S3 Mock (MA)

Q1a) $\bar{X} \sim N\left(100, \frac{14^2}{10}\right)$

b) $P(|\bar{X}-100| > 5) = 2P(\bar{X}-100 > 5)$

By symmetry as $E(\bar{X}-100) = 0$

$= 2P(\bar{X} > 105) = 2\left[P\left(Z > \frac{105-100}{\frac{14}{\sqrt{10}}}\right)\right]$

$= 2\left[P(Z > 1.13)\right] = 2\left[1 - P(Z < 1.13)\right]$

$= \boxed{0.2584}$

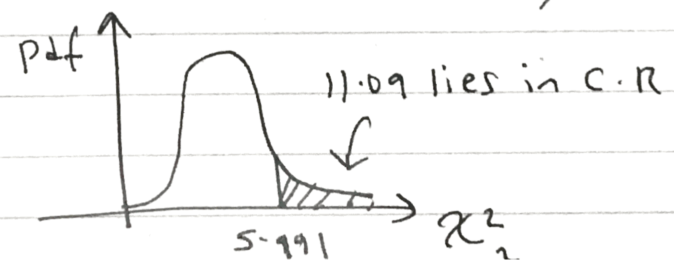
Q2) H_0: There is no association between type & cover.

H_1: There is an association between type & cover.

$\nu = (\text{rows}-1)(\text{columns}-1) = (3-1)(2-1) = 2$

$\chi^2_2(5\%) = \text{critical value} = 5.991$

$X^2 = \sum \frac{(O-E)^2}{E} = 11.09$



$11.09 > 5.991$

$\therefore$ Result is significant

Reject H_0

Evidence suggests there is association between type of book and cover type

Q3a) $H_0: \mu_{sp} = \mu_{st}$ sp → special

$H_1: \mu_{sp} > \mu_{st}$ st → standard

critical value (5%, 1-tail) $= \pm 1.6449$

$$\text{Test Stat} = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$

$$= \frac{75 - 64 - (0)}{\sqrt{\frac{22^2}{100} + \frac{31^2}{80}}}$$

$$= 2.68$$

$2.68 > 1.6449$ $\therefore$ Result is significant

Reject H_0.

Evidence suggests that the special diet is more effective in reducing blood cholesterol.

b) - Sample variance = population variance

- sample sizes are large enough to apply C.L.T to assume sample means are normally distributed.

Q4) H_0: Poisson distribution is a suitable model

H_1: Poisson distribution is not a suitable model.

$$\hat{\lambda} = \frac{\text{total breakdowns}}{\text{total days}} = \frac{52}{80} = 0.65$$

so $X \sim Po(0.65)$ where X = no. of breakdowns daily.

Expected frequencies $(E_i) = 80 \times P(X = x)$

(80 days)

$$E_0 = 80P(X=0) = 80\left[e^{-0.65}\right] = 41.76$$

$$E_1 = 80P(X=1) = 80\left[e^{-0.65}(0.65)\right] = 27.15$$

$$E_2 = 80P(X=2) = 80\left[\frac{e^{-0.65}(0.65^2)}{2!}\right] = 8.82$$

$$E_3 = 80 - (\sum E_i) = 2.27$$

Remember, expected frequencies must all be greater than 5 for the test statistic X^2 to be approximated well by the chi-squared distribution(s).

So pool final two groups:

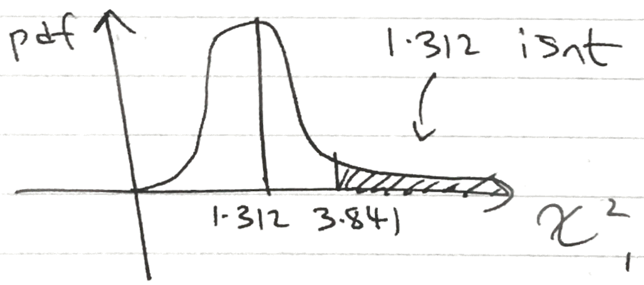
Breakdowns	0	1	≥ 2
O_i	38	32	10
E_i	41.76	27.15	11.09
$\frac{(O-E)^2}{E}$	0.3385	0.8664	0.1071

$$X^2 = \sum \frac{(O-E)^2}{E} = 1.312$$

$$\nu = 3 - 1 - 1 = 1$$

$\therefore$ critical value $= \chi^2_1 (5\%) = 3.841$

$$1.312 < 3.841$$



$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests Poisson distribution is a suitable model.

$X \sim N(8, 2^2)$ $Y \sim N(14, 3^2)$

Q5a) $E(R) = E(X) + 4E(Y)$

$= 8 + 4(14) = \boxed{64}$

b) $Var(R) = Var(X) + 4^2 Var(Y)$

$= 2^2 + 4^2(3^2) = \boxed{148}$

c) $P(R<41)$; $R \sim N(64, 148)$

$P(R < 41) = P\left(Z < \frac{41-64}{\sqrt{148}}\right)$

$= P(Z < -1.89)$

$= 1 - P(Z < 1.89) = \boxed{0.0294}$

d) $S = Y_1 + Y_2 + Y_3 - \frac{1}{2}X$

$Var(S) = 3Var(Y) + \frac{1}{4}Var(X)$

$= 3(3^2) + \frac{1}{4}(2^2) = \boxed{28}$

Remember, $Y_1 + Y_2 + Y_3 \neq 3Y$

So do not square the 3.

Q6a) Stratified Sampling

b) - population divides naturally into mutually exclusive groups. (subjects).

- advantage is that sub-groups and stratas can be individually analysed.

c) $\text{mean} = \frac{\sum fx}{n} = \frac{12(12.6) + 12(14.1) + 8(10.2)}{32}$

$= \boxed{12.56}$

d) 95% C I: $\left[\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}\right]$

$\left[12.56 \pm 1.96\left(\frac{2.48}{\sqrt{32}}\right)\right]$

$[11.7, \quad 13.4]$

e) 12 is inside the confidence interval so the suggestion aligns with the current time spent by these students.

(PMCC)

Q7a) $H_0 : p = 0$ critical value : ± 0.7887
$H_1 : p > 0$ (1%, 1-tail)

$0.774 < 0.7887$
$\therefore$ Result is insignificant
Accept H_0.
Evidence suggests that
there is no positive correlation present.

b)

Gymnast	Ability	Perfomance	R_A	R_P	d	d^2
A	8.5	6.2	3	2	1	1
B	8.6	7.5	4	5	1	1
C	9.5	8.2	8	7	1	1
D	7.5	6.7	2	3	1	1
E	6.8	6.0	1	1	0	0
F	9.1	7.2	5	4	1	1
G	9.4	8.0	7	6	1	1
H	9.2	9.1	6	8	2	4
						10

$\sum d^2 = 10 \quad \therefore \quad r_s = 1 - \frac{6(10)}{8(63)}$

$= \boxed{0.881}$

c) $H_0 : p = 0$ critical value : ± 0.8333
$H_1 : p > 0$ (1%, 1-tail)

$0.881 > 0.8333$
$\therefore$ Result is significant
Reject H_0.
Evidence suggests there is a (+ve) correlation.

d) Technical Ability / Artistic Performance may not be jointly normally distributed – this is required when using PMCC but not with a rank correlation coefficient.