S3 June 2017 (MA)

Q1a) more representative of the entire population.

b) No. from each group = $\frac{\text{group size}}{\text{population size}}$ x sample size.

managers = $\frac{72}{720} \times 40 = \boxed{4}$

drivers = $\frac{108}{720} \times 40 = \boxed{6}$

admins = $\frac{180}{720} \times 40 = \boxed{10}$

warehouse = $\frac{360}{720} \times 40 = \boxed{20}$

c) Label all managers 1-72 then use random numbers to select 4 managers. in range.

Q2a) Angle(degrees) range = 360.

Expected = $\frac{\text{Group range}}{\text{Total range}} \times 100$ $(n = 100)$.

$a = \frac{45}{360} \times 100 = 12.5$

$b = \frac{90}{360} \times 100 = 25$

$c = \frac{135}{360} \times 100 = 37.5$

b) H_0: Continuous Uniform Distribution is a suitable model.

H_1: Continuous Uniform Distribution is not a suitable model.

Angle	0-45	45-90	90-180	180-315	315-360
Frequency	18	16	18	29	19
Expected	12.5	12.5	25	37.5	12.5
$\frac{(O-E)^2}{E}$	2.42	0.98	1.96	1.93	3.38

$$X^2 = \sum \frac{(O-E)^2}{E} = 10.67$$

$$\nu = 5-1 = 4$$

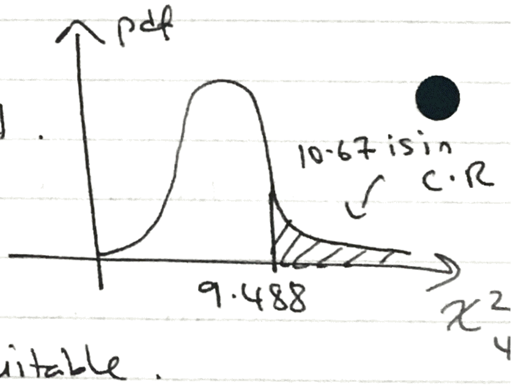
$\therefore$ critical value $= \chi^2_4(5\%) = 9.488$

$10.67 > 9.488$

$\therefore$ Result is significant.

Reject H_0.

Evidence suggests a continuous uniform distribution is not suitable.



Q3a)

Skater	Senior	Junior	d	d^2
A	1	3	2	4
B	2	1	1	1
C	4	5	1	1
D	3	2	1	1
E	6	6	0	0
F	5	4	1	1
				8

$\sum d^2 = 8 \quad \therefore \; r_s = 1 - \dfrac{6(8)}{6(35)}$

$= \boxed{0.771}$

b) $H_0 : \rho = 0$
$H_1 : \rho > 0$

agreement indicated by +ve correlation

Critical value : ± 0.8286
(5%, 1-tail)

$0.771 < 0.8286$

$\therefore$ Result is insignificant
Accept H_0.
Evidence suggests that no positive correlation exists – no agreement.

c) Ineffective as result of test was insignificant

Q4a) expected no. $= \dfrac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$

	Under	About Right	Over
Male	17.28	24	30.72
Female	18.72	26	33.28

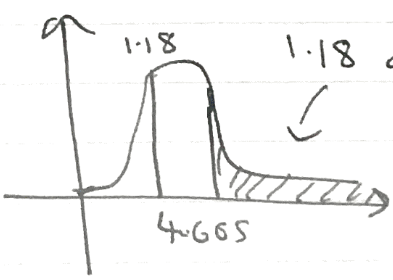
b) H_0: Perceived weight is independent of gender.

H_1: Perceived weight is not independent of gender.

O_i	E_i	$\frac{(O-E)^2}{E}$
20	17.28	0.428
22	24	0.167
30	30.72	0.0169
16	18.72	0.395
28	26	0.154
34	33.28	0.0156
		1.18

$\nu = (3-1)(2-1) = 2$
$= (\text{columns} - 1)(\text{rows} - 1)$

critical value $= \chi^2_2\,(10\%) = 4.605$



1.18 does not lie in C.R

$1.18 < 4.605$

$\therefore$ Result is insignificant
Accept H_0
Evidence suggests perceived body weight is independent of gender.

c)

O_i	E_i	$\frac{(O-E)^2}{E}$
36	50	3.92
50	50	0
64	50	3.92
		7.84

[expected values will all be 50 if weight types are chosen equally]

$$X^2 = \sum \frac{(O-E)^2}{E} = 7.84$$

$$\nu = 3-1 = 2$$

$$\chi^2_2(2.5\%) = 7.378$$
$$\chi^2_2(1\%) = 9.210$$

hence smallest significance level where result is significant is $\boxed{2.5\%}$

Q5a) mean $= \frac{\sum x}{n} = \frac{60}{15} = \boxed{4}$

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$$

$$= \frac{1}{14}\left(1946 - \frac{(60)^2}{15}\right)$$

$$= 121.86\ldots = \boxed{122}$$

bi) 95% C.I : $\left[\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}\right]$

$$\left[121.86 \pm 1.96\left(\frac{10}{\sqrt{15}}\right)\right]$$

$$\left[-1.06 \quad , \quad 9.06\right]$$

as a time: $[08:30 \quad , \quad 08:40]$ to nearest min.

ii) times of buses are sampled independently of each other.

c) 08:31 is within the confidence interval so Paul's belief is not supported.

Q6a) $H_0: \mu_{new} - \mu_{old} = 8$

$H_1: \mu_{new} - \mu_{old} > 8$

critical value: ± 1.6449
(5%, 1-tail)

$$\text{Test Statistic} = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$

$$= \frac{83 - 74 - (8)}{\sqrt{\frac{7}{50} + \frac{6}{40}}}$$

$$= 1.86$$

$1.86 > 1.6449$

$\therefore$ Result is significant

Reject H_0.

Evidence suggests that the engineer's claim is correct.

b) Allows us to assume the sample means for new/old batteries are normally distributed as. n is large.

Q7a) let M = medium bags (weight), $M \sim N(520, 10^2)$

and L = large bags (weight), $L \sim N(1510, 20^2)$

$$P(\text{required}) = P(L > M_1 + M_2 + M_3 + 15)$$

$$= P(L - (M_1 + M_2 + M_3 + 15) > 0)$$

let $A = L - (M_1 + M_2 + M_3) - 15$,

$$E(A) = 1510 - 3(520) - 15 = -65$$

$$Var(A) = Var(L) + 3Var(M) \quad \boxed{(M_1 + M_2 + M_3 \neq 3M)}$$

$$= 20^2 + 3(10^2) = 700$$

$$\therefore A \sim N(-65, 700)$$

$$P(\text{required}) = P(A > 0) = P\left(Z > \frac{0 - -65}{\sqrt{700}}\right)$$

$$= P(Z > 2.46) = 1 - P(Z < 2.46)$$

$$= \boxed{0.0069}$$

b) $P(\text{required}) = P(L < 3M) = P(L - 3M < 0)$

let $B = L - 3M$,

$E(B) = E(L) - 3E(M)$

$= 1510 - 3(520) = -50$

$Var(B) = Var(L) + 3^2 Var(M)$

$= 20^2 + 3^2(10^2) = 1300$

$\therefore B \sim N(-50, 1300)$

$P(\text{required}) = P(B < 0) = P\left(Z < \frac{0 - -50}{\sqrt{1300}}\right)$

$= P(Z < 1.39) = \boxed{0.9177}$

c) $\left[P(M > 520)\right]^5 = P(\bar{M} > d)$

$\bar{M} \sim N\left(520, \frac{10^2}{5}\right)$ by C.L.T

$P(M > 520) = 0.5$ (since 520 is the mean)

$\therefore 0.5^5 = P(\bar{M} > d)$

$P(\bar{M} > d) = P\left(Z > \frac{d - 520}{\frac{10}{\sqrt{5}}}\right) = 0.5^5$

$P\left(Z > \frac{\sqrt{5}(d - 520)}{10}\right) = 0.03125$

$\therefore P\left(Z < \frac{\sqrt{5}(d - 520)}{10}\right) = 0.96875$

$P(Z < z) = 0.96875$ is not in the normal distribution tables so find the closest probability.

from tables : $P(Z < 1.86) = 0.9686$

$$\therefore 1.86 = \frac{\sqrt{5}(d - 520)}{10}$$

$$\frac{1.86 \times 10}{\sqrt{5}} = d - 520$$

$$d = \frac{1.86(10)}{\sqrt{5}} + 520$$

$$\boxed{d \simeq 528.3}$$