

S3 June 2010 (MA)

(Q1) $H_0: \mu = 80$ $n = 100$ critical value: ± 1.6449
 $H_1: \mu > 80$ $\bar{x} = 83$ (5%, 1-tail)
 $\sigma = 15$

$$\text{Test Stat} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{83 - (80)}{\frac{15}{\sqrt{100}}} = 2.00 //$$

$$2.00 > 1.6449$$

\therefore Result is significant (reject H_0).
 Evidence suggests that the
 director's claim is correct.

(Q2a) $P \sim N(90, 9)$

$J \sim N(91, 12)$

$P(\text{required}) = P(J < P) = P(J - P < 0)$

$J - P \sim N(1, 21) \rightarrow \begin{cases} E(J - P) = 91 - 90 = 1 \\ \text{Var}(J - P) = 12 + 9 = 21 \end{cases}$

$\therefore P(J - P < 0) = P\left(Z < \frac{0 - 1}{\sqrt{21}}\right) = P(Z < -0.22)$

$= 1 - P(Z < 0.22) = 1 - 0.5871$

$= \boxed{0.4129}$

2 minutes



$$b) P(\text{required}) = P(P_1 + \dots + P_{60} + 120 < J_1 + \dots + J_{60})$$

$$= P(P_1 + \dots + P_{60} + 120 - (J_1 + \dots + J_{60}) < 0)$$

$$\text{let } A = P_1 + \dots + P_{60} + 120 - (J_1 + \dots + J_{60}),$$

$$E(A) = 60(90) + 120 - (9)(60) = 60 //$$

$$\text{Var}(A) = 60(9) + 60(12) = 1260 //$$

(remember $\text{Var}(aX + b) = a^2 \text{Var}(X)$)

note: $P_1 + \dots + P_{60} \neq 60P$ so do not square the 60 when calculating variance of A

$$\therefore A \sim N(60, 1260)$$

$$P(\text{required}) = P(A < 0) = P\left(Z < \frac{-60}{\sqrt{1260}}\right)$$

$$= P(Z < -1.69) = 1 - P(Z < 1.69)$$

$$= 1 - 0.9545 = \boxed{0.0455}$$

● (Q3a) $X \sim N(w, 0.5^2)$ where $X = \underline{\text{measured width}}$

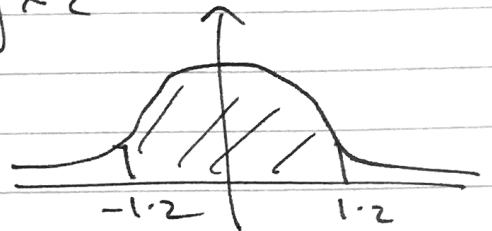
$$P(\text{required}) = P(w - 0.6 < X < w + 0.6)$$

$$\Rightarrow P\left(\frac{w - 0.6 - w}{0.5} < Z < \frac{w + 0.6 - w}{0.5}\right)$$

$$\Rightarrow P(-1.2 < Z < 1.2)$$

$$\Rightarrow [P(Z < 1.2) - 0.5] \times 2$$

$$\Rightarrow [0.8849 - 0.5] \times 2$$



$$\Rightarrow \boxed{0.7698}$$

b) so 16 samples of X are taken.

● By central limit theorem... $\bar{X} \sim N\left(w, \frac{0.5^2}{16}\right)$

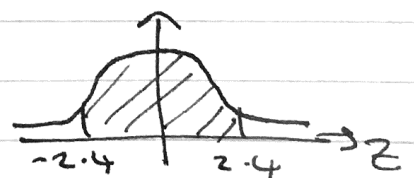
$$P(\text{required}) = P(w - 0.3 < \bar{X} < w + 0.3)$$

$$= P\left(\frac{w - 0.3 - w}{\frac{0.5}{4}} < Z < \frac{w + 0.3 - w}{\frac{0.5}{4}}\right)$$

$$= P(-2.4 < Z < 2.4)$$

$$= [P(Z < 2.4) - 0.5] \times 2 = 2[0.9918 - 0.5]$$

$$= \boxed{0.9836}$$



(1% each tail, c.v = 2.3263)

$$c) 98.1\% \text{ C.I.} \Rightarrow \left[\bar{x} \pm 2.3263 \left(\frac{s}{\sqrt{n}} \right) \right]$$

$$\Rightarrow \left[35.6 \pm 2.3263 \left(\frac{0.5}{\sqrt{16}} \right) \right]$$

$$\Rightarrow [35.3, 35.9]$$

Q4a)

Position	Distance b	Depth s	R_b	R_s	d	d^2
A	100	60	1	1	0	0
B	200	75	2	2	0	0
C	300	85	3	4	1	1
D	400	76	4	3	1	1
E	500	110	5	6	1	1
F	600	120	6	7	1	1
G	700	104	7	5	2	4
						<u>8</u>
						8

$$\sum d^2 = 8 \quad \therefore r_s = 1 - \frac{6(8)}{7(48)}$$

$$= \boxed{0.857}$$

$$b) H_0: p = 0$$

$$H_1: p > 0$$

critical value: ± 0.8929
(1%, 1-tail)

$$0.857 < 0.8929$$

↑
depth increases as distance increases.

this is the claim made.

Result is insignificant.
Accept H_0 .

Insufficient evidence
to support the
researcher's claim.

● (Q5) H_0 : Relative state of finances is independent of income range (no association).

H_1 : Relative state of finances is not independent of income range (association exists).

$$\text{Expected no.} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$$

<u>EXPECTED</u> :	Worse	Same	Better	
Under £15k	10.54	10.54	12.92	34
≥ £15k	20.46	20.46	25.08	66
	31	31	38	<u>100</u>

O_i	E_i	$\frac{(O-E)^2}{E}$
14	10.54	1.1358
11	10.54	0.0200
9	12.92	1.1893
17	20.46	0.5851
20	20.46	0.0103
29	25.08	0.6126
		<u>3.5531..</u>

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 3.55 //$$

$$\nu = (\text{rows} - 1)(\text{columns} - 1) = (2-1)(3-1) = 2$$

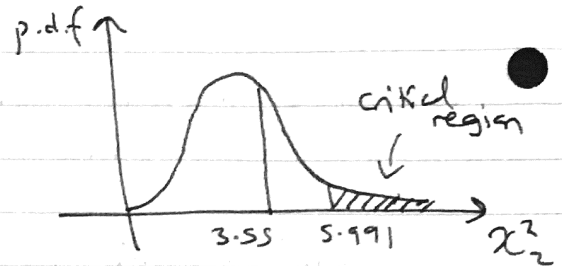
$$\therefore \text{critical value} = \chi^2_{2(5\%)} = 5.991 //$$

$$3.55 < 5.991$$

∴ Result is insignificant.

Accept H_0 .

Evidence suggests state of finances is independent of income.



Q6) H_0 : C.U.D is a good fit for these data.
 H_1 : C.U.D is not a good fit for these data.

Expected no. of items = $\frac{b-a}{n} \times 228$
 in each cell

where n = total range (12).

b = upper class limit

a = lower class limit

so... [eg for 9-12 class, $E_i = \frac{12-9}{12} \times 228 = 57$]

Distance	0-1	1-2	2-4	4-6	6-9	9-12
O_i	22	15	44	37	52	58
E_i	19	19	38	38	57	57
$\frac{(O_i - E_i)^2}{E_i}$	$\frac{9}{19}$	$\frac{16}{19}$	$\frac{18}{19}$	$\frac{1}{38}$	$\frac{25}{57}$	$\frac{1}{57}$

$$\text{Test Stat} = \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 2.75 //$$

$$r = \text{no. of cells} - 1 = 6 - 1 = 5$$

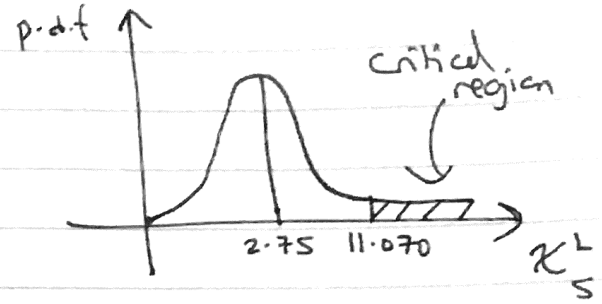
(after pooling)

$$\therefore \text{critical value} = \chi^2_5 (5\%) = 11.070$$

$2.75 < 11.070$
 \therefore Result is insignificant.

Accept H_0 .

Evidence suggests that
 C.U.D is a good fit for
 these data.



(Q7a) Label full-time staff 1-6000
 Label part-time staff 1-4000

$$\text{No. of full-time staff needed} = \frac{6000}{10000} \times 200 = 120$$

$$\text{No. of full part-time staff needed} = \frac{4000}{10000} \times 200 = 80$$

. Use random numbers to pick 120 full-time
 staff and 80 part-time staff.

b) Stratified ensures that each group is fairly
 represented \rightarrow reflects population structure.

c) $H_0: \mu_F = \mu_P$ F indicates full-time
 $H_1: \mu_F \neq \mu_P$ P indicates part-time.

critical value: 2.5758
 (2-tail, 1%)

$$\text{Test Stat} = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$= \frac{52 - 50}{\sqrt{\frac{21}{80} + \frac{19}{80}}} = 2.828\dots$$

$$2.828 > 2.5758$$

\therefore Result is significant.

Reject H_0 .

Evidence suggests means are different for full-time & part-time staff.

d) allows us to assume sample means \bar{F} and \bar{P} are normally distributed.

e) sample variance = population variance.
(Test Statistic by definition uses population variance)

$$f) \quad 2.53 < 2.5758$$

Result is insignificant.

Accept H_0 .

Insufficient evidence to suggest a difference in mean scores of full-time & part-time staff.

g) The training course has improved the awareness of part-time staff substantially as now there is an insignificant difference between full-time and part-time staff test scores. In (c) before the training, full-time staff were more aware as a group.