (7)	9(7)	7(7)	21
Survived > 4 years	5 (6)	6 (6)	7(6)	18
Totals	20	20	20	60

$\frac{RT \times CT}{GT}$ M1

6×7 A1

3×6 A1

H₀: Treatment & Survival are independent (not associated) B1 both

H₁: Treatment & Survival are not independent (associated)

$\alpha = 0.05$

$\nu = (3-1)\times(3-1) = 4$ B1

CR: $\chi^2 > 9.488$ B1√

$$\sum \frac{(O-E)^2}{E} = \frac{9}{7} + \frac{4}{7} + \frac{1}{7} + \frac{4}{7} + \frac{4}{7} + 0 + \frac{1}{6} + 0 + \frac{1}{6}$$ Using $\sum \frac{(O-E)^2}{E}$ M1

Any 2 values A1

$= 3.47619....$ AWRT 3.48 A1

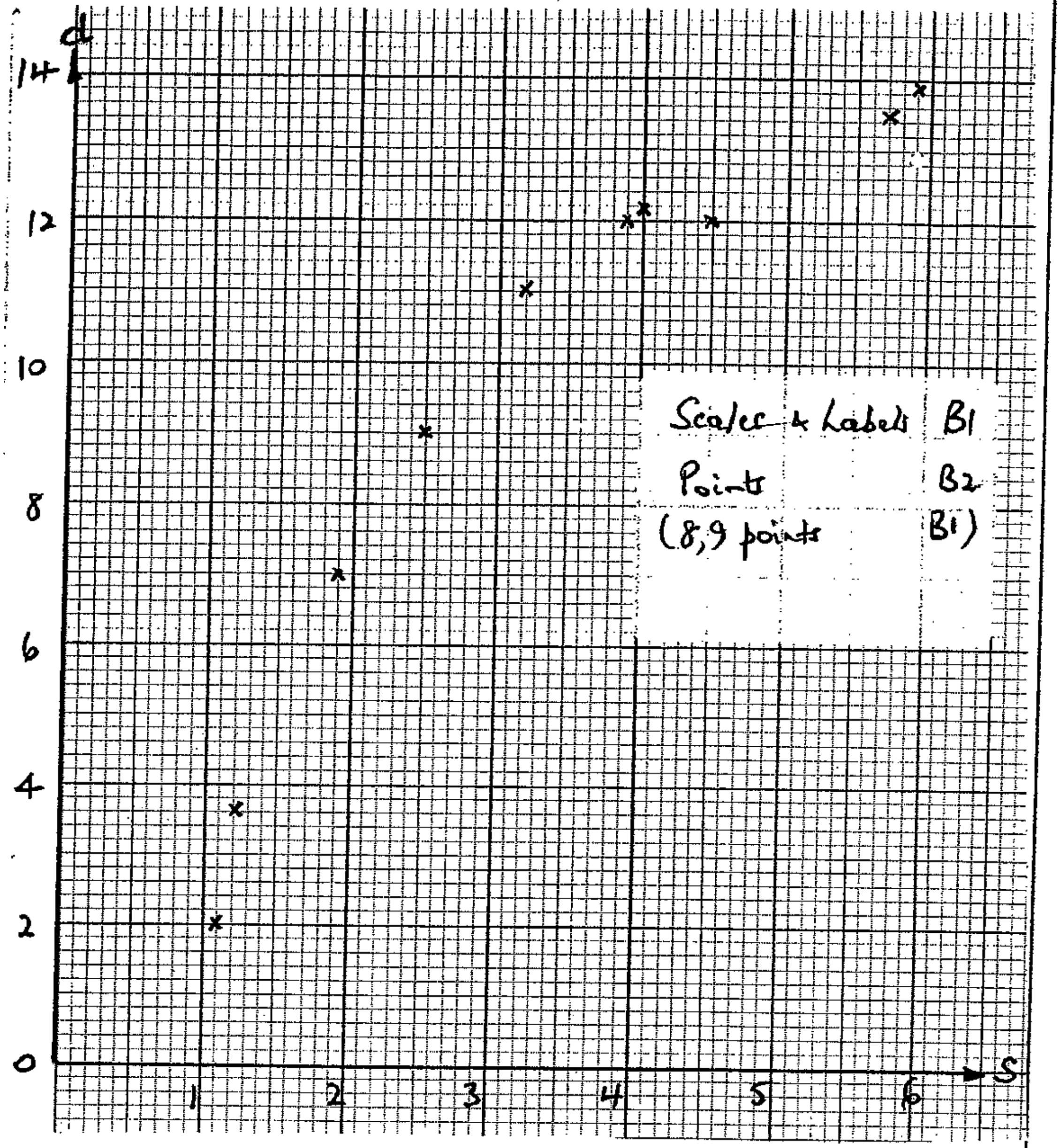
Since 3.47619... is NOT in the critical region (ie <9.488) there is insufficient evidence to reject H₀.

There is no evidence of association between treatment and length of survival.

Comparison M1

Conclusion A1√ (11)

4 (a) NB No graph paper ⇒ 0/3



Scales & Labels B1

Points B2

(8,9 points B1)

(3)

(b) Linear association between s and d B1 (1)

(c) $S_{ss} = 141.51 - \frac{33.9^2}{10} = 26.589$; $S_{dd} = 152.444$; $S_{sd} = 59.524$ B1; B1; B1 (3)

(d) $r = \frac{59.524}{\sqrt{152.444 \times 26.589}}$ M1

$= 0.93494....$ AWRT 0.935 A1 (2)

(e) $H_0: \rho = 0$; $H_1: \rho > 0$ B1

Critical Value at 1% = 0.7155 B1

Reject H_0; Levels of serum & disease are positively correlated B1 (3)

(f) Linear correlation significant but scatter diagram looks non-linear. B1 (1)

---

5. H_0: Poisson distribution is a suitable model both B1

H_1: Poisson distribution is not a suitable model

$$\hat{\lambda} = \frac{(0\times 99) + (1\times 65) + \dots + (4\times 2)}{200} = \frac{153}{200} = 0.765$$ M1 A1

Using $P(X=x) = \frac{0.765^x e^{-0.765}}{x!}$ where X represents the number of restarts gives

$200 \times P(X=x)$ M1

X	Observed Frequency	Expected Frequency		
0	99	93.06678....		
1	65	71.19604....	0,1,2	A1, A1 (-1 e.e.)
2	22	27.23250....		
3	12 } 14	6.94428... } 8.50468		A1
≥4	2	1.56040...		

$\nu = 4-1-1 = 2$; CR: $\chi^2_2 > 5.991$ from Poisson B1; B1√

$\nu = 4-1 = 3$ CR: $\chi^2 > 7.815$ from Poisson (0.765)

$$\sum \frac{(O-E)^2}{E} = 5.47368\dots$$ Use of $\sum(O-E)^2/E$ M1

5.40 – 5.50 A1

5.47 is not in the critical region. Number of computer failures per day can be modelled by a Poisson distribution A1√ (12)

6.	(a) Let X represent repair time		
	$\therefore \sum x = 1435 \quad \therefore \bar{x} = \frac{1435}{5} = \underset{\sim}{287}$		B1
	$\sum x^2 = 442575 \quad \therefore s^2 = \frac{1}{4}\left\{442575 - \frac{1435^2}{5}\right\}$		M1 A1
	$= \underset{\sim}{7682.5}$		A1 (4)
	(b) $P(\lvert\mu - \hat{\mu}\rvert) < 20 = 0.95$	Use of 19.20 or 40 with their σ & $\sqrt{n}$	M1
		1.96	B1
	$\therefore \frac{20}{\sigma/\sqrt{n}} = 1.96$		A1
	$\therefore n = \frac{1.96^2\sigma^2}{20^2} = \frac{1.96^2 \times 100^2}{400} = \underset{\sim}{96.04}$	Solving for n	M1 A1
	$\therefore$ Sample size $(\geq)$ 97 required		A1 (6)
7.	Let $W = C_1 - C_2$ NB $W = C_1 + C_2 \Rightarrow$ M1 A0 M1 only		
	(a) $\therefore W \sim N(0, 16)$	Normal	M1
		0; 16	A1; A1
	$\therefore P(\lvert W\rvert > 6) = 2P(W > 6)$		M1
	NB W = C−L treat as MR Prob = 0.4346 $\quad = 2 \times P\left(Z > \frac{6-0}{\sqrt{16}}\right)$	Standardising, their σ	M1
	$= 2 \times P(Z > 1.5)$		
	$= 2 \times (1 - 0.9332) = \underset{\sim}{0.1336}$		A1 (6)
	(b) Let $W = C - L$		
	$\therefore W \sim N(5, 25)$	5; 25	B1; B1
	$P(W > 0) = P\left(Z > \frac{\pm 5}{\sqrt{25}}\right)$		M1 A1
	$= P(Z < 1)$		M1 $(p > 0.5)$
	$= \underset{\sim}{0.8413}$		A1 (6)

	(c) Let $W = C_1 + \cdots + C_{24} + B$	
	$\therefore E(W) = 24 \times 350 + 100 = \underline{8500}$	B1
	$Var(W) = 24 \times 8 + 2^2 = \underline{196}$	B1
	$P(8510 \leq W \leq 8520) = P\left(\frac{8510-8500}{\sqrt{196}} \leq Z \leq \frac{8520-8500}{\sqrt{196}}\right)$	M1
	$= P(0.71\ldots \leq Z \leq 1.43\ldots\ldots)$ AWRT	A1√ A1√
	$= 0.9236 - 0.7611$	
	$= \underline{0.1625}$ 0.161 − 0.163	A1 (6)
	(d) <u>All</u> random variables are <u>independent</u>.	B1 (1)

18/06/05