S3 Specimen (MA)

Q1a) Label members 1–240 then use random numbers to select first member from range 1–8 then select every 8^{th} member afterwards

$$\left(\frac{240}{30} = 8 \right)$$

b) It is quicker.

Q2a) $n = 16 \rightarrow \bar{P} \sim N\left(110, \frac{8^2}{16}\right)$ (by C.L.T)

so $\boxed{\bar{P} \sim N(110, 4)}$

b) $P(110 < \bar{P} < 113) = P(\bar{P} < 113) - 0.5$

$= P\left(Z < \frac{113 - 110}{2}\right) - 0.5$

$= P(Z < 1.50) - (0.5)$

$= 0.9332 - 0.5 = \boxed{0.4332}$

Q3a) let T = time for all tasks

then $T = A + B + C$

$E(T) = 225 + 165 + 185 = 575$

$Var(T) = 38^2 + 23^2 + 27^2 = 2702$

$\therefore T \sim N(575, 2702)$

$P(\text{required}) = P(533 < T < 655)$

$= P(T < 655) - P(T < 533)$

$= P\left(Z < \frac{655-575}{\sqrt{2702}}\right) - P\left(Z < \frac{533-575}{\sqrt{2702}}\right)$

$= P(Z < 1.54) - P(Z < -0.81)$

$= 0.9382 - [1 - 0.7910]$

$= \boxed{0.7292}$

b) $P(B > C) = P(B - C > 0)$

let $F = B - C$,

$E(F) = 165 - 185 = -20$

$Var(F) = 23^2 + 27^2 = 1258$

$$\therefore F \sim N(-20, 1258)$$

$$P(\text{required}) = P(F > 0) = P\left(Z > \frac{20}{\sqrt{1258}}\right)$$

$$= P(Z > 0.56)$$

$$= 1 - P(Z < 0.56)$$

$$= 1 - 0.7123 = \boxed{0.2877}$$

Q4a)

Club	Position	R_{avg}	d	d^2
A	1	2	1	1
B	2	1	1	1
C	3	8	5	25
D	4	5	1	1
E	5	3	2	4
F	6	6	0	0
G	7	7	0	0
H	8	4	4	16
				48 //

$$\sum d^2 = 48$$

$$\therefore r_s = 1 - \frac{6(48)}{8(63)} = \boxed{0.4286}$$

b) $H_0 : \rho = 0$ critical value: ± 0.7381 //

$H_1 : \rho \neq 0$ (5%, 2-tail)

$0.4286 < 0.7381$

$\therefore$ Result is insignificant
Accept H_0.
Evidence suggests that there is no correlation between league position and attendance.

c) Award tied ranks an "average rank" then use the PMCC on the ranked data.

Q5a) $P(X = x) = \frac{1}{6}$ for $x = 1, 2, 3, 4, 5, 6$.

b) Discrete Uniform Distribution

c) H_0: Discrete uniform distribution is a suitable model
H_1: Discrete uniform distribution is not a suitable model.

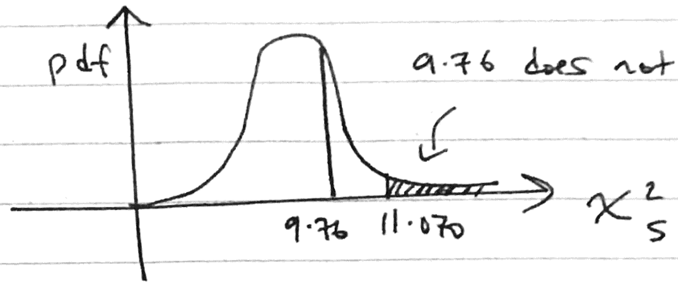
expected value $= \frac{300}{6} = 50$.

No.	1	2	3	4	5	6
E_i	50	50	50	50	50	50
O_i	41	49	52	58	37	63
$\frac{(O-E)^2}{E}$	1.62	0.02	0.08	1.28	3.38	3.38

$$X^2 = \sum \frac{(O-E)^2}{E} = 9.76$$

$\nu = 6 - 1 = 5$ $\therefore$ critical value $= \chi^2_5(5\%) = 11.070$

$9.76 < 11.070$



$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests that Discrete Uniform Distribution is a suitable model.

(Q6a) $H_0: \mu_{LOW} = \mu_{HIGH}$

critical value $= \pm 1.96$
(5%, 2-tail)

$H_1: \mu_{LOW} \neq \mu_{HIGH}$

Test Stat $= \dfrac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\dfrac{\sigma_X^2}{n_X} + \dfrac{\sigma_Y^2}{n_Y}}}$

$= \dfrac{7.42 - 6.40}{\sqrt{\dfrac{8.13^2}{400} + \dfrac{6.69^2}{300}}} = 1.82$

$1.82 < 1.96$
$\therefore$ Result is insignificant
Accept H_0.
Evidence indicates no difference in mean amounts spent on tobacco in both groups.

b) n is large so we can assume sample means for both groups are normally distributed

Q7) H_0: There is no association between gender and passing a driving test at first attempt.

H_1: There is an association between gender and passing a driving test at first attempt.

expected no. = $\dfrac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$.

expected:

	Pass	Fail	
Male	27.5	22.5	50
Female	27.5	22.5	50
	55	45	$\boxed{100}$

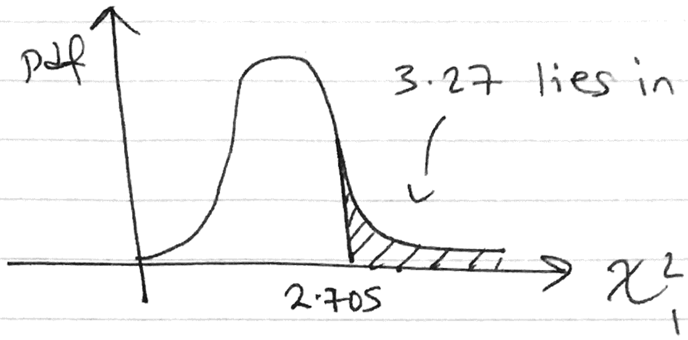
O_i	E_i	$\frac{(O-E)^2}{E}$
23	27.5	0.7364
27	22.5	0.9000
32	27.5	0.7364
18	22.5	0.9000
		3.27

$X^2 = \sum \frac{(O-E)^2}{E} = 3.27$

$\nu = (\text{rows}-1)(\text{columns}-1) = (2-1)(2-1) = 1$

critical value $= \chi^2_1 (10\%) = 2.705$

$3.27 > 2.705$



$\therefore$ Result is significant
Reject H_0.
Evidence suggests an association exists between gender and passing a driving test first time.

(Q8a) $\sum T = 85.2$
$\sum T^2 = 906.18$

$$\text{mean} = \frac{\sum T}{n} = \boxed{7.10} \quad \left(= \frac{85.2}{12}\right)$$

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$$

$$= \frac{1}{11}\left(906.18 - \frac{(85.2)^2}{12}\right) = \boxed{27.4}$$

b) 90% C.I : $\left[\bar{x} \pm 1.6449\left(\frac{\sigma}{\sqrt{n}}\right)\right]$

$$\left[7.10 \pm 1.6449\left(\frac{5.1}{\sqrt{12}}\right)\right]$$

$$[4.68 \ , \ 9.52]$$

c) 4 is not inside the interval so claim is likely inaccurate