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General Certificate of Education (A-level)
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Mathematics MS03

(Specification 6360)

Statistics 3

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1 (a)	H_0: $p = 0.25$ (25%) H_1: $p > 0.25$	B1		Both
	SL $\alpha = 0.02$ (2%)			
	CV $z =$ **2.05 to 2.06**	B1		AWFW (2.0537) Allow 2.32 to 2.33 if H_1: $p \neq 0.25$
	$\hat{p} = \frac{108}{375} =$ **0.288**	B1		CAO
	$z = \frac{0.288 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{375}}} =$ **1.70**	M1 A1		Allow use of 0.288 in denominator AWRT
	or			$P(X \geq 108 \mid n = 375, p = 0.25) =$ **0.052**
	$z = \frac{108(-0.5) - 93.75}{\sqrt{375 \times 0.25 \times 0.75}} =$ **1.70 (or 1.64)**	(M1) (A1)		Allow use of 0.288 in denominator AWRT
	Thus, **no evidence**, at 2% level, **to support** consumer report's claim	AF1	6	F on CV and z-value **or** F on 2% and probability
(b)	Can be considered to be a **random sample**	B1	1	
		Total	**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2 (a)	98% $\Rightarrow$ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	CI for λ is: $\hat{\lambda} \pm z \times \sqrt{\hat{\lambda}}$ **or** $\bar{x} \pm z \times \sqrt{\frac{\bar{x}}{n}}$	M1		Form; allow $\hat{\lambda} \pm z \times \sqrt{\frac{\hat{\lambda}}{n}}$
	ie $108 \pm 2.3263 \times \sqrt{108}$	AF1		F on z only; allow $108 \pm z \times \sqrt{\frac{108}{13}}$
	or $\frac{108}{13} \pm 2.3263 \times \sqrt{\frac{108}{13^2}}$	(AF1)		F on z only; allow $\frac{108}{13} \pm z \times \sqrt{\frac{108}{13}}$
	Dividing by 13 or equivalent to obtain a **correct** numerical expression	A1		May be implied
	Thus **8.31 ± 1.86** **or** **(6.45, 10.2)**	A1	5	AWRT
	Note: For incorrect numerical expressions the maximum marks are B1 M1 AF1 A0 A0 (3)			
(b)	1 per 24 hours $\Rightarrow$ 7 per week			
	CI includes 7	BF1		F on (a); must use 7 **or** 1 v CI/7
	No reason, at 2% level, **to dispute** station officer's claim	Bdep1	2	Or equivalent Dependent on BF1
		Total	**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3 (a)(i)	P(G) = **0.15**	B1	1	CAO
(ii)	$P(A \cap \leq 1) = 0.60 \times 0.55 = \mathbf{0.33}$	B1	1	CAO
(iii)	$P(\leq 24) = (0.60 \times 0.80) + (0.25 \times 0.85) + (0.15 \times 0.75)$	M1		May be implied
	$= 0.48 + 0.2125 + 0.1125 = \mathbf{0.805}$	A1	2	CAO
(iv)	$P(B \mid \leq 24) = \dfrac{P(B \cap \leq 24)}{P(\leq 24)}$	M1		Used; may be implied
	$= \dfrac{0.25 \times 0.85}{(\text{iii})} = \dfrac{0.2125}{0.805}$	AF1		F on (iii)
	= **0.264**	A1	3	AWRT
(b)(i)	$P(3 \text{ @ } B \mid \leq 24) = [(a)(iv)]^3$	M1		Used; may be implied
	= **0.018 to 0.0185**	A1	2	AWFW (0.01839)
(ii)	$P(\text{same station} \mid \leq 24)$ $= [P(A \mid \leq 24)]^3 + (b)(i) + [P(G \mid \leq 24)]^3$	M1		Used; may be implied
	$= \left(\dfrac{0.48}{0.805}\right)^3 + (b)(i) + \left(\dfrac{0.1125}{0.805}\right)^3$	M1 M1		At least 1 term correct; allow (b)(i) providing it is a (cond prob)^3 All 3 terms correct
	$= 0.2120 + 0.0184 + 0.0027 = \mathbf{0.233}$	A1	4	AWRT (0.23312)
		Total	**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	95% $\Rightarrow$ z = **1.96**	B1		CAO (AWRT from calculator)
	Require $2 \times \frac{1.96\sigma}{\sqrt{n}} \le 0.2\mu$	M1		Used; may be implied Allow 'no 2 ×' Allow '= sign' throughout
	Thus $2 \times \frac{1.96}{\sqrt{n}} \times \frac{\mu}{2} \le 0.2\mu$	M1		Use of $\sigma = \frac{\mu}{2}$; may be implied Allow 'no 2 ×'
	Thus $\sqrt{n} \ge \frac{1.96}{2}$	M1		Attempt at solution for $\sqrt{n}$ or n
	Thus $n \ge 96.04$			
	Thus, to nearest 10; n = **100**	A1	5	CAO
		Total	**5**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5	E-mails are selected: **randomly** **independently**	 B1 B1		
	99% $\Rightarrow$ z = **2.57 to 2.58**	B1		AWFW (2.5758)
	$\hat{p}_G = \frac{72}{160}$ = **0.45** $\hat{p}_H = \frac{102}{250}$ = **0.408**	B1		CAO both; ignore notation
	Approximate CI for $p_G - p_H$ is:			
	$(\hat{p}_G - \hat{p}_H) \pm z \times \sqrt{\frac{\hat{p}_G(1-\hat{p}_G)}{n_G} + \frac{\hat{p}_H(1-\hat{p}_H)}{n_H}}$	M1 m1		Form used Standard deviation term
	Thus: $(0.45 - 0.408) \pm$ $1.96 \times \sqrt{\frac{0.45 \times 0.55}{160} + \frac{0.408 \times 0.592}{250}}$	AF1		Or equivalent F on $\hat{p}_G$, $\hat{p}_H$ and z
	Thus: **0.042 ± 0.129** **or** **(–0.09, 0.17)**	A1	8	CAO/AWRT **or** AWRT
	Note: If a pooled estimate of variance is used, then the maximum marks are B1 B1 B1 B1 M1 m0 AF0 A0 (5)			
		Total	**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\mathrm{V}(X_1 + X_2) = \mathrm{V}(X_1) + \mathrm{V}(X_2) + 2\mathrm{Cov}(X_1, X_2)$	M1		Used
	Thus: $140^2 = 120^2 + 120^2 + 2\mathrm{Cov}(X_1, X_2)$	A1		
	Thus $2\mathrm{Cov}(X_1, X_2) =$ 19600 – 14400 – 14400			
	= **–4600**	A1	3	CAO AG
(ii)	$\mathrm{E}(X_1 - X_2) = 1000 - 1000 = \mathbf{0}$	B1		CAO; may be implied
	$\mathrm{V}(X_1 - X_2) = 120^2 + 120^2 - (2 \times -4600)$ = **38000** **or** $\mathrm{Sd}(X_1 - X_2)$ = **194 to 195**	B1		CAO AWFW (194.936)
	P(Difference > 250) =			
	$\mathrm{P}(\lvert X_1 - X_2 \rvert > 250) = 2 \times \mathrm{P}\left(Z > \frac{250 - 0}{\sqrt{38000}}\right)$	M1		Standardising 250 using c's mean & c's standard deviation Allow 'no 2 ×'
	= 2 × P(Z > 1.28)			
	= 2 × [1 – (0.899 to 0.901)]	m1		Area change; allow 'no 2 ×'
	= **0.2(00)**	A1	5	AWRT (0.19968)
(b)	$Y + B$ has: Mean = **2500**	B1		CAO
	and Variance = $140^2 + 40^2$ = **21200** **or** Standard deviation = **145 to 146**	B1		CAO AWFW (145.602)
	$\mathrm{P}(Y + B < 2750) = \mathrm{P}\left(Z < \frac{2750 - 2500}{\sqrt{21200}}\right)$	M1		Standardising 2750 using c's mean & c's standard deviation
	= P(Z < **1.72**)	A1		AWRT; ignore inequality and sign
	= **0.957**	A1	5	AWRT (0.95701)
		Total	**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$X \sim \text{Po}(\lambda)$			
(a)(i)	$\text{E}(X(X-1)) = \sum_{x=0}^{\infty} x(x-1) \times \frac{e^{-\lambda}\lambda^x}{x!} =$	M1		Used; ignore limits until A1
	$\lambda^2 e^{-\lambda} \times \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} =$	M1		Factor of at least λ^2 Division of $x!$ by $x(x-1)$
	$\lambda^2 e^{-\lambda} \times e^{\lambda} = \boldsymbol{\lambda^2}$	A1	3	Fully correct convincing solution AG
(ii)	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$ $= \text{E}[X(X-1)] + \text{E}(X) - [\text{E}(X)]^2$	M1		Used (Other derivations are possible)
	$= \lambda^2 + \lambda - \lambda^2 = \boldsymbol{\lambda} = \textbf{E}(\boldsymbol{X})$	A1	2	CAO either AG
(b)(i)	$\text{E}(Z) = 4 \times 2.5 + 30 = \textbf{40}$	B1		CAO
	$\text{Var}(Z) = 4^2 \times 2.5$	M1		Use of $\text{V}(aX) = a^2\,\text{V}(X)$ Ignore '+30'
	$= \textbf{40}$ $\{= \text{E}(Z)\}$	A1	3	CAO AG
	Note: $4 \times 2.5 + 30 = 4^2 \times 2.5 \Rightarrow$ B1 M1 A0 plus value of 40 quoted $\Rightarrow$ B1 M1 A1			
(ii)	No values less than 30 are possible No odd values are possible Only even values are possible Only values of 30, 34, 38, etc, are possible	B1	1	Or equivalent
		Total	**9**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	H_0: $\mu_A = \mu_B$ H_1: $\mu_A \neq \mu_B$	B1		Both; allow suffices of 1 & 2 or X & Y
	SL $\alpha = 0.05$ (5%) CV $z = \pm$**1.96**	B1		CAO (AWRT from calculator) Allow (+)1.96
	$z = \dfrac{\bar{x}-\bar{y}}{\sigma \times \sqrt{\dfrac{1}{n_A}+\dfrac{1}{n_B}}}$	M1		Attempted use; or equivalent Allow σ_A and σ_B
	$z = \dfrac{3770-3695}{285\times\sqrt{\dfrac{1}{80}+\dfrac{1}{120}}}$	A1		
	= **1.82**	A1		AWRT (1.82321)
	No evidence, at 5% level, to suggest that **there is a difference**	AF1	6	F on CV and z-value
(ii)	**Large samples** (so CLT is applicable)	B1	1	
(b)(i)	$z = \dfrac{\bar{x}-\bar{y}}{285\times\sqrt{\dfrac{1}{80}+\dfrac{1}{120}}} = \pm 1.96$	M1		Equating z-term to 1.96
	Thus $(\bar{x}-\bar{y}) = \pm 1.96 \times 41.13616$ $= \pm$**80.63**	A1	2	Requires a convincing deduction AG
(ii)	P(Type II error) = P(accept H_0 | H_0 false) =	B1		Used or stated; may be implied
	$\mathrm{P}\left(-80.63 < (\bar{x}-\bar{y}) < 80.63 \mid \mu_A - \mu_B = 125\right)$ = $\mathrm{P}\left(Z < \dfrac{80.63-125}{285\times\sqrt{\dfrac{1}{80}+\dfrac{1}{120}}}\right)$	M1		Accept $(\bar{x}-\bar{y}) < 80.63$ –80.63 $\Rightarrow$ z = –5.00 $\Rightarrow$ probability $\approx$ 0
	= P(Z < –**1.08**)	A1		AWRT; ignore sign
	= **0.14**	A1	4	AWRT (0.14038)
		Total	**13**	
	TOTAL		**75**	