S3 June 2014 (MA)

Q1a) This is a sample where every other possible sample of size n has an equal chance of being selected.

b) When its not possible to obtain a sampling frame.

c i) A list of all the students.

ii) Create a numbered list of all students (1–75). Then use random numbers to select 8 students. Pick only numbers in the range 1–75 and the numbers picked will correspond to the students in the list.

Q2ai) Yes – no population parameters included, only known data

ii) No – contains μ, a population parameter.

iii) No – contains μ and σ, both population parameters.

b) $E\left(\frac{3X_1 - X_{20}}{2}\right) = \frac{1}{2}\left[E(3X_1 - X_{20})\right]$

$= \frac{1}{2}\left[3E(X_1) - E(X_{20})\right]$

$= \frac{1}{2}\left[3\mu - \mu\right] = \mu$. (unbiased estimator of μ)

$$\text{Var}\left(\frac{3X_1 - X_{20}}{2}\right) = \frac{1}{4}\text{Var}(3X_1 - X_{20})$$

$$= \frac{9}{4}\text{Var}(X_1) + \frac{1}{4}\text{Var}(X_{20})$$

$$= \frac{9}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \boxed{\frac{5\sigma^2}{2}}$$

Q3) H_0: There is no association between gender and happiness.

H_1: There is an association between gender and happiness.

$$\text{Expected no.} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$$

EXPECTED :

	NH	FH	VH
F	13.51	41.77	30.71
M	8.49	26.23	19.29

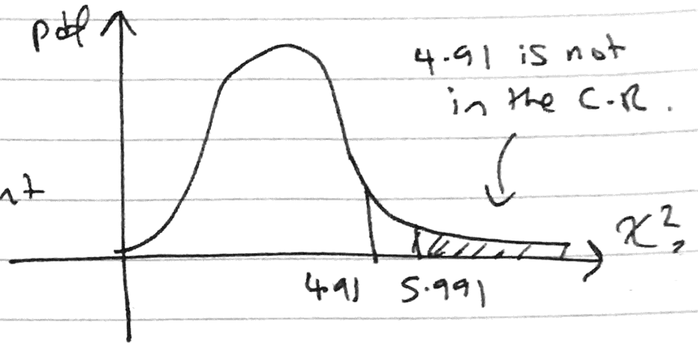
O:	E:	$\frac{(O-E)^2}{E}$
9	13.51	1.508
43	41.77	0.0361
34	30.71	0.351
13	8.49	2.402
25	26.23	0.0575
16	19.29	0.560

$$X^2 = \sum \frac{(O-E)^2}{E} = 4.91$$

$$\nu = (\text{rows} - 1)(\text{columns} - 1) = (2-1)(3-1) = 2$$

$\therefore$ critical value $= \chi^2_2(5\%) = 5.991$

$\therefore$ Result $\quad 4.91 < 5.991$
is insignificant
Accept H_0.
Evidence suggests gender is independent of happiness.



(Q4) $\quad A = B + 4C - 3D$

$$E(A) = E(B) + 4E(C) - 3E(D)$$
$$= 6 + 4(7) - 3(4) = 22$$
$$Var(A) = Var(B) + 4^2\,Var(C) + 3^2\,Var(D)$$
$$= 2^2 + 4^2(3^2) + 3^2(1.5^2)$$

$$= 168.25$$

$$\therefore A \sim N(22, 168.25)$$

$$P(A < 45) = P\left(Z < \frac{45-22}{\sqrt{168.25}}\right)$$

$$= P(Z < 1.77) = \boxed{0.9616}$$

Q5a) - fixed number of seeds in a row.
- only two outcomes → either a seed germinates or it does not.

b) $p = \dfrac{\text{no. of geminated seeds}}{\text{total no. of seeds}}$

$= \dfrac{0(2)+1(6)+2(11)+3(19)+4(25)+5(32)+6(16)+7(9)}{120 \times 7}$

$= \dfrac{504}{840} = \boxed{0.6}$

c) $s = 120 \times P(X=2)$ where $X \sim B[7, 0.6]$
X = no. of seeds geminating in one row.

$= 120\left[\binom{7}{2}(0.6^2)(0.4^5)\right]$

$= \boxed{9.29}$

$t = 120 - (\sum E_i) = \boxed{34.84}$

H_0: Binomial is a suitable model
H_1: Binomial is not a suitable model.

d) Remember that expected frequencies must all be greater than 5 for the test statistic X^2 to be approximated well by the chi-squared distribution(s).

So pool first 3 cells and last 2...

No. of...	0 – 2	3	4	5	6 – 7
E:	11.55	23.22	34.84	31.35	19.04
O:	19	19	25	32	25
$\frac{(O-E)^2}{E}$	4.805	0.767	2.779	0.013	1.866

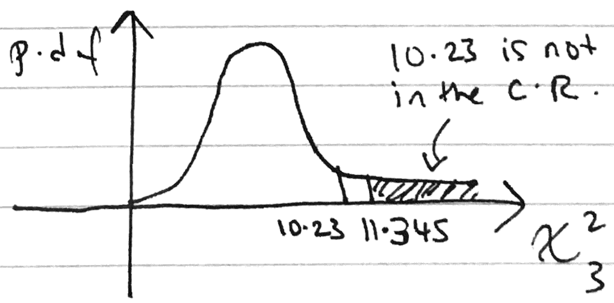
$$X^2 = \sum \frac{(O-E)^2}{E} = 10.23$$

$$\nu = 5 - 1 - 1 = 3$$

[subtract an extra '1' as we calculated p]

$\therefore$ critical value $= \chi^2_3(1\%) = 11.345$

$$10.23 < 11.345$$



$\therefore$ Result is insignificant.
Accept H_0.
Binomial is a suitable model.

Q6a) if $\bar{X}$ is an unbiased estimator of μ then $E(\bar{X}) = \mu$.

$$E(\bar{X}) = E\left(\frac{1}{n}(X_1 + X_2 + \dots + X_n)\right)$$

$$= \frac{1}{n}\left[E(X_1) + E(X_2) + \dots + E(X_n)\right]$$

$X_1, X_2, X_3, \dots, X_n$ all have an expected value of μ.

$$\therefore \frac{1}{n}\left[\mu + \mu + \dots + \mu\right] = E(\bar{X}) = \frac{n \times \mu}{n}$$

$$= \mu$$

b) $$\frac{\sum x}{n} = \text{mean} = \frac{1001}{5} = \boxed{200.2}$$

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$$

$$= \frac{1}{4}\left(200469 - \frac{(1001)^2}{5}\right)$$

$$= \boxed{17.2}$$

c) $\overline{X}$ = estimated population mean.

what we want is ...

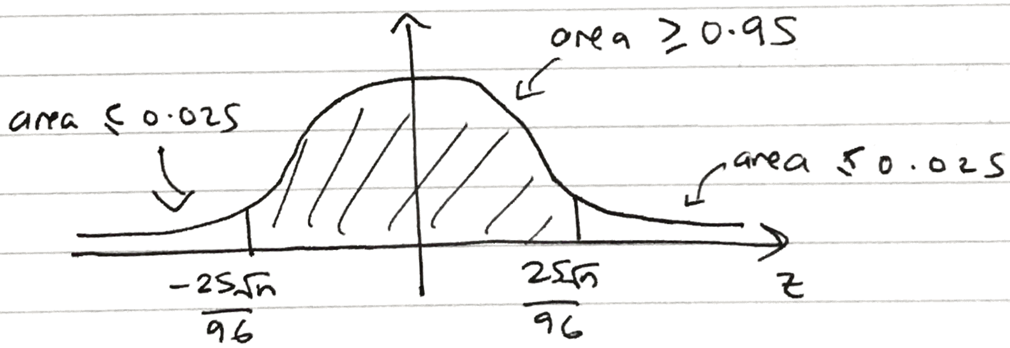
$$P(-1.25 < \overline{X} - \mu < 1.25) \geq 0.95$$

$$P(-1.25 + \mu < \overline{X} < 1.25 + \mu) \geq 0.95$$

Now by C.L.T, $\overline{X} \sim N\left(\mu, \frac{4.8^2}{n}\right)$

$$P\left(\frac{-1.25+\mu-\mu}{\frac{4.8}{\sqrt{n}}} < Z < \frac{1.25+\mu-\mu}{\frac{4.8}{\sqrt{n}}}\right) \geq 0.95$$

$$P\left(-\frac{25\sqrt{n}}{96} < Z < \frac{25\sqrt{n}}{96}\right) \geq 0.95$$



the 'middle' area ≥ 0.95 $\therefore$ the area to the right (or left) ≤ 0.025

so we can say: $P\left(Z > \frac{25\sqrt{n}}{96}\right) \leq 0.025$

the z-value corresponding to $(p = 0.025) = 1.96$

$\therefore \frac{25\sqrt{n}}{96} \geq 1.96$ (as z-value increases, $P(Z>z)$ decreases)

$$\text{so } \sqrt{n} \geq \frac{1.96(96)}{25}$$

$$n \geq 566 \ldots$$

$$\therefore \boxed{n_{min} = 57} \quad \text{(next integer)}$$

Q7a) $P(X < \mu - 30) = 0.0005$

$$P\left(Z < \frac{\mu - 30 - \mu}{\sigma}\right) = 0.0005$$

$$P\left(Z > \frac{30}{\sigma}\right) = 0.0005.$$

$\therefore$ from tables, $3.2905 = \frac{30}{\sigma}$

$$\sigma = \frac{30}{3.2905} = \boxed{9.117g}$$

b) $H_0: \mu = 1000$
$H_1: \mu < 1000$

$$\bar{x} = \frac{\sum x}{n} = 999.54$$

critical value : ± 2.3263
(1%, 1-tail)

$$\text{Test Statistic} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{999.54 - 1000}{\frac{9.117}{\sqrt{10}}}$$

$$= -0.160$$

$-0.160 > -2.3263$

$\therefore$ Result is insignificant

Accept H_0. Mean is not less than 1kg.

Q8a) $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{9.3433}{\sqrt{0.0632 \times 1957.5556}}$

$= \boxed{0.840}$

b) $H_0 : \rho = 0$ critical value : $\pm\ 0.5822$ (5%, 1-tail)

$H_1 : \rho > 0$

$0.840 > 0.5822$

$\therefore$ Result is significant.

Reject H_0.

Evidence suggests there is a positive correlation between man's height and weight.

c)

Man	y	Peter	R_y	R_p	d	d^2
A	75	1	1	1	0	0
B	76	4	2	4	2	4
C	100	2	7	2	5	25
D	77	6	3	6	3	9
E	90	3	4	3	1	1
F	95	8	5	8	3	9
G	110	5	8	5	3	9
H	96	9	6	9	3	9
I	120	7	9	7	2	4
						70

$\sum d^2 = 70 \quad \therefore \quad r_s = 1 - \dfrac{6(70)}{9(80)} = \boxed{0.417}$

d) $H_0 : \rho = 0$
$H_1 : \rho > 0$

ability to correctly order men would mean a positive correlation.

critical value : 0.600
(5%, 1-tail)

$0.417 < 0.600$

$\therefore$ Result is ~~sig~~ insignificant.
Accept H_0.
Peter does not have the stated ability.