Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MS03

Unit Statistics 3

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions in the spaces provided.

1 A consumer report claimed that more than 25 per cent of visitors to a theme park were dissatisfied with the catering facilities provided.

In a survey, 375 visitors who had used the catering facilities were interviewed independently, and 108 of them stated that they were dissatisfied with the catering facilities provided.

(a) Test, at the 2% level of significance, the consumer report's claim. *(6 marks)*

(b) State an assumption about the 375 visitors that was necessary in order for the hypothesis test in part **(a)** to be valid. *(1 mark)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

2 The number of emergency calls received by a fire station may be modelled by a Poisson distribution.

During a given period of 13 weeks, the station received a total of 108 emergency calls.

(a) Construct an approximate 98% confidence interval for the average **weekly** number of emergency calls received by the station. *(5 marks)*

(b) Hence comment on the station officer's claim that the station receives an average of one emergency call **per day**. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

3 An IT help desk has three telephone stations: Alpha, Beta and Gamma. Each of these stations deals only with telephone enquiries.

The probability that an enquiry is received at Alpha is 0.60 .
The probability that an enquiry is received at Beta is 0.25 .
The probability that an enquiry is received at Gamma is 0.15 .

Each enquiry is resolved at the station that receives the enquiry. The **percentages** of enquiries resolved within various times at **each** station are shown in the table.

		Time		
		$\leqslant$ 1 hour	**$\leqslant$ 24 hours**	**$\leqslant$ 72 hours**
Station	**Alpha**	55	80	100
	Beta	60	85	100
	Gamma	40	75	100

For example:
80 per cent of enquiries received at Alpha are resolved within 24 hours;
25 per cent of enquiries received at Alpha take between 1 hour and 24 hours to resolve.

(a) Find the probability that an enquiry, selected at random, is:

(i) resolved at Gamma; *(1 mark)*

(ii) resolved at Alpha within 1 hour; *(1 mark)*

(iii) resolved within 24 hours; *(2 marks)*

(iv) received at Beta, given that it is resolved within 24 hours. *(3 marks)*

(b) A random sample of 3 enquiries was selected.

Given that all 3 enquiries were resolved within 24 hours, calculate the probability that they were all received at:

(i) Beta; *(2 marks)*

(ii) the same station. *(4 marks)*

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QUESTION PART REFERENCE

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4 The waiting time at a hospital's A&E department may be modelled by a normal distribution with mean μ and standard deviation $\dfrac{\mu}{2}$.

The department's manager wishes a 95% confidence interval for μ to be constructed such that it has a width of at most 0.2μ.

Calculate, to the nearest 10, an estimate of the minimum sample size necessary in order to achieve the manager's wish. *(5 marks)*

QUESTION PART REFERENCE

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5 An examination of 160 e-mails received by Gopal showed that 72 had attachments.

An examination of 250 e-mails received by Haley showed that 102 had attachments.

Stating **two** necessary assumptions about the selection of e-mails, construct an approximate 99% confidence interval for the difference between the proportion of e-mails received by Gopal that have attachments and the proportion of e-mails received by Haley that have attachments. *(8 marks)*

QUESTION PART REFERENCE

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6 The weight, X grams, of a dressed pheasant may be modelled by a normal random variable with a mean of 1000 and a standard deviation of 120.

Pairs of dressed pheasants are selected for packing into boxes. The total weight of a pair, $Y = X_1 + X_2$ grams, may be modelled by a normal distribution with a mean of 2000 and a standard deviation of 140.

(a) (i) Show that $\text{Cov}(X_1, X_2) = -4600$. *(3 marks)*

(ii) Given that $X_1 - X_2$ may be assumed to be normally distributed, determine the probability that the difference between the weights of a selected pair of dressed pheasants exceeds 250 grams. *(5 marks)*

(b) The weight of a box is independent of the total weight of a pair of dressed pheasants, and is normally distributed with a mean of 500 grams and a standard deviation of 40 grams.

Determine the probability that a box containing a pair of dressed pheasants weighs less than 2750 grams. *(5 marks)*

QUESTION PART REFERENCE

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7 (a) The random variable X has a Poisson distribution with $\mathrm{E}(X) = \lambda$.

(i) Prove, from first principles, that $\mathrm{E}(X(X-1)) = \lambda^2$. *(3 marks)*

(ii) Hence deduce that $\mathrm{Var}(X) = \mathrm{E}(X)$. *(2 marks)*

(b) The random variable Y has a Poisson distribution with $\mathrm{E}(Y) = 2.5$.

Given that $Z = 4Y + 30$:

(i) show that $\mathrm{Var}(Z) = \mathrm{E}(Z)$; *(3 marks)*

(ii) give a reason why the distribution of Z is **not** Poisson. *(1 mark)*

QUESTION PART REFERENCE

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8 The tensile strength of rope is measured in kilograms.

The standard deviation of the tensile strength of a particular design of 10 mm diameter rope is known to be 285 kilograms. A retail organisation, which buys such rope from two manufacturers, A and B, wishes to compare their ropes for mean tensile strength.

The mean tensile strength, $\bar{x}$, of a random sample of 80 lengths from manufacturer A was 3770 kilograms.

The mean tensile strength, $\bar{y}$, of a random sample of 120 lengths from manufacturer B was 3695 kilograms.

(a) (i) Test, at the 5% level of significance, the hypothesis that there is no difference between the mean tensile strength of rope from manufacturer A and that of rope from manufacturer B. *(6 marks)*

(ii) Why was it **not** necessary to know the distributions of tensile strength in order for your test in part **(a)(i)** to be valid? *(1 mark)*

(b) (i) Deduce that, for your test in part **(a)(i)**, the critical values of $(\bar{x} - \bar{y})$ are ± 80.63, correct to two decimal places. *(2 marks)*

(ii) In fact, the mean tensile strength of rope from manufacturer A exceeds that of rope from manufacturer B by 125 kilograms.

Determine the probability of a Type II error for a test of the hypothesis in part **(a)(i)** at the 5% level of significance, based upon a random sample of 80 lengths from manufacturer A and a random sample of 120 lengths from manufacturer B. *(4 marks)*

QUESTION PART REFERENCE

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END OF QUESTIONS