



$$\sum d^2 = 24 \quad \therefore r_s = 1 - \frac{6(24)}{8(63)}$$

$$= \boxed{0.714}$$

d)  $H_0: \rho = 0$       critical value:  $0.6429$  //  
 $H_1: \rho > 0$       (5%, 1-tail)

$$0.714 > 0.6429$$

$\therefore$  Result is significant.

Reject  $H_0$ .

Evidence suggests a +ve correlation exists between c and a.

e) Data likely not jointly normally distributed, which is required to use PMCC.

- (Q2a)
- List all ticket numbers for standard/premium
  - Use random numbers to select a sample for both standard and premium ticket holders.
  - These sample sizes must be proportionate to the total population (ie all ticket holders at the concert).

b)  $H_0: \mu_p - \mu_s = 6$  where  $\begin{cases} p = \text{premium} \\ s = \text{standard} \end{cases}$

$H_1: \mu_p - \mu_s > 6$

$$\text{Test Statistic} = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$= \frac{23 - 15 - (6)}{\sqrt{\frac{10^2}{80} + \frac{8^2}{55}}}$$

$$= 1.189 \dots$$

critical value:  $\pm 1.6449$   
(5%, 1-tail)

So  $\dots$   $1.189 \dots < 1.6449$   
 $\therefore$  Result is insignificant.  
 Accept  $H_0$ .  
 Evidence suggests manager's claim is incorrect.

c) No - sample sizes are large so C.L.T applies.

$$\begin{aligned}
 \text{(Q3a)} \quad \left. \begin{array}{l} \sum x = 6.2 \\ \sum x^2 = 9.78 \\ n = 4 \end{array} \right\} \begin{array}{l} \text{mean} = \frac{\sum x}{n} = \frac{6.2}{4} = \boxed{1.55} \\ \text{Variance} = s^2 = \frac{1}{n-1} \left( \sum x^2 - \frac{(\sum x)^2}{n} \right) \\ = \frac{1}{3} \left( 9.78 - \frac{(6.2)^2}{4} \right) \\ = \boxed{0.057} \end{array}
 \end{aligned}$$

b)  $T \sim N(\mu, 0.5^2)$  where  $T = \text{repair time}$

$\bar{T}$  = estimate of population mean...

$$\bar{T} \sim N\left(\mu, \frac{0.5^2}{n}\right) \quad \text{By C.L.T}$$

$$P(\text{required}) \Rightarrow P(|\bar{T} - \mu| < 0.1) \geq 0.99$$

$$P(|\bar{T} - \mu| < 0.1) = P(-0.1 < \bar{T} - \mu < 0.1) \geq 0.99$$

$$P(-0.1 + \mu < \bar{T} < 0.1 + \mu) \geq 0.99$$

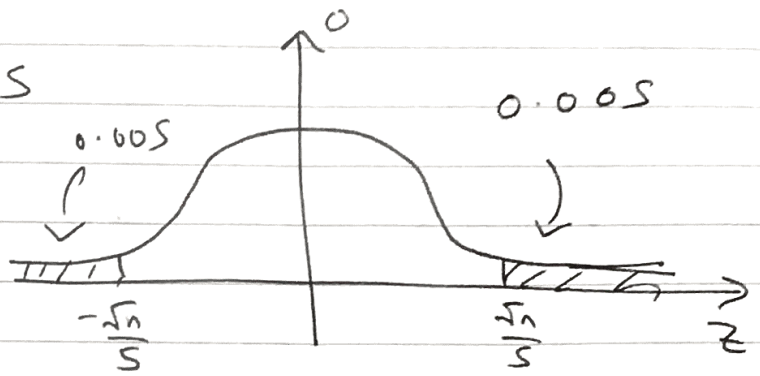
$z$

$$\rightarrow P\left(\frac{-0.1 + \mu - \mu}{\frac{0.5}{\sqrt{n}}} < z < \frac{0.1 + \mu - \mu}{\frac{0.5}{\sqrt{n}}}\right) \geq 0.99$$

$$P\left(-\frac{\sqrt{n}}{5} < z < \frac{\sqrt{n}}{5}\right) \geq 0.99$$

Because Area under unshaded region  $> 0.99!$

so  $P(Z > \frac{\sqrt{n}}{5}) \leq 0.005$



but  $P(Z > 2.5758) = 0.005$

this means that  $\frac{\sqrt{n}}{5} \geq 2.5758$

$$\sqrt{n} \geq 5 \times 2.5758$$

$$n \geq (5 \times 2.5758)^2$$

$$n \geq 165.87 \dots$$

so  $n_{\min} = 166$

(Q4a) width =  $2(2.5758) \frac{\sigma}{\sqrt{n}} = 2(2.5758) \frac{\sigma}{\sqrt{120}}$

'critical' value

=  $0.47 \sigma$

b)  $H_0: \mu = 6$  } 6 is in the interval!  
 $H_1: \mu \neq 6$  }

sig level: 10%

so Result is insignificant  
 Accept  $H_0 \rightarrow \mu = 6$



$$c) \text{ width of given interval} = 6.25 - 5.14 \\ = 1.11$$

$$\text{width of given interval} = 2(1.6449) \left( \frac{\sigma}{\sqrt{100}} \right)$$

$$[P(Z > 1.6449) = 0.05]$$

$$\text{So } \frac{2(1.6449)\sigma}{10} = 1.11$$

$$\sigma = \frac{1.11 \times 10}{2 \times 1.6449} = \boxed{3.37}$$

$$(Q5a) C \sim N(1000, 250^2)$$

$$L \sim N(2800, 650^2)$$

$$P(\text{required}) = P(L > 3C) = P(L - 3C > 0)$$

$$\text{let } A = L - 3C, \quad E(A) = 2800 - 3(1000) \\ = -200 //$$

$$\text{Var}(A) = \text{Var}(L) + 9\text{Var}(C) \\ = 650^2 + 9(250^2) \\ = 985000 //$$

$$\therefore A \sim N(-200, 985000)$$

$$P(A > 0) = P\left(Z > \frac{200}{\sqrt{985000}}\right) = P(Z > 0.20)$$

$$= 1 - P(Z < 0.20) = \boxed{0.4207}$$

b) let  $B = 8C + 3L$ ,

$$E(B) = 8(1060) + 3(2800) = \underline{\underline{16400}}$$

$$\begin{aligned} \text{Var}(B) &= 8^2 \text{Var}(C) + 3^2 \text{Var}(L) \\ &= 64(250^2) + 3^2(650^2) = \underline{\underline{1767500}} \end{aligned}$$

$$P(\text{required}) = P(B > 20000)$$

$$= P\left(z > \frac{20000 - 16400}{\sqrt{1767500}}\right)$$

$$= P(z > 2.71) = 1 - P(z < 2.7)$$

$$\approx \boxed{0.0035}$$

c) - selection of cars/tonnes is random  
OR - weights of cars/tonnes are independent.

$$X \sim B[4, 0.5]$$

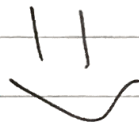
(Q6a)  $H_0$ :  $B[4, 0.5]$  is a suitable model.

$H_1$ :  $B[4, 0.5]$  is not a suitable model.

No.	0	1	2	3	4
$O_i$	12	45	36	39	18
$E_i$	$150 \times P(X=0)$	$150 \times P(X=1)$	$150 \times P(X=2)$	$150 \times P(X=3)$	$150 \times P(X=4)$

$$E_i = 150 \times P(X=i) //$$

because experiment is repeated 150 times.



No.	0	1	2	3	4
$O_i$	12	45	36	39	18
$E_i$	9.375	37.5	56.25	37.5	9.375
$\frac{(O_i - E_i)^2}{E_i}$	0.735	1.50	7.29	0.66	7.935

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 17.5 //$$

$$\gamma = 5 - 1 = 4 \quad \therefore \text{critical value} = \chi^2_4(1\%) = 13.277 //$$

$$17.5 > 13.277$$

$\therefore$  Result is significant.

Reject  $H_0$

$B[4, 0.5]$  isn't a suitable model!



$$b) \hat{p} = \frac{\text{total dices showing even no.}}{\text{total dice rolls}}$$

$$= \frac{0(12) + 1(45) + 2(36) + 3(39) + 4(18)}{4 \times 150}$$

$$= \boxed{0.51}$$

$$c) d = 150 P(X=2) = 150 \times \left[ \binom{4}{2} (0.51)^2 (1-0.51)^2 \right]$$

$$= \boxed{56.2}$$

$$e = 150 - (\sum \epsilon_i) = \boxed{10.2}$$

d)  $H_0: B[4, 0.51]$  is a suitable model

$H_1: B[4, 0.51]$  is not a suitable model.

$$e) \gamma = 5 - 1 - 1 = 3 \rightarrow \chi^2_3(1 \cdot 1 \cdot) = 11.345 //$$

(Subtract an extra 1  
as p was calculated.)

$$16.9 > 11.345$$

$\therefore$  Result is significant.  
Reject  $H_0$ .

$B[4, 0.51]$  is not a  
suitable model!