S3 June 2011 (MA)

Q1) if X follows any distribution with mean μ and variance σ^2 and a sample $X_1, X_2, \ldots, X_n$ is drawn then provided n is large the sample mean $\bar{X} \simeq \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Q2a)

Town	h	c	R_h	R_c	d	d^2
A	14	52	1	4	3	9
B	20	45	5	3	2	4
C	16	43	2	2	0	0
D	18	42	3	1	2	4
E	37	61	7	6	1	1
F	19	82	4	7	3	9
G	24	55	6	5	1	1
						28

$\sum d^2 = 28.$

$$r_s = 1 - \frac{6(28)}{7(48)} = \boxed{0.5000}$$

b) $H_0: \rho = 0$
$H_1: \rho \neq 0$

the alternative to no correlation is a correlation (either +ve or -ve).

critical value: ± 0.7857
(5%, 2-tail)

$0.5 < 0.7857$

$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests the councillor is correct.

Q3) H_0: Type of defect is independent of the shift that manufactured the component.

H_1: Type of defect is dependent upon the shift that manufactured the component.

$$\text{Expected no.} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$$

EXPECTED :

	D_1	D_2	
First Shift	47.25	15.75	63
Second Shift	56.25	18.75	75
Third Shift	46.5	15.5	62
	150	50	$\boxed{200}$

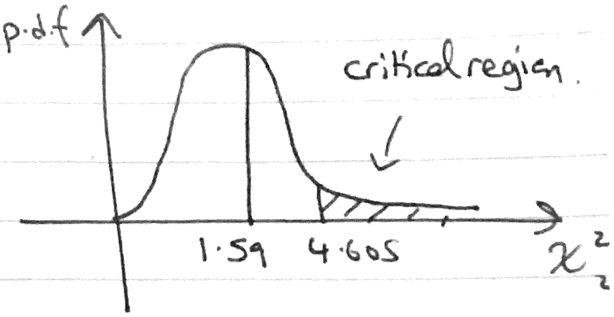
O_i	E_i	$\frac{(O-E)^2}{E}$
45	47.25	0.1071
18	15.75	0.3214
55	56.25	0.0278
20	18.75	0.0833
50	46.50	0.2634
12	15.50	0.7903
		1.59 //

ν = (rows − 1) (columns − 1) = (3−1)(2−1) = 2 //

∴ critical value = χ^2_2 (10%) = 4.605 //

$1.59 < 4.605$

$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests that the manager's belief is incorrect.



(Q4a) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{5320}{80} = \boxed{66.5}$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$$

$$= \frac{1}{79}\left(392000 - \frac{(5320)^2}{80}\right) = \boxed{484}$$

b) $H_0: \mu_1 = \mu_2$
$H_1: \mu_1 < \mu_2$

let μ_1 indicate mean without music and μ_2 indicate mean with music.

critical value $= \pm 1.6449$.
(5%, 1-tail)

$$\text{Test Stat} = \frac{\bar{x} - \bar{y} - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$= \frac{69.0 - 66.5}{\sqrt{\frac{484}{80} + \frac{46.44}{60}}} = 0.68$$

$0.68 < 1.6449$

$\therefore$ Result is insignificant.
Accept H_0.
Insufficient evidence to suggest mean money spent is greater with music.

Q5a) - Hurricanes occur independently of each other
- Hurricanes occur at a constant rate

b) $$\frac{0(0)+1(2)+2(5)+3(17)+4(20)+5(12)+6(12)+7(12)}{80}$$

$= \text{mean} = \boxed{4.4875}$

$= \left(\frac{\text{total hurricanes}}{\text{total years}}\right)$

c) $X \sim Po(4.4875)$ where X = no. of hurricanes in 1 year.

$P(X=2) \times 80 = r$

for 80 years

$$\therefore r = 80\left[\frac{e^{-4.4875}(4.4875^2)}{2}\right] = \boxed{9.06}$$

so $s = 80 - \sum E_i = \boxed{15.20}$

d) H_0: Poisson $(\lambda = 4.4875)$ is a suitable model for these data.

H_1: Poisson $(\lambda = 4.4875)$ is not a suitable model for these data.

remember, expected frequencies need to be greater than 5 for the test statistic X^2 to be approximated well by the chi-squared distribution (s).

So pool first 3 cells to give:

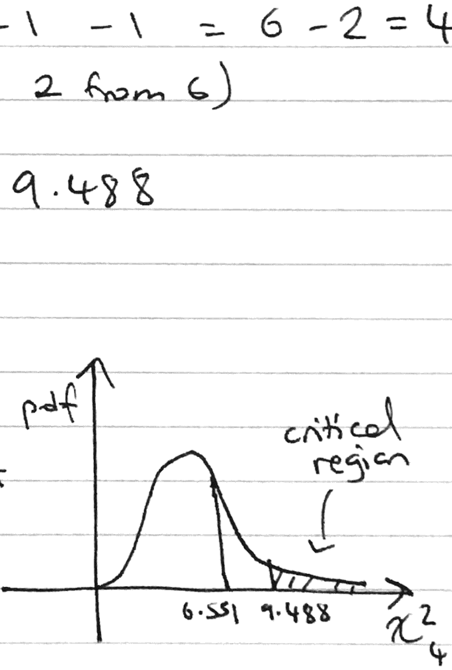
h	0-2	3	4	5	6	≥ 7
E_i	14	13.55	15.20	13.65	10.21	13.39
O_i	7	17	20	12	12	12
$\frac{(O-E)^2}{E}$	3.500	0.878	1.516	0.199	0.314	0.144

$$X^2 = \sum \frac{(O-E)^2}{E} = 6.551$$

ν = no. of cells after pooling -1 -1 $= 6 - 2 = 4$
(we calculated λ so subtract 2 from 6)

critical value $= \chi^2_4(5\%) = 9.488$

$6.551 < 9.488$
$\therefore$ Result is insignificant
Accept H_0.
Evidence suggests that Poisson $(\lambda = 4.4875)$ is a suitable model.



Q6a) $L \sim N(20, 5^2)$

a sample of 6 batteries $= L_1 + L_2 + \dots + L_6 = A$

$E(A) = 6(20) = \boxed{120} =$ mean

$Var(A) = 6(5^2) = 150 \quad \therefore$ s.d $= \sqrt{150} = \boxed{12.2}$

$\uparrow$

$(L_1 + L_2 + \dots + L_6 \neq 6L)$

b) $P(\text{required}) = P(A > 110) = P\left(Z > \frac{110 - 120}{12.2}\right)$

$= P(Z > -0.82) = P(Z < 0.82)$

$= \boxed{0.7939}$

c) $B \sim N(35, 8^2)$

$P(\text{required}) = P(A > 4B) = P(A - 4B > 0)$

$E(A - 4B) = E(A) - 4E(B)$

$= 120 - 4(35) = -20$

$Var(A - 4B) = Var(A) + 4^2 Var(B)$

$= 150 + 16(64)$

$= 1174$

$\therefore (A - 4B) \sim N(-20, 1174)$

$\Rightarrow P(A - 4B > 0) = P\left(Z > \frac{20}{\sqrt{1174}}\right)$

$\Rightarrow P(Z > 0.58) = 1 - P(Z < 0.58)$

$= \boxed{0.2797}$

Q7a) $H_0 : \mu = 250$ critical value : ± 1.6449.
$H_1 : \mu < 250$ (5%, 1-tail)

$$\text{Test Stat} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{248 - 250}{\frac{5.4}{\sqrt{90}}}$$

$$= -3.513$$

$$-3.513 < -1.6449$$

$\therefore$ Result is significant (Reject H_0)

Evidence suggests manager's claim is justified.

b) 98% C.I : $\left[\bar{x} \pm 2.3263\left(\frac{\sigma}{\sqrt{n}}\right)\right]$

(1% each tail)

$$\left[248 \pm 2.3263\left(\frac{5.4}{\sqrt{90}}\right)\right]$$

$$[247, 249]$$

c) 250 is above the upper limit of confidence interval so the manager should ask "Roastie's" manufactures to either reduce the stated weight to a value in the range [247, 249] or increase the weight of each packet

d) $\bar{X} \sim N\left(\mu, \frac{3^2}{n}\right)$ for a sample of size n by the C.L.T

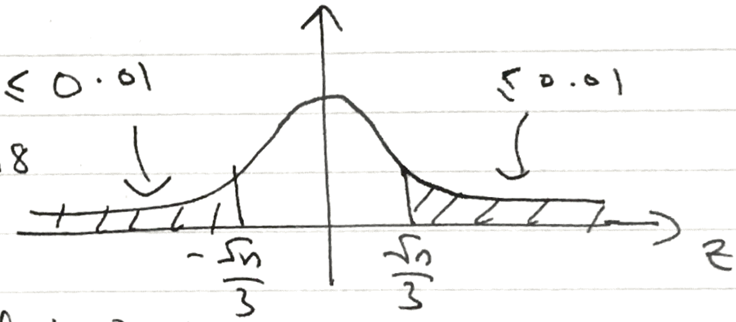
$$P(|\bar{X} - \mu| < 1) \geqslant 0.98$$

$$P(-1 < \bar{X} - \mu < 1) \geqslant 0.98$$

$$P(\mu - 1 < \bar{X} < \mu + 1) \geqslant 0.98$$

$$P\left(\frac{\mu-1-\mu}{\frac{3}{\sqrt{n}}} < Z < \frac{\mu+1-\mu}{\frac{3}{\sqrt{n}}}\right) \geqslant 0.98$$

$$P\left(-\frac{\sqrt{n}}{3} < Z < \frac{\sqrt{n}}{3}\right) \geqslant 0.98$$



the middle area $\geqslant 0.98$ so the area to the right (or left) is less than or equal to 0.01.

hence $P\left(Z > \frac{\sqrt{n}}{3}\right) \leqslant 0.01$

the z-value 2.3263 corresponds to $p = 0.01$

hence $\frac{\sqrt{n}}{3} \geqslant 2.3263$

as z increases the probability to the right decreases. so '$\geqslant$' applies.

$$\sqrt{n} \geqslant 3(2.3263)$$

$$\Rightarrow n \geqslant 48.7\ldots \text{ so } \boxed{n_{min} = 49}$$