AQA Maths Statistics 3

Mark Scheme Pack

2006-2016

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2006 examination – June series

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

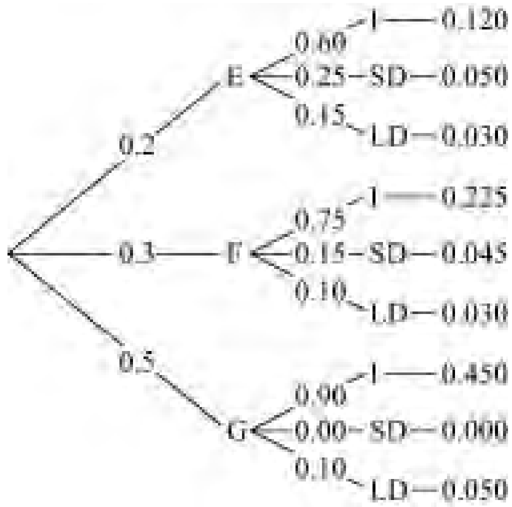
MS03

Q	Solution	Marks	Total	Comments
1(a)	$\hat{p} = \frac{209}{250} = 0.836$	B1		CAO
	95% CI ⇒ $z = 1.96$	B1		CAO
	CI for p:			
	$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1 M1		Variance term Use of: $\hat{p} \pm z \times \sqrt{(\text{Var}(\hat{p}))}$
	ie $0.836 \pm 1.96 \times \sqrt{\frac{0.836 \times 0.164}{250}}$	A1√		√ on $\hat{p}$ and z; not on n
	ie 0.836 ± 0.046			
	or (0.790, 0.882)	A1	6	AWRT; accept 0.79
(b)	Value of 0.8 (80%) **is within** CI	B1√ ↑ dep		√ on CI
	Council's clam **is supported** (at 5% level)	B1√	2	√ on CI
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$r = 0.819$ to 0.82	B3		AWFW
	or			
	$r = 0.81$ to 0.83	(B2)		AWFW
	or			
	$r = 0.8$ to 0.85	(B1)		AWFW
	Attempt at Σx Σx^2 Σy Σy^2 Σxy			989, 99321 1717, 296101 170956
	or attempt at S_{xx} S_{yy} S_{xy}	(M1)		1508.9, 1292.1, 1144.7
	Attempt at a correct formula for r	(m1)		
	$r = 0.819$ to 0.82	(A1)	3	AWFW
(b)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both
	SL $\alpha = 0.01$ (1%) SS $n = 10$			
	CV $r = 0.7155$	B1		AWFW 0.715 to 0.716
	Calculated r > Tabulated r	M1		Comparison
	Evidence (at 1% level) of a positive correlation between heart rate and systolic blood pressure	A1√	4	√ on r and CV
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3				
(a)(i)	$P(G \cap I) = 0.5 \times 0.9 = 0.45$	B1	1	CAO; or equivalent
(ii)	$P(I) = (i) + P(E \cap I) + P(F \cap I)$	M1		3 possibilities
	$= 0.45 + (0.2 \times 0.6) + (0.3 \times 0.75)$	A1		≥ 1 correct **new** term
	$= 0.45 + 0.12 + 0.225 = 0.795$	A1	3	CAO; or equivalent
(iii)	$P(G \mid I) = \dfrac{P(G \cap I)}{P(I)}$	M1		Attempted use of Bayes' Theorem
	$= \dfrac{(i)}{(ii)} = \dfrac{0.45}{0.795} = 0.566$	m1 A1	3	AWRT; or equivalent
(b)	$P(E \mid SD) = \dfrac{P(E \cap SD)}{P(SD)}$	M1		Correct use of Bayes' Theorem
	$= \dfrac{0.2 \times 0.25}{(0.2 \times 0.25) + (0.3 \times 0.15)} = \dfrac{0.05}{0.05 + 0.045}$	A1 A1		Numerator (B1 if no Bayes' Theorem) Denominator (B1 if no Bayes' Theorem)
	$= \dfrac{0.05}{0.095} = 0.526$	A1	4	AWRT; or equivalent
	Total		**11**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$E(R) = (6 \times 0.1) + (7 \times 0.6) + (8 \times 0.3)$			
	$= 0.6 + 4.2 + 2.4 = 7.2$	B1		CAO
	$E(R^2) = (3.6 + 29.4 + 19.2) = 52.2$	B1		CAO
	$\text{Var}(R) = E(R^2) - (E(R))^2$	M1		Use of
	$= 52.2 - 51.84 = 0.36$	A1	4	CAO
(b)(i)	$E(T) = 7.2 + 10.9 = 18.1$	B1√		√ on E(*R*)
	$\text{Cov}(R, S) = \rho_{RS} \times \sqrt{\text{Var}(R) \times \text{Var}(S)}$	M1		Use of; or equivalent May be scored in (ii)
	$\text{Var}(T) = \text{Var}(R) + \text{Var}(S) + 2\text{Cov}(R, S)$	M1		Use of; or equivalent May be scored in (ii)
	$= 0.36 + 1.69 + 2 \times \frac{2}{3}\sqrt{0.36 \times 1.69}$			
	$= 0.36 + 1.69 + 1.04 = 3.09$	A1	4	CAO
(ii)	$E(D) = 10.9 - 7.2 = 3.7$	B1√		√ on E(*R*)
	$\text{Var}(D) = \text{Var}(S) + \text{Var}(R) - 2\text{Cov}(S, R)$			
	$= 1.69 + 0.36 - 2 \times \frac{2}{3}\sqrt{1.69 \times 0.36}$			
	$= 1.69 + 0.36 - 1.04 = 1.01$	B1	2	CAO
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5	Letters/week ~ Po(12.25)			
(a)	Letters/4-week ~ N(49, 49)	B1		CAO; mean = variance = 49
	$P(42 \le X_P \le 54) = P(41.5 < X_N < 54.5)$	M1		Use of ±0.5
	$= P\left(\frac{41.5-49}{7} < Z < \frac{54.5-49}{7}\right)$	M1		Standardising (41.5, 42 or 42.5) or (53.5, 54 or 54.5) with C's μ and $\sqrt{\mu}$
	$= P(-1.07 < Z < 0.79)$			
	$= \Phi(0.79) - (1 - \Phi(1.07))$	m1		Area change
	= 0.78524 – 1 + 0.85769			
	= 0.641 to 0.644	A1	5	AWFW
(b)(i)	98% CI ⇒ z = 2.3263	B1		AWFW 2.32 to 2.33
	CI for λ/16-week: $\hat{\lambda} \pm z\sqrt{\hat{\lambda}}$	M1		Use of expression
	ie $248 \pm 2.3263 \times \sqrt{248}$			
	or $15.5 \pm 2.3263 \times \sqrt{\frac{15.5}{16}}$	A1√		√ on z
	ie 248 ± 36.6 or 15.5 ± 2.3	M1		Division by 16 somewhere
	or (13.2, 17.8)	A1	5	AWRT
(ii)	Value of 12.25 (196) **is below** CI	B1√		√ on CI; must use 12.25 (196)
		↑ dep		
	Rosa's belief **is supported**	B1√		√ on CI
	Total		**12**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\text{E}(X) = \sum x \times \text{P}(X = x)$	M1		Use of
	$= \sum_{x=0}^{\infty} x \times \frac{\text{e}^{-\lambda}\lambda^x}{x!} = \lambda \times \sum_{x=1}^{\infty} \frac{\text{e}^{-\lambda}\lambda^{x-1}}{(x-1)!}$	M1		Factor of λ Cancelling of x (Ignore change in limits)
	$= \lambda \times \sum \text{P}(X = x) = \lambda \times 1 = \lambda$	M1		AG; must be clear
	$\text{G}(t) = \text{e}^{\lambda t - \lambda}$ or $\text{M}(t) = \text{e}^{\lambda \text{e}^t - \lambda}$	(B1)		Either CAO
	Alternative $\text{E}(X) = \left.\frac{\text{d}\,\text{G}(t)}{\text{d}t}\right\|_1$ or $\left.\frac{\text{d}\,\text{M}(t)}{\text{d}t}\right\|_0$	(M1)		Use of either
	$\left[\lambda \text{e}^{\lambda t - \lambda}\right]_1$ or $\left[\lambda \text{e}^t \text{e}^{\lambda \text{e}^t - \lambda}\right]_0 = \lambda$	(A1)	3	AG; correct derivation
(b)	$\text{E}(X(X-1)) = \sum_{x=0}^{\infty} x(x-1) \times \frac{\text{e}^{-\lambda}\lambda^x}{x!}$	M1		Use of
	$= \lambda^2 \times \sum_{x=2}^{\infty} \frac{\text{e}^{-\lambda}\lambda^{x-2}}{(x-2)!}$	M1		Factor of λ^2 Cancelling of $x(x-1)$ (Ignore change in limits)
	$= \lambda^2 \times \sum \text{P}(X = x) = \lambda^2 \times 1 = \lambda^2$	M1		AG; must justify
	$\text{Var}(X) = \text{E}(X^2) - (\text{E}(X))^2$ $= \text{E}(X(X-1)) + \text{E}(X) - (\text{E}(X))^2$	M1		
	$= \lambda^2 + \lambda - \lambda^2 = \lambda$	A1		AG; must be clear
	Alternative $\text{Var}(X) =$ $\left.\frac{\text{d}^2\,\text{G}(t)}{\text{d}^2 t}\right\|_1 + \lambda - \lambda^2$ or $\left.\frac{\text{d}^2\,\text{M}(t)}{\text{d}^2 t}\right\|_0 - \lambda^2$	(M2)		use of either
	$= \left[\lambda^2 \text{e}^{\lambda t - \lambda}\right]_1 + \lambda - \lambda^2 = \lambda$	(A2)		AG; correct derivation
	or $= \left[\lambda \text{e}^t \text{e}^{\lambda \text{e}^t - \lambda} + \lambda^2 \text{e}^{2t} \text{e}^{\lambda \text{e}^t - \lambda}\right]_0 - \lambda^2 = \lambda$	(A1)	5	AG; correct derivation
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\bar{y} = 1193$	B1	1	CAO
(b)	H_0: $\mu_Y - \mu_X = 200$	B1		200 is not necessary
	H_1: $\mu_Y - \mu_X > 200$	B1		200 is necessary
	SL $\alpha = 0.01$ (1%)			
	CV $z = 2.3263$	B1		AWFW 2.32 to 2.33
	$z = \dfrac{(\bar{y}-\bar{x})-200}{\sqrt{\dfrac{\sigma_Y^2}{n_Y}+\dfrac{\sigma_X^2}{n_X}}} = \dfrac{(1193-936)-200}{\sqrt{\dfrac{65^2}{10}+\dfrac{45^2}{20}}}$	M1 M1 A1√		Numerator; 200 is not necessary Denominator √ on (a)
	= 2.48 to 2.5	A1		AWFW
	Evidence (at 1% level) to support the claim	A1√	8	√ on z and CV
(c)(i)	$\text{CV}(\bar{y}-\bar{x})$: $200 + z(\text{denominator in (b)})$	M1		May be scored in (b)
	ie $200 + 2.3263 \times \sqrt{523.75}$			
	(= 253.24)	A1	2	AG; must justify
(ii)	Power = 1 – P(Type II error)	M1		Use of
	= 1 – P(accept H_0 | H_0 false)	M1		Use of; or equivalent
	$= 1 - P\left(Z < \dfrac{253.24-275}{\sqrt{523.75}}\right)$	M1		Standardising 253.24 using 275 and C's denominator in (b)
	= 1 – Φ(–0.95) = Φ(0.95)	m1		Area change
	= 0.83	A1	5	AWRT
(iii)	Probability of **accepting** that difference in mean weights **is more than 200** grams	B1		Not in context ⇒ max of 2
	when, in fact, **it is 275** grams	B1		
	is 0.83 (or 83%)	B1√	3	√ on (ii)
	Total		**19**	
	TOTAL		**75**	

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MS03

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1(a)	Samples are independent or random	B1		
	98% $\Rightarrow$ $z = 2.3263$	B1		AWFW 2.32 to 2.33
	CI for $\mu_1 - \mu_2$ is:			
	$(\bar{x}_S - \bar{x}_A) \pm z \times \sqrt{\frac{s_S^2}{n_S} + \frac{s_A}{n_A}}$	M1 A1		Form Allow: sigmas, *A&B* or 1&2 and $n - 1$ Correct
	$(19268 - 17896)$ $\pm 2.3263 \times \sqrt{\frac{7321^2}{175} + \frac{8205^2}{225}}$	A1√		√ on z only $s_P = 7830$ to 7850
	ie $1372 \pm (1805$ to $1820)$ or (–450 to –430, 3170 to 3200)	A1	6	$1372 \pm (1830$ to $1845)$ AWFW
(b)	Confidence interval includes zero so (at 5% level) Mean starting salaries may be equal	B1√ ↑dep↑ B1√	 2	√ on CI; OE √ on CI; OE
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)	P(≥ 18 | Road) = 0.85	B1	1	CAO; OE; **not** 85
(b)	P(18 to 64) = P(Route) × P(18 to 64 | Route) =	M1		Use of 3 possibilities, each the product of 2 probabilities
	(0.25 × 0.80)+(0.60 × 0.35)+(0.55 × 0.40)	A1		At least 1 term correct
	= 0.20 + 0.21 + 0.22 = 0.63	A1	3	CAO; OE
(c)	P(FR ∩ >64) = P(FR) × P(>64 | FR)			
	= 0.35 × 0.15	B1		Correct expression
	= 0.052 to 0.053	B1	2	AWFW (0.0525)
(d)	P(FR | >64) = $\dfrac{(c)}{P(>64)}$ =	M1 M1		$\dfrac{\text{answer}(c)}{\sum(3\times2)\text{ probabilities}}$
	$\dfrac{0.0525}{(0.25\times0.05)+(0.35\times0.15)+(0.40\times0.35)}$	A1		At least 2 terms correct
	$= \dfrac{0.0525}{0.0125+0.0525+0.1400} = \dfrac{0.0525}{0.205}$	A1		CAO
	$= 0.256$ or $\dfrac{21}{82}$	A1	5	AWRT/CAO; OE
	Total		**11**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$H_0 : p_K = p_S$ $H_1 : p_K \neq p_S$	B1		Both; OE; allow A&B or 1&2
	SL $\alpha = 0.05$ CV $\lvert z \rvert = 1.96$	B1		CAO
	$\hat{p} = \dfrac{(150\times0.28)+(250\times0.34)}{400}$	M1		Used
	$= \dfrac{127}{400}$ or 0.317 to 0.318	A1		CAO/AWFW (0.3175)
	$z = \dfrac{(\hat{p}_K - \hat{p}_S) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_K}+\dfrac{1}{n_S}\right)}}$	M1		Used; accept unpooled denominator
	$\lvert z \rvert = \dfrac{\lvert 0.28-0.34 \rvert}{\sqrt{0.3175\times0.6825\left(\dfrac{1}{150}+\dfrac{1}{250}\right)}}$	A1√		√ on $\hat{p}$; accept no pooling
	$= \lvert 1.24 \rvert$ to $\lvert 1.25 \rvert$	A1		AWFW; $\lvert 1.26 \rvert$ to $\lvert 1.27 \rvert$
	Thus accept H_0 as $\lvert z \rvert < 1.96$	A1√		√ on z and CV with same sign
	Thus no evidence, at 5% level, of a difference between two proportions of male customers in two salons	E1√	9	√ on z and CV with same sign In context and qualified
(b)	Zero since	B1		CAO
	Cannot make a Type I error when H_0 is false	B1	2	OE
	Total		**11**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	98% $\Rightarrow$ $z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI width is $2 \times \frac{z\sigma}{\sqrt{n}}$	M1		Used; allow $\frac{z\sigma}{\sqrt{n}}$
	Thus $2 \times \frac{2.5758 \times 0.08}{\sqrt{n}} = 0.05$	A1√		OE; √ on z; allow no '2 ×'
	Thus $\sqrt{n} = 8.24256$	m1		Solving for $\sqrt{n}$ or n
	Thus $n = 67.9 \Rightarrow 68$	A1√		AWRT; √ on z
	Thus, to nearest 5, $n = 70$	A1	6	CAO
	Total		**6**	
5	$D = \sum^{3} X_i - \sum^{2} Y_i$ or $D' = \sum^{2} Y_i - \sum^{3} X_i$	M1		Used or implied
	have means $\mu = 162 - 166 = -4$ $\mu = 166 - 162 = +4$	B1		CAO either
	and variance $\sigma^2 = (3 \times 2^2) + (2 \times 3^2) = 12 + 18$ $= 30$	M1 A1		Use of [a × Var(Z)]; implied CAO
	$P\left(\sum^{3} X_i < \sum^{2} Y_i\right) =$ $P(D < 0)$ or $P(D' > 0) =$	M1		Used or implied
	$P\left(Z > \frac{0-(-4)}{\sqrt{30}}\right)$ or $P\left(Z > \frac{0-(+4)}{\sqrt{30}}\right) =$	m1		Standardising 0 using μ and $\sqrt{\sigma^2}$
	$P(Z < +0.73)$ or $P(Z > -0.73) =$			
	0.767 to 0.768	A1	7	AWFW
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{E}(X) = \sum_{x=0}^{n} x \times \binom{n}{x} p^x (1-p)^{n-x}$	M1		Use of $\sum x \times \mathrm{P}(X=x)$
	$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$	M1		Expansion of ${}^n\mathrm{C}_x$; cancelling of x (Ignore limits)
	$= np \times \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$	M1		Factors of n and p (Ignore limits)
	$= np \times \sum \mathrm{P}(X=x) \mid \mathrm{B}(n-1,\ p) = np$	M1	4	AG; must be convincing
(ii)	$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$	M1		Used
	$= [\mathrm{E}(X^2) - \mathrm{E}(X)] + \mathrm{E}(X) - (\mathrm{E}(X))^2$ $= n(n-1)p^2 + np - n^2p^2$	m1		Attempted
	$= np(1-p)$	A1	3	AG; must be convincing
(iii)	Thus $np(1-p) = 3(1-p) = 2.97$	M1		Substituting μ in σ^2
	Thus $1-p = \frac{2.97}{3} = 0.99$			
	Thus $p = 0.01$ and $n = 300$	A1 A1	3	CAO CAO
(iv)	$\mathrm{B}(300,\ 0.01) \sim \mathrm{Po}(3)$	B1		CAO; PI
	$\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \le 2)$	M1		Must be applied to Poisson
	$= 1 - 0.4232 = 0.577$	A1	3	AWRT

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)			13	
(b)	$Y \sim \text{B}(500,\ 0.45)$ or $Y \sim$ (normal) with mean $\mu = 225$	B1		PI
	and variance $\sigma^2 = 123.75$ or standard deviation $\sigma = 11.124$	B1		AWFW 123 to 124 AWFW 11.05 to 11.15
	(At least) half $\Rightarrow$ $(\geq)$ 250	B1		CAO
	$\text{P}(Y_\text{B} \geq 250) = \text{P}(Y_\text{N} > 249.5) =$	B1		CAO
	$\text{P}\left(Z > \dfrac{249.5 - 225}{\sqrt{123.75}}\right) =$	M1		Standardising 249.5, 250 or 250.5 with c's μ and $\sqrt{\sigma^2}$
	$\text{P}(Z > 2.20) = 1 - \text{P}(Z < 2.20)$	m1		Area change
	$= 0.0138$ to 0.014	A1	7	
	Note: Use of $\dfrac{0.5 - 0.45}{\sqrt{0.000495}}$ $\Rightarrow$ max of 5 marks Use of $\dfrac{0.499 - 0.45}{\sqrt{0.000495}}$ $\Rightarrow$ max of 7 marks			Use of distribution of $\hat{p}$ Use of distribution of $\hat{p}$ with continuity correction
	Total		**20**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$H_0 : \lambda = 13$ $H_1 : \lambda < 13$	B1 B1		CAO; OE CAO; OE
	$P(R \le 10 \mid Po(13))$	M1		Used or implied
	$= 0.2517$	A1		AWFW 0.251 to 0.252
	Prob of $0.2517 > 0.10$ (10%) $z = -0.83$ to $-0.70 > -1.28$	M1		Comparison of prob with 0.10 Comparison of z with -1.28
	Thus no evidence, at 10% level, of a reduction in the mean value of R	A1√	6	√ on probability or z In 'context' and qualified
(b)	Require $P(R \le r \mid Po(13)) \approx 0.10$	M1		Stated or implied
	Critical Region is $R \le 8$ or $R < 9$	A1	2	Accept $R = 8$ May be scored in (a)
(c)	Require P(accept H_0 | H_0 false)	B1		OE; PI
	$= P(R > 8 \mid Po(6.5))$	M1		Use of Po(6.5)
	$= 1 - P(R \le 8 \mid Po(6.5))$	m1		
	$= 1 - 0.7916$			
	$= 0.208$ to 0.209	A1	4	AWFW (0.2084)
	Total		**12**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2008 examination - June series

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Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1 **(a)**	$r = \dfrac{1.3781}{\sqrt{7.0036 \times 0.8464}} =$	M1		Used
	0.56 to 0.57	A1	2	AWFW (0.56602)
(b)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both
	SL $\alpha = 0.01$ (1%) CV $r =$ **0.515 to 0.516**	B1		AWFW (0.5155)
	Calculated r > Tabulated r	M1		Comparison
	Evidence, at 1% level, of a positive correlation between x and y	A1√	4	ft on r and CV
	Special Case for part (b)			
	CV $t_{n-2}(0.99)$ 2.552	(B1)		
	$r\sqrt{\dfrac{n-2}{1-r^2}} = 2.913$	(B1)		
(c)	(Strong) evidence of a positive correlation between best performances of junior athletes in the long jump and in the high jump	B1√	1	ft on **(b)**; or equivalent
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2 (a)	$\hat{p} = \frac{132}{200}$ = **0.66**	B1		CAO; or equivalent
	98% ⇒ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	CI for p: $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Variance term
		M1		CI expression used
	ie $0.66 \pm 2.3263 \times \sqrt{\frac{0.66 \times 0.34}{200}}$	A1√		ft on $\hat{p}$ and z
	ie **0.66 ± 0.08** or **(0.58, 0.74)**	A1	6	AWRT; or equivalent
(b)	Value of 0.6 (60%) is **within** CI	B1√		ft on **(a)**
	Reason to **doubt** claim of more than 60%	B1√	2	dependent on previous B1 ft on **(a)**; or equivalent
	Total		**8**	
3	$H_0 : \mu_X = \mu_Y$ $H_1 : \mu_X \neq \mu_Y$	B1		Both
	SL α = 0.01(1%) CV z = (±) **2.57 to 2.58**	B1		AWFW (2.5758)
	$z = \frac{\lvert 157 - 162 \rvert}{\sqrt{\frac{4.5^2}{10} + \frac{5.7^2}{15}}}$ =	M1		Numerator
		M1		Denominator
	(±) **2.44 to 2.445**	A1		AWFW (2.4424)
	No evidence, at 1% level, to reject hypothesis that $\mu_X = \mu_Y$	A1√	6	ft on z, CV and signs; or equivalent
	Total		**6**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4 **(a)**	B(0.25) — E(0.30) 0.0750 B(0.25) — M(0.50) 0.1250 B(0.25) — W(0.20) 0.0500	B1		B, S & D with 3 probabilities
	S(0.60) — E(0.40) 0.0240 S(0.60) — M(0.45) 0.2700 S(0.60) — W(0.15) 0.0900	B2	3	$3 \times$ (E, M & W) each with 3 probabilities
	D(0.15) — E(0.55) 0.0825 D(0.15) — M(0.35) 0.0525 D(0.15) — W(0.10) 0.0150	(B1)		$\geq 1 \times$ (E, M & W) (each) with 3 probabilities
(b)(i)	P(E) = $(0.25 \times 0.3) + (0.6 \times 0.4) + (0.15 \times 0.55)$ = 0.075 + 0.24 + 0.0825 =	M1		≥ 1 term correct
	0.397 to 0.398 or 159/400	A1	2	AWFW/CAO (0.3975)
(ii)	$P(D \mid E) = \dfrac{0.0825}{(b)(i)} =$	M1		Or equivalent
	0.207 to 0.208 or 11/53	A1	2	AWFW/CAO (0.2075)
(c)	$X \sim B(10,\ (b)(ii))$	M1		Used
	$P(X = 4) = \binom{10}{4}(0.2075)^4(0.7925)^6 =$	A1√		ft on **(b)(ii)**
	0.0955 to 0.0975	A1	3	AWFW (0.09645)
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5 (a)	$\hat{\lambda}_A = \frac{3312}{184}$ = **18** $\hat{\lambda}_B = \frac{2760}{184}$ = **15**	B1		CAO both
	95% $\Rightarrow$ z = **1.96**	B1		CAO
	CI for $(\lambda_A - \lambda_B)$:	M1		Variance term
	$(\hat{\lambda}_A - \hat{\lambda}_B) \pm z\sqrt{\frac{\hat{\lambda}_A}{n_A} + \frac{\hat{\lambda}_B}{n_B}}$	M1		CI expression used
	ie $(18-15) \pm 1.96 \times \sqrt{\frac{18}{184} + \frac{15}{184}}$	A1√		ft on $\hat{\lambda}_A$, $\hat{\lambda}_B$ and z
	ie **3 ± 0.83** or **(2.17, 3.83)**	A1	6	AWRT
(b)	Calls from A and B are independent	B1	1	Or equivalent
(a)	*Alternative Solution*			
	$(3312-2760) \pm 1.96 \times \sqrt{3312+2760}$ =	(M2) (B1)		1.96
	ie 552 ± 152.73	(A1)		
	Dividing by 184	(M1)		
	ie **3 ± 0.83** or **(2.17, 3.83)**	(A1)		AWRT
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6 **(a)(i)**	$E(F) = 128 + 112 = \mathbf{240}$	B1		CAO
(ii)	$\text{Cov}(X, Y) = -0.4 \times \sqrt{50 \times 50} = \mathbf{-20}$	M1		Used; or equivalent
	$\text{Var}(F) = 50 + 50 + (2 \times -20) = \mathbf{60}$	M1 A1	4	$V(X) + V(Y) + 2\text{Cov}(X,Y)$ used CAO; **AG**
(b)(i)	$E(T) = 240 + 75 = \mathbf{315}$	B1√		ft on **(a)(i)**
	$\text{Var}(T) = 60 + 36 = \mathbf{96}$	B1	2	CAO
(ii)	$E(M) = 240 - (3 \times 75) = \mathbf{15}$	B1√		ft on **(a)(i)**
	$\text{Var}(M) = 60 + \{(-3^2) \times 36\}$ $= 60 + 324 = \mathbf{384}$	M1 A1	3	$V(F) + 3^2V(S)$ used CAO
(c)(i)	$P(T > 300) = P\left(Z > \dfrac{300-315}{\sqrt{96}}\right)$	M1		Standardising 300 or 300.5 using **(b)(i)**
	$= P(Z > -1.53) = P(Z < 1.53)$	m1		Area change
	$= \mathbf{0.936 \text{ to } 0.938}$	A1	3	AWFW
(ii)	$P\left(S > \dfrac{X+Y}{3}\right) =$	M1		Used; or equivalent
	$P(3S > X + Y) = P(3S > F) =$	M1		Attempt to change to M
	$P(F - 3S < 0) = P(M < 0)$	A1		Or equivalent
	$= P\left(Z < \dfrac{0-15}{\sqrt{384}}\right)$	M1		Standardising 0 using **(b)(ii)**
	$= P(Z < -0.765) = 1 - P(Z < 0.765)$	m1		Area change
	$= \mathbf{0.22(0) \text{ to } 0.225}$	A1	6	
	Total		**18**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\mathrm{E}\big(X(X-1)\big)=\sum_{x=0}^{\infty}x(x-1)\times\frac{\mathrm{e}^{-\lambda}\lambda^x}{x!}$ =	M1		$\sum x(x-1)\times \mathrm{P}(X=x)$ used Ignore limits until A1
	$\sum_{x=2}^{\infty}\frac{\mathrm{e}^{-\lambda}\lambda^x}{(x-2)!}$ =	M1		$\frac{x(x-1)}{x!}=\frac{1}{(x-2)!}$ used
	$\lambda^2\mathrm{e}^{-\lambda}\sum_{x=2}^{\infty}\frac{\lambda^{x-2}}{(x-2)!}$ =	M1		Factor of $\lambda^2\mathrm{e}^{-\lambda}$ used
	$(\lambda^2\mathrm{e}^{-\lambda})\times(\mathrm{e}^{\lambda})$ = $\boldsymbol{\lambda^2}$	A1	4	Fully correct derivation; **AG**
(ii)	Var(X) = E(X(X – 1)) + E(X) – (E(X))^2	M1		Used
	= $\lambda^2+\lambda-\lambda^2$ = $\boldsymbol{\lambda}$	A1	2	Fully correct derivation; **AG**
(b)(i)	E(D) = 5 – 2 = **3** Var(D) = 5 + 2 = **7**	B1 B1	2	CAO CAO
(ii)	E(F) = (2×5) + 10 = **20** Var(F) = $2^2\times5$ = **20**	B1 M1 A1	3	CAO $2^2\mathrm{V}(X_1)$ + 0 CAO
(iii)	D: Mean ≠ Variance	B1		Negative values possible
	F: Values < 10 impossible Odd values impossible	B1	2	$2X_1 = X_1+X_1$ is not sum of independent Po variables
(c)	B ~ Po(5) C ~ Po(2)			
	T = 24 × (5 + 2) ~ Po(**168**)	B1		CAO
	T ~ approx N(168, 168)	M1		Normal with $\mu = \sigma^2$
	$\mathrm{P}(T_{\mathrm{Po}}>175)\approx\mathrm{P}(T_{\mathrm{N}}>175.5)$	B1		175.5
	= $\mathrm{P}\left(Z>\frac{175.5-168}{\sqrt{168}}\right)$ = $\mathrm{P}(Z>0.58)$ =	M1		Standardising 174.5, 175 or 175.5 with $\mu = \sigma^2$
	$1-\mathrm{P}(Z<0.58)$ = **0.28(0) to 0.283**	m1 A1	6	Area change AWFW
	Total		**19**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2009 examination - June series

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Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1(a)	$\hat{p}_1 = \frac{102}{150} = 0.68$ $\hat{p}_2 = \frac{36}{80} = 0.45$	B1		Both CAO
	99% (0.99) $\Rightarrow$ z = 2.57 to 2.58	B1		AWFW (2.5758)
	CI for $(p_1 - p_2)$ is $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	M1 m1		Use of $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\text{attempted variance}}$ Use of correct expression for variance
	Thus $(0.68 - 0.45) \pm 2.5758 \times \sqrt{\frac{0.68 \times 0.32}{150} + \frac{0.45 \times 0.55}{80}}$	A1F		F on $\hat{p}_1$, $\hat{p}_2$ and z
	Hence 0.23 ± (0.173 to 0.174) **or** (0.056 to 0.057, 0.403 to 0.404)	A1	6	CAO & AWFW (accept 0.17) AWFW (accept 0.06 & 0.4) **Note:** Pooling of variances Maximum of B1 B1 M1
(b)	Whole of confidence interval is **above 0 or zero** so	B1F		F on (a) Or equivalent
	Disagree with claim / claim appears **doubtful**	B1F	2	F on (a) Or equivalent Dependent on previous B1F
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	P(B & B) = $(0.30 \times 0.80) + (0.55 \times 0.10) + (0.15 \times 0.30)$	M1		Use of **3 possibilities** each the product of **2 probabilities**
	$= 0.24 + 0.055 + 0.045 = 0.34$	A1	2	CAO; **AG**
(ii)	P(HB $\cap$ Coastal) = 0.55×0.65	M1		Can be implied by **correct** answer
	= 143/400 or 0.357 to 0.358	A1	2	CAO/AWFW (0.3575)
(iii)	P(Coastal | HB) = $\dfrac{P(\text{Coastal} \cap \text{HB})}{P(\text{HB})}$	M1 M1		$\dfrac{\text{answer to (ii)}}{\sum(3 \times 2)\text{ probabilities}}$
	$= \dfrac{0.3575}{(0.3 \times 0.15) + (0.3575) + (0.15 \times 0.5)}$	A1F		F on (ii)
	$= \dfrac{0.3575}{0.4775}$ = 143/191 or 0.747 to 0.75	A1	4	CAO/AWFW (0.74869)
(b)	P(City | HB) = $\dfrac{0.3 \times 0.15}{P(\text{HB})} = \dfrac{0.045}{0.4775} = \dfrac{90}{955}$	M1		
	P(Country | HB) = $\dfrac{0.15 \times 0.5}{P(\text{HB})} = \dfrac{0.075}{0.4775} = \dfrac{30}{191}$	M1		Or $\left(1 - (a)(iii) - \dfrac{0.045}{0.4775}\right)$
	Thus Probability = $\dfrac{0.045}{P(\text{HB})} \times \dfrac{0.3575}{P(\text{HB})} \times \dfrac{0.075}{P(\text{HB})}$	M1		Multiplication of 3 different probabilities
	Multiplied by $3! = 6$	B1		CAO
	$= 0.09424 \times 0.74869 \times 0.15707 \times 6$			
	= 0.063 to 0.068	A1	5	AWFW (0.06649)
	Total		**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3	98% (0.98) CI $\Rightarrow$ $z = 2.32$ to 2.33	B1		AWFW (2.3263)
	CI width is $2 \times z \times \sqrt{\frac{p(1-p)}{n}}$	M1		Used; allow $z \times \sqrt{\frac{p(1-p)}{n}}$
	$p = 0.35$ or 0.50	B1		
	Thus $2 \times 2.3263 \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.1$	A1F		Or equivalent F on z; allow no multiplier of 2 and/or $p = 0.50$
	Thus $\sqrt{n} = \frac{2 \times 2.3263}{0.1} \times \sqrt{0.35 \times 0.65}$	m1		Solving for $\sqrt{n}$ or n
	Thus $n = 492.5$ $(p = 0.35)$ or $n = 541.2$ $(p = 0.50)$			
	Thus to nearest 10 $n = 500$ or 490	A1	6	Either
	Notes: No '$\times 2$' gives $n = 123.1$ No '$\times 2$' and $p = 0.50$ gives $n = 135.3$			
	Total		**6**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	$H_0: \mu_X - \mu_Y = 15$	B1		Or equivalent Accept $H_0: \mu_X - \mu_Y = 0$
	$H_1: \mu_X - \mu_Y > 15$	B1		Or equivalent
	SL α = 1% (0.01)			
	CV z = 2.32 to 2.33	B1		AWFW (2.3263) If H_1 involves '≠' then accept 2.57 to 2.58 (2.5758)
	CV t = 2.35 to 2.36	(B1)		AWFW If H_1 involves '≠' then accept 2.60 to 2.62
	$z = \dfrac{(\bar{x}-\bar{y})-15}{\sqrt{\dfrac{s_X^2}{n_X}+\dfrac{s_Y^2}{n_Y}}}$ **or** $z/t = \dfrac{(\bar{x}-\bar{y})-15}{\sqrt{s_P^2\left(\dfrac{1}{n_X}+\dfrac{1}{n_Y}\right)}}$	M1		Used Allow 'no –15'
	$s_P^2 = \dfrac{(64\times3.4^2)+(74\times2.8^2)}{65+75-2}$ $= \dfrac{1320}{138} = 9.56522$			$s_P = 3.09277$
	$z = \dfrac{(40.7-24.4)-15}{\sqrt{\dfrac{3.4^2}{65}+\dfrac{2.8^2}{75}}} = \dfrac{1.3}{\sqrt{0.28238}}$	A1 A1		Numerator; allow 'no –15' Denominator
	= 2.44 to 2.45	A1		AWFW (2.4464) 'no –15' gives z = 30.674
	OR			
	$z/t = \dfrac{(40.7-24.4)-15}{\sqrt{\dfrac{1320}{138}\left(\dfrac{1}{65}+\dfrac{1}{75}\right)}} = \dfrac{1.3}{\sqrt{0.27469}}$	(A1) (A1)		Numerator; allow 'no –15' Denominator
	= 2.48	(A1)		AWRT (2.4804) 'no –15' gives z = 31.100
	Thus evidence, at 1% level, to support Holly's belief	A1F	8	F on z and CV
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5	$X \sim B(n,\ p)$			
(a)	$\text{Var}(X) = E(X^2) - [E(X)]^2$	M1		Used; may be implied
	$= E[X(X-1)] + E(X) - [E(X)]^2$ $= n(n-1)p^2 + np - n^2p^2$	M1		Rearranging & substitution
	$= np - np^2 = np(1-p)$	A1		Or equivalent
	OR			
	$E[X(X-1)] = E(X^2) - E(X)$ $= n(n-1)p^2 = n^2p^2 - np^2$	(M1)		Expansion & substitution
	$\text{Var}(X) = E(X^2) - [E(X)]^2$	(M1)		Used; may be implied
	$= \{n^2p^2 - np^2 + E(X)\} - n^2p^2$			
	$= np - np^2 = np(1-p)$	(A1)	3	Or equivalent
(b)(i)	Mean $= np = 36$ SD $= \sqrt{np(1-p)} = 4.8$	B1		Both CAO
	Thus $36(1-p) = 4.8^2$	M1		Attempt to solve for p or n
	Thus $n = 100$ & $p = 0.36$	A1	3	Both CAO
(ii)	$P(30 < X < 40) =$			
	$P\left(Z < \dfrac{39.5-36}{4.8}\right) - P\left(Z < \dfrac{30.5-36}{4.8}\right) =$	M1 B1		Standardising (39.5, 40 or 40.5) or (29.5, 30 or 30.5) with 36 and 4.8 and/or $(36-x)$ Use of 39.5 & 30.5
	$P(Z < 0.73) - P(Z < -1.15) =$			
	$P(Z < 0.73) - [1 - P(Z < 1.15)] =$	m1		Area change
	$0.76730 - [1 - (0.87286 \text{ to } 0.87493)] =$			
	0.64 to 0.643	A1	4	AWFW (0.64112)
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$E(X) = \underline{\mathbf{2.2}}$	B1		CAO
	$Var(X) = E(X^2) - 2.2^2 =$	M1		Used; or equivalent
	$6.8 - 4.84 = 1.96$	A1	3	CAO
(b)(i)	$E(S) = E(X) + 2.0 = 4.2$	B1F		F on (a)
	$Var(S) = Var(X) + 1.5 + 2 \times (-0.43)$	M1		Used for S or D
	$= 2.6$	A1F		F on (a)
(ii)	$E(D) = E(X) - 2.0 = 0.2$	B1F		F on (a)
	$Var(D) = Var(X) + 1.5 - 2 \times (-0.43)$			
	$= 4.32$	A1F	5	F on (a)
(c)	$\underline{L \sim N(2.31,\ 0.89^2) \quad M \sim N(2.04,\ 0.43^2)}$			
	$T = L + M \sim N(\mathbf{4.35},\ \mathbf{0.977})$	B1 B1		Both CAO; SD = 0.98843
	$P(T > 5) = P\left(Z > \frac{5 - 4.35}{\sqrt{0.977}}\right)$	M1		Standardising 5 or 5.01 using C's mean & SD
	$= P(Z > 0.66) = 1 - P(Z < 0.66)$	m1		Area change
	0.25 to 0.26	A1	5	AWFW (0.25540)
	Total		**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$\underline{X_D \sim \text{Po}(24)}$			
(a)	$T = X_{\Sigma D} \sim \text{Po}(\mathbf{144})$	B1		CAO
	Thus $T \sim$ approx N(144, 144)	M1		Normal with $\mu = \sigma^2$
	$P(T_{Po} \le 150) \approx P(T_N < \mathbf{150.5})$	B1		CAO
	$= P\left(Z < \frac{150.5 - 144}{12}\right)$	M1		Standardising (149.5, 150 or 150.5) with $\mu > 24$ and $\sqrt{\mu}$
	$= P(Z < 0.54) = 0.705$ to 0.71	A1	5	AWFW (0.70598)
(b)(i)	H_0: λ(or mean) = 2 (or 10) H_1: λ(or mean) > 2 (or 10)	B1		Both; or equivalent
	$P(Y \ge 17) = 1 - P(Y \le 16)$	M1		Accept $1 - P(Y \le 17)$
	$= 1 - 0.0.9730 = 0.027$	A1		AWRT
	< 0.10 (10%) [z = 2.05 to 2.38 > 1.2816]	M1		Comparison of probability with 0.1 Comparison of z with 1.2816 or 1.6449
	Thus evidence, at 10% level, of increase in mean daily number of requests	A1F	5	F on probability or on z
(ii)	CV of Y is such that $P(Y \ge \text{CV}) \le 0.10$ (10%)	M1		Can be implied by 13, 14 or 15 Accept $P(Y = \text{CV}) = 0.10$
	Thus $P(Y \le \text{CV} - 1) \ge 0.90$	M1		Can be implied by 13, 14 or 15 Accept $P(Y = \text{CV}) = 0.90$
	Thus CV = 15	A1	3	CAO
(iii)	Power = 1 − P(Type II error) = 1 – P(accept H_0 | H_0 false) = P(accept H_1 | H_1 true)	B1		Or equivalent Stated or implied use
	$\lambda = 5 \times 3 = 15$	B1		Stated or implied use of Po(15)
	Thus power = $P(Y \ge \text{CV})$	M1		Attempt at a probability based on C's CV from (ii) and Po(15)
	$= P(Y \ge 15) = 1 - P(Y \le 14)$ $= 1 - 0.4657 = 0.53$ to 0.54	A1	4	AWFW (0.5343)
	Total		**17**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education
June 2010

Mathematics MS03

Statistics 3

Mark Scheme

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Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1	H_0: $\rho = 0$ H_1: $\rho \neq 0$	B1		Both
	SL $\alpha = 0.05$ (5%) CV $r = (\pm)$ **0.404**	B1		AWRT (0.4044) H_1: $\rho > 0 \Rightarrow r = 0.3438$
	Calculated $r = 0.336 <$ Tabulated r	M1		Comparison
	No evidence, at 5% level, **of a correlation** between stem length and cup diameter of matsutake mushrooms	A1F	4	F on CV At 5% level, **accept hypothesis of no correlation**
	Total		**4**	
2(a)	99% $\Rightarrow$ z = **2.57 to 2.58**	B1		AWFW (2.5758)
	CI for $\mu_R - \mu_D$ is $\left(\bar{x}_R - \bar{x}_D\right) \pm z \times \sqrt{\frac{s_R^2}{n_R} + \frac{s_D^2}{n_D}}$	M1 A1		Form Allow $\left(\frac{ns^2}{n-1}\right)$ or $(n-1)$ Correct expression
	ie $(225 - 219) \pm 2.5758\sqrt{\frac{5^2}{50} + \frac{8^2}{75}}$	A1F		Or equivalent F on z only
	ie **6 ± 3 or (3, 9)** **Note:** Use of pooled $s^2 = 5961/123 = 48.46341 \Rightarrow$ $6 \pm 3.3 \Rightarrow$ max of B1 M1 A0 A1F A0 (3)	A1	5	CAO/AWRT or AWRT
(b)	**CI does not** include **0/zero**	B1F		F on (a)
	Evidence of a **difference** in mean weights	B1F dep	2	F on (a) Dependent on CI but not on 0/zero
(c)	Price, size, quality, taste, presentation, organic, marketing, stall position, etc	B1	1	Any sensible reason
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3	H_0: $\lambda_T = \lambda_S$ H_1: $\lambda_T > \lambda_S$	B1		Both
	SL $\alpha = 0.02$ (2%)			
	CV z = **2.05 to 2.06**	B1		AWFW (2.0537)
	or H_1 $\lambda_T \neq \lambda_s$ $\Rightarrow$ z = **2.32 to 2.33**	(B1)		AWFW (2.3263)
	$\bar{s} = \dfrac{940}{40}$ = **23.5** $\quad \bar{t} = \dfrac{1560}{60}$ = **26**	B1		Both CAO; may be implied
	Pooled value, $\bar{p} = \dfrac{2500}{100}$ = **25**	B1		CAO
	$z = \dfrac{\lvert 23.5 - 26 \rvert}{\sqrt{25\left(\dfrac{1}{40} + \dfrac{1}{60}\right)}}$ **or** $z = \dfrac{\lvert 23.5 - 26 \rvert}{\sqrt{\left(\dfrac{23.5}{40} + \dfrac{26}{60}\right)}}$	M1		
	z = **2.44 to 2.45 or** z = **2.47 to 2.48**	A1		Either AWFW (2.449 or 2.474)
	Evidence, at 2% level, **to agree** with Tina's claim	A1F	7	F on CV and z-value
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	|———A+(0.90) 0.09 |——— S(0.10)———A–(0.02) 0.002	B1		S and NS with Ps or %s
	| | | B(0.08)——— +(0.98) 0.00784	B1		2 × (A+ and A–) with Ps or %s
	| |——— –(0.02) 0.00016 M	B1		2 × (B) with Ps or %s
	| |———A+(0.01) 0.009 |———NS(0.90)———A–(0.80) 0.72	B1		2 × (B+ and B–) with Ps or %s
	| B(0.19)——— +(0.01) 0.00171 |——— –(0.99) 0.16929	(B2,1)	4	Basic shape with labels, but without Ps or %s
	Note: The following BF and AF marks are dependent on an essentially correctly-shaped tree diagram			
(b)(i) **(A)**	P(S and –) = 0.002 + 0.00016 = **0.00216**	B1F		F on (a); otherwise CAO
(B)	P(NS and +) = 0.009 + 0.00171 = **0.01071**	B1F	2	F on (a); otherwise AWRT 0.0107
(ii)	E(N) = 10000 × [(A) + (B)]	M1		Or equivalent
	= 128.6 to 128.7 ⇒ **130**	A1F	2	CAO
(c)(i)	$\text{P(S} \mid +) = \dfrac{\text{P(S and +)}}{\text{P}(+)} =$	M1		Used
	$\dfrac{0.09+0.00784}{0.09+0.00784+0.009+0.00171} = \dfrac{0.09784}{0.10855}$	A1F		F on (a) Otherwise correct
	= **0.901 to 0.902**	A1		AWRT (0.90134)
(ii)	$\text{P(NS} \mid -) = \dfrac{\text{P(NS and } -)}{\text{P}(-)} =$	(M1)		Used; only if not scored in (i)
	$\dfrac{0.72+0.16929}{0.002+0.00016+0.72+0.16929} = \dfrac{0.88929}{0.89145}$	A1F		F on (a) and/or denominator (c)(i) Otherwise correct
	= **0.997 to 0.998** **Special cases:** Only numerators correct ⇒ (M1) B1 B1 Only denominators correct ⇒ (M1) B1 B1	A1	5	AWFW (0.99758)
	Total		**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\text{E}(T) = 2 \times 350 + 2 \times 210 = \mathbf{1120}$	B1		CAO
	$\text{Cov}(W, H) = \sqrt{5\times4} \times 0.75 =$	M1		Used; may be implied
	3.34 to 3.36	A1		AWFW (3.3541)
	$\text{Var}(T) = (2^2 \times 5) + (2^2 \times 4) + (2 \times 2 \times 2 \times 3.3541)$	M1		Used Ignore 3rd expression
	$= 20 + 16 + 26.8328 =$ **62.7 to 62.9**	A1	5	AWFW (62.8328)
(b)	$L = T_1 + T_2 + T_3 + T_4$			
	Mean of L = **4480**	B1F		CAO; F on E(T)
	Variance of L = **4** × Var(T)	M1		
	= **250.8 to 251.6** SD of L = **15.8 to 15.9**	A1F		Either AWFW; F on Var (T) (251.3312) (15.8534)
	$\text{P}(L < 4500) = \text{P}\left(Z < \frac{4500-4480}{\sqrt{251.3312}}\right)$	M1		Standardising 4500 using C's mean and SD
	$= \text{P}(Z < 1.25 \text{ to } 1.27)$			
	= **0.894 to 0.898**	A1		AWFW (0.89645)
	Alternative Solution: Use of $\overline{T}$ rather than L			
	Mean of $\overline{T}$ = **1120**	(B1F)		CAO; F on E(T)
	Variance of $\overline{T}$ = Var(T) ÷ **4**	(M1)		
	= **15.6 to 15.8** SD of $\overline{T}$ = **3.95 to 3.97**	(A1F)		Either AWFW; F on Var(T) (15.7082) (3.9634)
	$\text{P}(\overline{T} < 1125) = \text{P}\left(Z < \frac{1125-1120}{\sqrt{15.7082}}\right)$	(M1)		Standardising 1125 using C's mean and SD
	$= \text{P}(Z < 1.25 \text{ to } 1.27)$			
	= **0.894 to 0.898**	(A1)	5	AWFW (0.89645)
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\hat{p} = \frac{28}{175}$ = **0.16**	B1		CAO; or equivalent
	95% ⇒ z = **1.96**	B1		AWRT
	Approximate CI for p is $\hat{p} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Used
	ie $0.16 \pm 1.96\sqrt{\frac{0.16 \times 0.84}{175}}$	A1F		Or equivalent F on $\hat{p}$ and z
	ie **0.16 ± 0.054 or (0.106, 0.214)**	A1	5	CAO/AWRT or AWRT (0.0543)
(ii)	**CI does** include **0.2** (20%)	B1F		F on (i)
	No evidence to support councils' claim	B1F dep	2	F on (i) Dependent on CI and on 0.2
(b)(i)	H_0: p = 0.40 (40%) H_1: p < 0.40	B1		Both
	Using B (50, 0.4) (40%)	M1		May be implied
	P(X ≤ 16) = **0.156**	A1		AWRT (0.1561)
	Calculated probability > 0.10 (10%)	M1		Comparison
	No evidence, at 10% level, **to support** council's claim **Special Case:** Normal approximation z = **–1.15**(47) B1 CV = **–1.28**(16) B1 Conclusion B1F Max of 4 marks	A1F	5	F on probability v 0.10 or 0.05 At 10% level, **accept** (at least) **40%** Allow B1 for hypotheses p = 0.123 to 0.125 v 0.10 B1 B1 F on z and CV
(ii)	Require P(X ≤ x) ≤ 0.10	M1		May be implied Ignore any reasoning if '15' stated
	⇒ CV = **15** (CR ≤ 15)	A1	2	CAO; or equivalent
(iii)	P(Type II error) = P(accept H_0 | H_0 false)	B1		Stated or used; or equivalent
	= P(X > CV or X ≥ CV)	M1		Attempt at a probability > or ≥ C's CV from (ii)
	= 1 – (**0.8369 or 0.7481**)	M1		Ignore '1 –'
	= **0.163**	A1	4	AWRT
	Total		**18**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$X \sim \text{Po}(\lambda)$			
(a)(i)	$\text{E}(X) = \sum_{x=0}^{\infty} x \times \frac{e^{-\lambda}\lambda^x}{x!} =$	M1		Used; ignore limits until A1
	$\lambda e^{-\lambda} \times \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} =$	M1		Factor of at least λ Division of $x!$ by x
	$\lambda e^{-\lambda} \times e^{\lambda} = \boldsymbol{\lambda}$	A1	3	AG; fully correct solution
(ii)	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$ $= \text{E}[X(X-1)] + \text{E}(X) - [\text{E}(X)]^2$	M1		Used (Other derivations are possible)
	$= \lambda^2 + \lambda - \lambda^2 = \boldsymbol{\lambda}$	A1	2	CAO
(b)(i)	$\text{P}(X = m) \geq \text{P}(X = m - 1)$ and $\text{P}(X = m) \geq \text{P}(X = m + 1) \Rightarrow$			
	$\frac{e^{-\lambda}\lambda^m}{m!} \geq \frac{e^{-\lambda}\lambda^{m-1}}{(m-1)!}$ and $\frac{e^{-\lambda}\lambda^m}{m!} \geq \frac{e^{-\lambda}\lambda^{m+1}}{(m+1)!}$	M1 M1		Use of $\text{Po}(\lambda)$ for $x = m$ Either inequality (accept = sign)
	$\boldsymbol{m \leq \lambda}$ **and** $\boldsymbol{m \geq \lambda - 1}$	A1	3	AG; fully correct solution
(ii)	Given $\lambda = 4.9 \Rightarrow m =$ **4**	B1		CAO
	$\text{P}(X = 4) = \frac{e^{-4.9}4.9^4}{4!} =$ **0.178 to 0.179**	B1	2	AWFW (0.178867)
(c)	Given SD $(Y) = 15.5 \Rightarrow$ $\lambda = \text{Var (Y)} = \mathbf{15.5^2} = \mathbf{240.25}$	B1		Either CAO
	Mode, $d =$ **240**	B1F		F on λ providing an integer value
	$\text{P}(Y_\text{P} \geq d) = \text{P}(Y_\text{N} > d - 0.5) =$	B1		Accept use of 'd'
	$\text{P}\left(Z > \frac{239.5 - 240.25}{15.5}\right) =$	M1		Standardising (d – 0.5, d or d + 0.5) with 15.5^2 and 15.5; do **not** accept use of 'd'
	$\text{P}(Z > -0.05) =$ **0.515 to 0.52**	A1	5	AWFW (0.5193)
	Total		**15**	
	TOTAL		**75**	

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General Certificate of Education (A-level)
June 2011

Mathematics MS03

(Specification 6360)

Statistics 3

Final

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1 (a)	H_0: $p = 0.25$ (25%) H_1: $p > 0.25$	B1		Both
	SL $\alpha = 0.02$ (2%)			
	CV z = **2.05 to 2.06**	B1		AWFW (2.0537) Allow 2.32 to 2.33 if H_1: $p \neq 0.25$
	$\hat{p} = \dfrac{108}{375} = \mathbf{0.288}$	B1		CAO
	$z = \dfrac{0.288 - 0.25}{\sqrt{\dfrac{0.25 \times 0.75}{375}}} = \mathbf{1.70}$	M1 A1		Allow use of 0.288 in denominator AWRT
	or			$P(X \geq 108 \mid n = 375, p = 0.25) = \mathbf{0.052}$
	$z = \dfrac{108(-0.5) - 93.75}{\sqrt{375 \times 0.25 \times 0.75}}$ = **1.70 (or 1.64)**	(M1) (A1)		Allow use of 0.288 in denominator AWRT
	Thus, **no evidence**, at 2% level, **to support** consumer report's claim	AF1	6	F on CV and z-value **or** F on 2% and probability
(b)	Can be considered to be a **random sample**	B1	1	
		Total	**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2 (a)	98% $\Rightarrow$ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	CI for λ is: $\hat{\lambda} \pm z \times \sqrt{\hat{\lambda}}$ **or** $\bar{x} \pm z \times \sqrt{\frac{\bar{x}}{n}}$	M1		Form; allow $\hat{\lambda} \pm z \times \sqrt{\frac{\hat{\lambda}}{n}}$
	ie $108 \pm 2.3263 \times \sqrt{108}$	AF1		F on z only; allow $108 \pm z \times \sqrt{\frac{108}{13}}$
	or $\frac{108}{13} \pm 2.3263 \times \sqrt{\frac{108}{13^2}}$	(AF1)		F on z only; allow $\frac{108}{13} \pm z \times \sqrt{\frac{108}{13}}$
	Dividing by 13 or equivalent to obtain a **correct** numerical expression	A1		May be implied
	Thus **8.31 ± 1.86** **or** **(6.45, 10.2)**	A1	5	AWRT
	Note: For incorrect numerical expressions the maximum marks are B1 M1 AF1 A0 A0 (3)			
(b)	1 per 24 hours $\Rightarrow$ 7 per week			
	CI includes 7	BF1		F on (a); must use 7 **or** 1 v CI/7
	No reason, at 2% level, **to dispute** station officer's claim	Bdep1	2	Or equivalent Dependent on BF1
		Total	**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3 (a)(i)	P(G) = **0.15**	B1	1	CAO
(ii)	$P(A \cap \le 1) = 0.60 \times 0.55 =$ **0.33**	B1	1	CAO
(iii)	$P(\le 24) = (0.60 \times 0.80) + (0.25 \times 0.85) + (0.15 \times 0.75)$	M1		May be implied
	$= 0.48 + 0.2125 + 0.1125 =$ **0.805**	A1	2	CAO
(iv)	$P(B \mid \le 24) = \dfrac{P(B \cap \le 24)}{P(\le 24)}$	M1		Used; may be implied
	$= \dfrac{0.25 \times 0.85}{(\text{iii})} = \dfrac{0.2125}{0.805}$	AF1		F on (iii)
	= **0.264**	A1	3	AWRT
(b)(i)	$P(3 \text{ @ } B \mid \le 24) = [(a)(iv)]^3$	M1		Used; may be implied
	= **0.018 to 0.0185**	A1	2	AWFW (0.01839)
(ii)	$P(\text{same station} \mid \le 24)$ $= [P(A \mid \le 24)]^3 + (b)(i) + [P(G \mid \le 24)]^3$	M1		Used; may be implied
	$= \left(\dfrac{0.48}{0.805}\right)^3 + (b)(i) + \left(\dfrac{0.1125}{0.805}\right)^3$	M1 M1		At least 1 term correct; allow (b)(i) providing it is a (cond prob)^3 All 3 terms correct
	$= 0.2120 + 0.0184 + 0.0027 =$ **0.233**	A1	4	AWRT (0.23312)
		Total	**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	95% $\Rightarrow$ $z = \mathbf{1.96}$	B1		CAO (AWRT from calculator)
	Require $2 \times \dfrac{1.96\sigma}{\sqrt{n}} \le 0.2\mu$	M1		Used; may be implied Allow 'no 2 ×' Allow '= sign' throughout
	Thus $2 \times \dfrac{1.96}{\sqrt{n}} \times \dfrac{\mu}{2} \le 0.2\mu$	M1		Use of $\sigma = \dfrac{\mu}{2}$; may be implied Allow 'no 2 ×'
	Thus $\sqrt{n} \ge \dfrac{1.96}{2}$	M1		Attempt at solution for $\sqrt{n}$ or n
	Thus $n \ge 96.04$			
	Thus, to nearest 10; $n = \mathbf{100}$	A1	5	CAO
		Total	**5**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5	E-mails are selected: **randomly** **independently**	 B1 B1		
	99% $\Rightarrow$ z = **2.57 to 2.58**	B1		AWFW (2.5758)
	$\hat{p}_G = \frac{72}{160}$ = **0.45** $\quad \hat{p}_H = \frac{102}{250}$ = **0.408**	B1		CAO both; ignore notation
	Approximate CI for $p_G - p_H$ is:			
	$(\hat{p}_G - \hat{p}_H) \pm z \times \sqrt{\frac{\hat{p}_G(1-\hat{p}_G)}{n_G} + \frac{\hat{p}_H(1-\hat{p}_H)}{n_H}}$	M1 m1		Form used Standard deviation term
	Thus:			
	$(0.45 - 0.408) \pm$ $1.96 \times \sqrt{\frac{0.45 \times 0.55}{160} + \frac{0.408 \times 0.592}{250}}$	AF1		Or equivalent F on $\hat{p}_G$, $\hat{p}_H$ and z
	Thus:			
	0.042 $\pm$ 0.129 **or** **(–0.09, 0.17)**	A1	8	CAO/AWRT **or** AWRT
	Note: If a pooled estimate of variance is used, then the maximum marks are B1 B1 B1 B1 M1 m0 AF0 A0 (5)			
		Total	**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6 (a)(i)	$V(X_1+X_2)=V(X_1)+V(X_2)+2Cov(X_1,X_2)$	M1		Used
	Thus: $140^2 = 120^2+120^2+2Cov(X_1,X_2)$	A1		
	Thus $2Cov(X_1,X_2)$ = 19600 – 14400 – 14400			
	= **–4600**	A1	3	CAO AG
(ii)	$E(X_1-X_2) = 1000-1000 =$ **0**	B1		CAO; may be implied
	$V(X_1-X_2) = 120^2+120^2-(2\times-4600)$ = **38000** **or** $Sd(X_1-X_2)$ = **194 to 195**	B1		CAO AWFW (194.936)
	P(Difference > 250) = $P(\lvert X_1-X_2\rvert>250) = 2\times P\left(Z>\frac{250-0}{\sqrt{38000}}\right)$	M1		Standardising 250 using c's mean & c's standard deviation Allow 'no 2 ×'
	= 2 × P(Z > 1.28)			
	= 2 × [1 – (0.899 to 0.901)]	m1		Area change; allow 'no 2 ×'
	= **0.2(00)**	A1	5	AWRT (0.19968)
(b)	$Y + B$ has: Mean = **2500**	B1		CAO
	and Variance = 140^2+40^2 = **21200** **or** Standard deviation = **145 to 146**	B1		CAO AWFW (145.602)
	P($Y + B$ < 2750) = $P\left(Z<\frac{2750-2500}{\sqrt{21200}}\right)$	M1		Standardising 2750 using c's mean & c's standard deviation
	= P(Z < **1.72**)	A1		AWRT; ignore inequality and sign
	= **0.957**	A1	5	AWRT (0.95701)
		Total	**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$X \sim \text{Po}(\lambda)$			
(a)(i)	$E(X(X-1)) = \sum_{x=0}^{\infty} x(x-1) \times \frac{e^{-\lambda}\lambda^x}{x!} =$	M1		Used; ignore limits until A1
	$\lambda^2 e^{-\lambda} \times \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} =$	M1		Factor of at least λ^2 Division of $x!$ by $x(x-1)$
	$\lambda^2 e^{-\lambda} \times e^{\lambda} = \boldsymbol{\lambda^2}$	A1	3	Fully correct convincing solution AG
(ii)	$\text{Var}(X) = E(X^2) - [E(X)]^2$ $= E[X(X-1)] + E(X) - [E(X)]^2$	M1		Used (Other derivations are possible)
	$= \lambda^2 + \lambda - \lambda^2 = \boldsymbol{\lambda} = \mathbf{E(X)}$	A1	2	CAO either AG
(b)(i)	$E(Z) = 4 \times 2.5 + 30 = \mathbf{40}$	B1		CAO
	$\text{Var}(Z) = 4^2 \times 2.5$	M1		Use of $V(aX) = a^2 V(X)$ Ignore '+30'
	$= \mathbf{40}$ $\{= E(Z)\}$	A1	3	CAO AG
	Note: $4 \times 2.5 + 30 = 4^2 \times 2.5 \Rightarrow$ B1 M1 A0 plus value of 40 quoted $\Rightarrow$ B1 M1 A1			
(ii)	No values less than 30 are possible No odd values are possible Only even values are possible Only values of 30, 34, 38, etc, are possible	B1	1	Or equivalent
		Total	**9**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	H_0: $\mu_A = \mu_B$ H_1: $\mu_A \neq \mu_B$	B1		Both; allow suffices of 1 & 2 or X & Y
	SL $\alpha = 0.05$ (5%) CV $z = \pm$**1.96**	B1		CAO (AWRT from calculator) Allow (+)1.96
	$z = \dfrac{\bar{x}-\bar{y}}{\sigma\times\sqrt{\dfrac{1}{n_A}+\dfrac{1}{n_B}}}$	M1		Attempted use; or equivalent Allow σ_A and σ_B
	$z = \dfrac{3770-3695}{285\times\sqrt{\dfrac{1}{80}+\dfrac{1}{120}}}$	A1		
	= **1.82**	A1		AWRT (1.82321)
	No evidence, at 5% level, to suggest that **there is a difference**	AF1	6	F on CV and z-value
(ii)	**Large samples** (so CLT is applicable)	B1	1	
(b)(i)	$z = \dfrac{\bar{x}-\bar{y}}{285\times\sqrt{\dfrac{1}{80}+\dfrac{1}{120}}} = \pm 1.96$	M1		Equating z-term to 1.96
	Thus $(\bar{x}-\bar{y}) = \pm 1.96\times 41.13616$ = **±80.63**	A1	2	Requires a convincing deduction AG
(ii)	P(Type II error) = P(accept H_0 | H_0 false) =	B1		Used or stated; may be implied
	$\mathrm{P}\left(-80.63<(\bar{x}-\bar{y})<80.63 \mid \mu_A-\mu_B=125\right)$ = $\mathrm{P}\left(Z<\dfrac{80.63-125}{285\times\sqrt{\dfrac{1}{80}+\dfrac{1}{120}}}\right)$	M1		Accept $(\bar{x}-\bar{y}) < 80.63$ –80.63 $\Rightarrow$ z = –5.00 $\Rightarrow$ probability $\approx$ 0
	= P(Z < –**1.08**)	A1		AWRT; ignore sign
	= **0.14**	A1	4	AWRT (0.14038)
		Total	**13**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MS03

(Specification 6360)

Statistics 3

Mark Scheme

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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1(a)	$r = \mathbf{0.665}$ **to** $\mathbf{0.666}$ $r = \mathbf{0.6}$ **to** $\mathbf{0.7}$	B2 (B1)	2	AWFW (0.66577) AWFW
	Alternative Use of a correct formula	(M1)		$\Sigma x = 20.40$ $\Sigma x^2 = 34.785$ $\Sigma xy = \mathbf{49.1155}$ $\Sigma y = 28.86$ $\Sigma y^2 = 69.4698$
	$r = \mathbf{0.665}$ **to** $\mathbf{0.666}$	(A1)		$S_{xx} = 0.105$ $S_{yy} = 0.0615$ $S_{xy} = \mathbf{0.0535}$
(b)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both; do not accept in terms of r but accept in words
	SL $\alpha = 0.01$ (1%)			
	CV $r = (+)\mathbf{0.658}$	B1		AWRT (0.6581) H_1: $\rho \neq 0 \Rightarrow$ CV $= (\pm)0.7079$
	Calculated $r = 0.666$ > Tabulated r	M1		Comparison
	Evidence, at 1% level, **of a positive correlation** between the neck length and the tail length of mature male giraffes	A1F	4	F on r and CV At 1% level, **reject hypothesis of no correlation**
	Total		**6**	
2(a)	H_0: $\mu_F = \mu_M$ H_1: $\mu_F \neq \mu_M$	B1		Both; allow suffices of 1 & 2 or X & Y
	SL $\alpha = 0.01$ (1%) CV $z = \mathbf{2.57}$ **to** $\mathbf{2.58}$	B1		AWFW (2.5758)
	$z = \dfrac{\overline{f} - \overline{m}}{\sqrt{\dfrac{s_F^2}{n_F} + \dfrac{s_M^2}{n_M}}} = \dfrac{22.0 - 21.6}{\sqrt{\dfrac{1.31^2}{50} + \dfrac{0.702^2}{75}}}$	M1 M1		Numerator Denominator; allow ÷49 and ÷74 but **not** pooling
	1.95 to 2(.0)	A1		AWFW (1.98)
	No evidence, at 1% level, to suggest that **there is a difference** between mean lengths	A1F	6	F on CV and z-value allow following pooling
(b)	Diameter, thickness, girth, width, weight, shape, colour, texture	B1	1	Accept any sensible alternative but **not** 'quality'
	Total		**7**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	P(S ∩ U) = 0.15 × 0.10 = **0.015**	B1	1	CAO
(ii)	P(O ∩ ≥2) = (0.40 × 0.50) + (0.45 × 0.40) + (0.15 × 0.70)	M1		≥1 term correct; may be implied
	= 0.20 + 0.18 + 0.105 = **0.485**	A1	2	CAO
(iii)	P(U) = (0.40 × 0.15) + (0.45 × 0.05) + (0.15 × 0.10) or (i)	M1		≥2 terms correct; may be implied
	= 0.06 + 0.0225 + 0.015 = **0.097 to 0.098**	A1	2	AWFW (0.0975)
(iv)	P(D | U) = $\frac{P(D \cap U)}{P(U)} = \frac{0.40 \times 0.15}{(iii)}$	M1		May be implied
	$= \frac{0.06}{0.0975}$ = **0.612 to 0.619**	A1	2	AWFW (0.61538)
(v)	P(S | O) = $\frac{0.15 \times (1 - 0.10)}{1 - (iii)} = \frac{0.135}{0.9025}$	M1 M1		Numerator Denominator
	= **0.149 to 0.15**	A1	3	AWFW (0.14958)
(b)	P(D ∩ T ∩ S | O) $= \frac{0.40 \times 0.85}{1 - (iii)} \times \frac{0.45 \times 0.95}{1 - (iii)} \times (v) \times 3!$	M1 M1 M1		≥2 terms correct in numerator (1 – (iii)) in denominator 3! or 6 or 3
	$= \frac{0.34 \times 0.4275 \times 0.135 \times 6}{0.9025^3}$ **or** = 0.3767 × 0.4737 × 0.1496 × 6			
	= **0.16**	A1	4	AWRT (0.16016)
	Total		**14**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	H_0: λ = 2.6 (650) H_1: λ > 2.6 (650)	B1		Both; accept μ instead of λ
	SL α = 0.05 (5%)			
	CV z = **1.64 to 1.65**	B1		AWFW (1.6449) Allow 1.96 iff H_1: $\lambda \neq 2.6$
	$\hat{\lambda} = \frac{688}{250}$ = **2.75(2)**	B1		AWRT (2.752) Can be implied by use of 688
	$z = \frac{2.752-2.6}{\sqrt{\frac{2.6}{250}}} = \frac{688-650}{\sqrt{650}}$ = **1.47 to 1.49**	M1 A1		Allow use of 2.752 or 688 or 687.5 in denominator AWFW
	p-value = **0.068 to 0.071** > 0.05	(M1)		Use of $P(X \geq 688 \mid \lambda = 650) = 0.072$ $\Rightarrow$ M0 A0 (M1) AF1
	No evidence, at 5% level, **to support** manager's suspicion	A1F	6	F on CV and z-value
	Total		**6**	
5(a)	$\hat{p} = \frac{68}{125}$ = **0.544**	B1		CAO; or equivalent
	98% $\Rightarrow$ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	Approximate CI for p is $0.544 \pm 2.3263 \times \sqrt{\frac{0.544 \times 0.456}{125}}$	M1 A1F		F on $\hat{p}$ and z
	0.544 ± (0.103 to 0.104) or **(0.44 to 0.441, 0.647 to 0.65)**	A1		CAO/AWFW May be implied by **correct** answer AWFW
	(44%, 65%)	A1	6	AWRT
(b)	Require $2 \times 2.3263 \times \sqrt{\frac{p(1-p)}{n}} \leq 0.1$ (10%)	M1 A1		Allow 'no 2' and FT on CI from (a) Allow p = 0.44 to 0.65
	Thus $\sqrt{n}$ = 22.1 to 23.3	M1		Attempted solution for $\sqrt{n}$ or n
	***n* = 490, 495, ..., 545**	A1	4	Must be to 'nearest 5'
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$M = U + V$			
	E(M) = 13 + 15 = **28**	B1		CAO
	V(M) = $3^2 + 6^2$	M1		
	$+ 2 \times 3 \times 6 \times (-0.6)$	M1		Allow 'no 2'
	= 9 + 36 – 21.6 = **23.4**	A1	4	CAO
(ii)	$D = W - 2U$			
	E(D) = 24 – 2 × 13 = **–2**	B1		CAO
	V(D) = $4^2 + (2^2 \times 3^2)$	M1		
	= 16 + 36 = **52**	A1	3	CAO
(iii)	$T = M + W + X$			
	E(T) = 28 + 24 + 9 = **61**	B1F		F on E(M) from (a)(i)
	V(T) = $23.4 + 4^2 + 2^2$ = **43.4**	B1F	2	F on V(M) from (a)(i)
(b)(i)	P(M = 30) = **0**	B1	1	CAO
(ii)	P($W > 2U$) = P($D > 0$) = $\mathrm{P}\left(Z > \dfrac{0-(-2)}{\sqrt{52}}\right)$	M1		Standardising 0 using c's E(D) & c's V(D) from (a)(ii)
	= P(Z > 0.28) = 1 – 0.61026	m1		Area change
	= **0.389 to 0.394**	A1	3	AWFW (0.39076)
(iii)	P(50 < T < 70) =			
	$\mathrm{P}\left(Z < \dfrac{70-61}{\sqrt{43.4}}\right) - \mathrm{P}\left(Z < \dfrac{50-61}{\sqrt{43.4}}\right)$	M1		Standardising 70 & 50 using c's E(T) & c's V(T) from (a)(iii)
	= P(Z < **1.37**) – P(Z < **–1.67**)	A1		AWRT either
	= 0.91466 – (1 – 0.95254)	m1		Area change
	= **0.865 to 0.87**	A1	4	AWFW (0.86657)
	Total		**17**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$X \sim B(n,\ p)$			
	$E(X) = \sum_{x=0}^{n} x \times \binom{n}{x} p^x (1-p)^{n-x}$	M1		Used; ignore limits until A1
	$= np \times \sum_{x=1}^{n-1} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1}(1-p)^{n-x}$	M1		Factor of np plus p^x to p^{x-1}, $n!$ to $(n-1)!$ and $x!$ to $(x-1)!$ Ignore limits until A1
	$= np \times \sum_{x=1}^{n-1} B(n-1,\ p) = \boldsymbol{np}$	A1	3	Fully correct and complete derivation AG
(ii)	$\text{Var}(X) = E(X^2) - [E(X)]^2$			
	$= E[X(X-1)] + E(X) - [E(X)]^2$	M1		Used (Other derivations are possible)
	$= n(n-1)p^2 + np - n^2p^2 = \boldsymbol{np(1-p)}$	A1	2	Fully correct and complete derivation
(b)(i)	$\binom{n}{m} p^m (1-p)^{n-m} \geq \binom{n}{m-1} p^{m-1}(1-p)^{n-m+1}$	M1		Stated or used
	$\frac{n!}{m!(n-m)!} p^{n-m} \geq \frac{n!}{(m-1)!(n-m+1)!}(1-p)^{n-m+1}$	m1		Expanding $\binom{n}{m}$ and $\binom{n}{m-1}$
	$(n-m+1)p \geq m(1-p)$ $np - mp + p \geq m - mp$	A1		Simplification
	$\boldsymbol{m \leq (n+1)p}$	A1	4	Fully correct and complete derivation AG
(ii)	B(10, 0.65)			
	$(n+1)p = 7.15$ and $(n+1)p - 1 = 6.15$	M1		Can be implied by '7'
	m = **7**	A1		CAO
	B(35, 0.5)			
	$(n+1)p = 18$ and $(n+1)p - 1 = 17$	(M1)		Can be implied by '17 or 18'
	m = **17 and 18**	A1	3	CAO; accept 'or'
(c)	$Y \sim B(4000,\ 0.00095) \approx$ **Po(3.8)**	B1		May be implied
	$\boldsymbol{k = 3}$	B1		CAO; may be implied
	$P(Y \leq 3)$ = **0.473 to 0.474**	B1	3	AWFW (0.4735)
	SC: $\boldsymbol{k = 4} \Rightarrow P(Y \leq 4)$ = **0.667 to 0.668** Award B1 B0 B1			AWFW (0.6678)
	Total		**15**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2013

Mathematics MS03

(Specification 6360)

Statistics 3

Final

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Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	98% $\Rightarrow$ $z =$ **2.32 to 2.33**	B1		AWFW (2.3263)
	Approximate CI for λ: $\hat{\lambda} \pm z\sqrt{\hat{\lambda}}$	M1		Used
	$392 \pm 2.3263 \times \sqrt{392}$	AF1		F on z
	Per shift $\Rightarrow$ $\div 12$	M1		
	Thus: **32.7 ± 3.8 or (28.8, 36.5)**	A1	5	AWRT
(b)	Per hour (weekday night) $\Rightarrow$ **(2.05 to 2.06, 2.6 to 2.61)**	BF1		F on (a)
	Per hour (weekend) = $\frac{136.8}{48}$ = **2.85**	B1		
	Thus evidence to **agree with claim**	BF1	3	F on comparison of value with CI Definitive conclusion $\Rightarrow$ BF0
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)	**A** **B** 	——— E(0.15) 0.135 	——— T(0.9)	——— T(0.75) 0.675 		——— L(0.10) 0.090
	——— T(0.35) 0.035 	——— L(0.1)
——— L(0.65) 0.065	B1 B1 B1	 3	Correct shape Correct labels Correct probabilities
(b)(i)	P(E ∪ T @ B) = 0.9 × 0.9 + 0.1 × 0.35	M1		1 – (0.09 + 0.065)
	= **<u>0.84 to 0.85</u>**	A1	2	AWFW (0.845)
(ii)	P(T @ A | T @ B) = $\dfrac{0.9\times0.75}{(0.9\times0.75+0.1\times0.35)}$	M1 m1		P(A | B) **used in (ii) or (iii)** $a \div (a+b)$ with at least a correct
	$=\dfrac{0.675}{0.71}$ = **<u>0.95 to 0.951</u>**	A1	3	AWFW (0.95070)
(iii)	P(L @ A | L′ @ B) = $\dfrac{0.1\times0.35}{\text{(i)}}$	AF1		F on (i)
	$=\dfrac{0.035}{0.845}$ = **<u>0.04 to 0.042</u>**	A1	2	AWFW (0.04142)
(c)	P((T @ A | L @ B) ∩ (T′ @ A | L @ B))			
	$\dfrac{0.9\times0.1}{1-0.845}\times\dfrac{0.1\times0.65}{1-0.845}\times 2$	M1 M1 M1		First expression (18/31) Second expression (13/31) × 2
	= **<u>0.486 to 0.49</u>**	A1	4	AWFW (0.48699)
	Total		**14**	

Q	Solution	Marks	Total	Comments
3(a)	95% $\Rightarrow$ $z = \underline{\mathbf{1.96}}$	B1		AWRT
	$\bar{x} = \underline{\mathbf{1026}}$ $\bar{y} = \underline{\mathbf{1045}}$	B1		Both CAO
	CI for $\mu_Y - \mu_X$ is $(\bar{y} - \bar{x}) \pm z\sqrt{\frac{\sigma_Y^2}{n_Y} + \frac{\sigma_X^2}{n_X}}$	M1 m1		Used Accept $(\bar{x} - \bar{y})$ throughout SD term
	ie $(1045 - 1026) \pm 1.96\sqrt{\frac{30^2}{8} + \frac{25^2}{10}}$	AF1		F on $\bar{x}$, $\bar{y}$ and z
	ie 19 ± 25.9 or $(-6.9, 44.9)$	A1		CAO & AWRT or AWRT
	ie **<u>20 ± 25 or (–5 or –10, 45)</u>**	B1	7	Rounding answer to nearest 5 kg
(b)	Fred used: machine X for sand and machine Y for gravel	B1		Apparent rounding to nearest 5 kg
	Use each machine for both	B1	2	OE
	Total		**9**	
4	$H_0: p_M - p_D = 0.10$ $H_1: p_M - p_D > 0.10$	B1 B1		If B0 B0, then award B1 for $p_M - p_D = 0$
	95% $\Rightarrow$ $z = \underline{\mathbf{1.64 \text{ to } 1.65}}$	B1		AWFW (1.6449)
	$z = \frac{(\hat{p}_M - \hat{p}_D) - 0.10}{\sqrt{\frac{\hat{p}_M(1-\hat{p}_M)}{n_M} + \frac{\hat{p}_D(1-\hat{p}_D)}{n_D}}} =$	M1 m1		Used; allow pooling and/or 'no –0.10' Denominator
	$\frac{(0.38 - 0.21) - 0.10}{\sqrt{\frac{0.38 \times 0.62}{250} + \frac{0.21 \times 0.79}{100}}} =$	A1		Correct expression but allow 'no –0.10'
	$\frac{0.07}{0.051} = \underline{\mathbf{1.37}}$	A1		AWRT (1.3724)
	No evidence, at 5% level, to suggest that the difference is more than 10 per cent	AF1	8	F on CV and z-value Definitive conclusion $\Rightarrow$ AF0
	Total		**8**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$L = X + Z$ $E(L) = 68 + 73$ **<u>141</u>**	B1		CAO
	$V(L) = 10^2 + 15^2 =$ **<u>325</u>**	B1	2	CAO
(ii)	$M = X + Y$ $E(M) = 68 + 25 =$ **<u>93</u>**	B1		CAO
	$V(M) = 10^2 + 5^2 + 2 \times 10 \times 5 \times (-0.8)$	M1		Allow 'no 2'
	$= 100 + 25 - 80 =$ **<u>45</u>**	A1	3	CAO
(b)(i)	Require: $P(L < 150) =$ $P\left(Z < \frac{150-141}{\sqrt{325}}\right)$	M1		Standardising 150 using c's E(L) & c's V(L) from (a)(i)
	$= P(Z < 0.5)$			(0.49923)
	= **<u>0.69 to 0.692</u>**	A1	2	AWFW (0.69119)
(ii)	Require: $P(X + Y > 105) = P(M > 105)$			
	$= P\left(Z > \frac{105-93}{\sqrt{45}}\right)$	M1		Standardising 105 using c's E(M) & c's V(M) from (a)(ii)
	$= P(Z > 1.79) = 1 - P(Z < 1.79)$	m1		**Correct** area change (1.78885) May be implied by a **correct** answer **or** by an answer **< 0.5**
	= **<u>0.036 to 0.038</u>**	A1	3	AWFW (0.03682)
	Total		**10**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\lambda = 6 \times 2.5 = \underline{\mathbf{15}}$	B1		CAO
	$\text{P}(W \le 18) = \underline{\mathbf{0.819\ \ to\ \ 0.82}}$	B1	2	AWFW (0.8195)
(ii)	$\text{P}(W > w) \le 0.05 \ \Rightarrow\ \text{P}(W \le w) \ge 0.95$	M1		Implied by a value of 21, 22 or 23
	$w = \underline{\mathbf{22}}$	A1	2	CAO
(b)(i)	$F \sim \underline{\mathbf{N(30,\ 30)}}$	B1		May be implied
	$\text{P}(F > 35) =$ $\text{P}\left(Z > \dfrac{35.5 - 30}{\sqrt{30}}\right) = \text{P}(Z > 1.00)$	M1 B1		Standardising (34.5, 35 or 35.5) with $\mu = \sigma^2$ 35.5 (1.00416)
	$= \underline{\mathbf{0.157\ \ to\ \ 0.16}}$	A1	4	AWFW (0.15765)
(ii)	$\text{P}(F > f) \le 5\% \ \Rightarrow$ $\text{P}\left(Z > \dfrac{(f + 0.5) - 30}{\sqrt{30}}\right) \le 0.05$	M1		Standardising ($f - 0.5$, f or $f + 0.5$) with $\mu = \sigma^2$
	$5\% \ \Rightarrow\ z = \underline{\mathbf{1.64\ \ to\ \ 1.65}}$	B1		AWFW (1.6449)
	So $f = \underline{\mathbf{39}}$	Adep1	3	CAO Dependent on $(f + 0.5)$ and on B1
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	H_0: $p = 0.50$ H_1: $p > 0.50$	B1 B1		Here or in (b)(i)
	$P(X \geq 29 \mid B(50, 0.50)) =$ **1 – (0.8389 or 0.8987)**	M1 M1		Use of B(50, 0.50); may be implied
	= **0.16 to 0.165**	A1		AWFW (0.16112)
	No evidence to support the claim	AF1	6	F on 10% and (p-value > 0.10) Definitive conclusion ⇒ AF0
(b)(i)	10% ⇒ z = **1.28**	B1		AWRT (1.2816)
	$z = \dfrac{\frac{271}{500} - 0.5}{\sqrt{\frac{0.5 \times 0.5}{500}}}$ = **1.87 to 1.89**	M1 A1		Accept use of $\hat{p}$ in denominator giving $z = 1.88511$ AWFW (1.87830)
	Evidence to support the claim	AF1	4	F on CV and z-value Definitive conclusion ⇒ AF0
(ii)	Power = 1 – P(Type II error) = 1 – P(accept H_0 | H_0 false) or P(reject H_0 | H_0 false) or P(accept H_1 | H_1 true)	B1		Any one stated or used
	$P\left(\hat{P} > 0.529 \mid B(500, 0.55)\right) =$	M1		Use of B(500, 0.55) M0 for use of 0.529 or 0.5
	$P\left(Z > \dfrac{0.529 - 0.55}{\sqrt{\frac{0.55 \times 0.45}{500}}}\right)$ = P(Z > –**0.94**)	M1 A1		Accept use of 0.529 in denominator giving $z = 0.94075$ but not use of 0.5 Ignore inequality and sign AWRT (0.94388)
	= **0.82 to 0.83**	A1	5	AWFW (0.82738)
	Total		**15**	
	TOTAL		**75**	

A-LEVEL

Mathematics

Statistics 3 – MS03

Mark scheme

6360

June 2014

Version/Stage: Final

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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

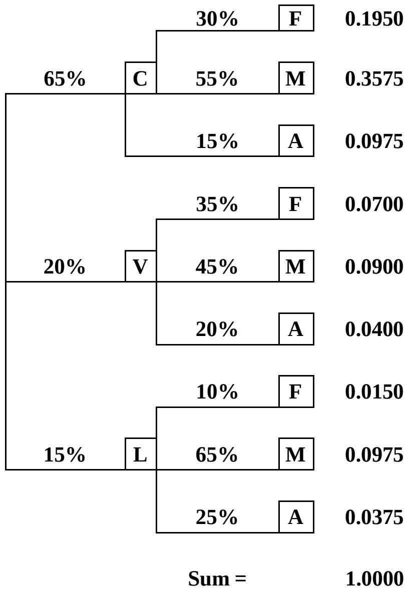
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1 (a)	96% $\Rightarrow$ z = **2.05 to 2.06**	B1		AWFW (2.0537)
	$\hat{p} = \frac{23}{200}$ = **0.115**	B1		CAO; or equivalent
	Approximate CI for p: $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Used
	$0.115 \pm 2.0537\sqrt{\frac{0.115 \times 0.885}{200}}$	AF1		F on $\hat{p}$ and z
	0.115 ± 0.046 or **(0.069, 0.161)**	A1	5	CAO/AWRT; AWRT
(b)	2 in 50 = $\frac{2}{50}$ = **0.04 < LCL or CI**	BF1		F on LCL or CI
	Thus evidence to **reject supplier's claim**	Bdep1	2	Dependent on BF1 Accept fairly definitive conclusion
		Total	**7**	

Q	Solution	Marks	Total	Comments
2	H_0: $\mu_B = \mu_G$ H_1: $\mu_B \neq \mu_G$	B1		At least H_1; allow suffices of 1 & 2 or X & Y, etc
	SL α = 0.05 (5%) CV z = (±)**1.96**	B1		AWRT (1.9600)
	$z = \frac{\lvert \bar{b} - \bar{g} \rvert}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_G^2}{n_G}}} = \frac{\lvert 21.35 - 21.90 \rvert}{\sqrt{\frac{0.5625}{20} + \frac{0.9025}{15}}}$	M1		Numerator
		M1		Denominator
	= (±)**1.85**	A1		Dependent on at least M1 M0 AWRT (1.8510) Ignore sign (p-value = 0.0642)
	Evidence, at 5% level, that $\boldsymbol{\mu_B = \mu_G}$ **or** **No evidence**, at 5% level, that $\boldsymbol{\mu_B \neq \mu_G}$	AF1	6	F on CV & z-value; consistent signs Definitive conclusion $\Rightarrow$ AF0 F on 5% & p-value; consistent areas
		Total	**6**	

Q	Solution	Marks	Total	Comments
3 (a)	 	B1 B1 B1	3	Shape; 3×3 branches Labels; C, V, L **and** ≥ 1 F, M, A Percentages or equivalent for C, V, L **and** ≥ 1 F, M, A
(b) (i)	$P((C \cup L) \cap M) = P(C \cap M) + P(L \cap M)$ $= (0.65 \times 0.55) + (0.15 \times 0.65)$ $= 0.3575 + 0.0975 =$ **<u>0.455 or 91/200</u>**	M1 A1	(2)	 CAO
(ii)	$P(L \mid A) = P(L \cap A) \div P(A)$ $= \dfrac{0.15 \times 0.25}{(0.65 \times 0.15) + (0.20 \times 0.20) + (0.15 \times 0.25)}$ $= \dfrac{0.0375}{0.0975 + 0.04 + 0.0375} = \dfrac{0.0375}{0.1750} =$ **<u>0.214</u>**	M1 M1 A1	(3)	Numerator Denominator AWRT (0.21429) CAO (3/14)
(iii)	$P(F' \mid C') = P(F' \cap C') \div P(C')$ $= \dfrac{0.2 \times (0.45 + 0.20) + 0.15(0.65 + 0.25)}{0.35}$ $= \dfrac{0.13 + 0.135}{0.35} = \dfrac{0.265}{0.35} =$ **<u>0.757</u>**	M1 M1 A1	(3)	Numerator Denominator AWRT (0.75714) CAO (53/70)
			8	
(c)	Prob $= P(C \mid F) \times P(V \mid F) \times P(L \mid F) \times 3! =$ $\dfrac{(0.65 \times 0.30) \times (0.20 \times 0.35) \times (0.15 \times 0.10)}{\left[(0.65 \times 0.30) + (0.20 \times 0.35) + (0.15 \times 0.10)\right]^3} \times 6$ $= \dfrac{(0.195 \times 0.07 \times 0.015) \text{ or } (0.00020475)}{0.28^3} \times 6$ $=$ **<u>0.056</u>**	M1 M1 M1 A1	4	Numerator Denominator $\times$ 3! or 6 AWRT (0.05596) CAO (351/6272)
		Total	**15**	

Q	Solution	Marks	Total	Comments
4 (a)	98% $\Rightarrow$ z = **<u>2.32 to 2.33</u>**	B1		AWFW (2.3263)
	CI for $\mu_E - \mu_G$: $(\bar{e} - \bar{g}) \pm z\sqrt{\frac{s_E^2}{n_E} + \frac{s_G^2}{n_G}}$	M1 m1		General form used Correct form used for SD Accept pooling
	$(42.6 - 39.7) \pm 2.3263\sqrt{\frac{6.2^2}{50} + \frac{5.3^2}{50}}$	AF1		F on z Pooling gives $2.3263\sqrt{1.3306}$
	<u>2.9 ± 2.7 or (0.2, 5.6)</u>	A1	5	AWRT
(b) (i)	**Random**	B1	1	CAO
(ii)	**Large samples** (both > 25 or 30) so can apply **Central Limit Theorem**	B1 Bdep1	2	Dependent on B1
		Total	**8**	

Q	Solution	Marks	Total	Comments
5 (a)(i)	Distribution of X is **symmetrical** around 4	B1		Accept calculation
	$E(X^2) = 0.2^2 \times 0.05 + ... + 6^2 \times 0.05$	M1		Must show method for $E(X^2)$
	= 0.20 + 2.25 + 6.40 + 6.25 + 1.80 = **<u>16.9</u>**	A1		CAO
	$Var(X) = E(X^2) - 4^2 = 16.9 - 16 =$ **<u>0.9</u>**	B1	4	**AG**; must show method for $Var(X)$
(ii)	Cov(X, Y) = 14.4 – 4 × 3.7 = **<u>–0.4</u>**	M1 A1		
	$\rho_{XY} = \frac{-0.4}{\sqrt{0.9 \times 0.61}}$ = **<u>–0.54</u>**	M1 AF1	4	Expression AWRT (–0.53985) F on Cov(X, Y)
(b) (i)&(ii)	$E(T)$ = **<u>7.7</u>** $E(D)$ = **<u>0.3</u>**	B1		CAO; both
	$Var(T) = 0.9 + 0.61 + 2 \times (-0.4)$	M1		Use of either $Var(X \pm Y) = Var(X) + Var(Y) \pm 2Cov(X, Y)$
	= **<u>0.71</u>**	A1		CAO
	$Var(D) = 0.9 + 0.61 - 2 \times (-0.4) =$ **<u>2.31</u>**	A1	4	CAO
		Total	**12**	

Q	Solution	Marks	Total	Comments
6 (a)	$\text{Var}\left(\overline{X}_A - \overline{X}_B\right) = \frac{18.8}{n} + \frac{18.8}{n}$	M1		Award for $\frac{18.8}{n}$ or $\frac{(2)\sigma^2}{n}$
	$=$ **<u>37.6/*n*</u>**	A1	2	OE
(b)	$99\% \Rightarrow z =$ **<u>2.57 to 2.58</u>**	B1		AWFW (2.5758)
	Require: $2 \times z \times \sqrt{\frac{37.6}{n}} \le 5$	M1		Award if "no 2", incorrect *z*-value, $\sqrt{\frac{18.8}{n}}$ or $\sqrt{\frac{(2)\sigma^2}{n}}$ or $\sqrt{\frac{c}{n}}$ from (a)
	$2 \times 2.5758 \times \sqrt{\frac{37.6}{n}} \le 5$	A1		Fully correct expression
	$n \ge \frac{4 \times 2.5758^2 \times 37.6}{25}$	m1		Attempt at solving equation involving $\sqrt{n}$ for n or $\sqrt{n}$
	$n =$ **<u>40</u>**	A1	5	CAO
Note	Accept equalities or strict inequalities throughout			
		Total	**7**	

Q	Solution	Marks	Total	Comments
7(a) (i)	$\underline{X \sim \text{Po}(\lambda)}$			
	$\text{E}(X) = \sum_{x=0}^{\infty} x \times \frac{e^{-\lambda}\lambda^x}{x!}$	M1		Used; ignore limits until A1 Accept a list of ≥3 terms summed
	$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$	M1		Factor of (at least) λ Division of $x!$ by x
	$= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda e^{-\lambda} e^{\lambda} = \underline{\boldsymbol{\lambda}}$ $\quad (y = x - 1)$	A1	3	**AG**; fully correct convincing solution with valid reason for (= λ)
(ii)	$\text{Var}(X) = \text{E}(X^2) - \lambda^2 = (\lambda^2 + \lambda) - \lambda^2 = \underline{\boldsymbol{\lambda}}$	B1	1	**AG**; fully correct convincing solution
(b)(i)	H_0: $\lambda = 10$ H_1: $\lambda > 10$	B1		Both; here or in (b)(ii)(A) and only mark available here if not exact test
	$P(X \geq 15 \mid \lambda = 10) = \mathbf{1 - (0.9165 \text{ or } 0.9513)}$	M1		
	= **<u>0.083 to 0.084</u>**	A1		AWFW (0.0835)
	Calculated p-value > 0.05 (5%)	m1		Comparison with 0.05
	No evidence, at 5% level, that $\boldsymbol{\lambda} > \mathbf{10}$	AF1	5	OE; F on p-value Definitive conclusion ⇒ AF0
(ii)(A)	5% ⇒ CV for z = **<u>1.64 to 1.65</u>**	B1		AWFW; seen anywhere (1.6449)
	$z = \frac{241(-0.5) - 200}{\sqrt{200 \text{ or } 241}}$ = **<u>2.86 to 2.9</u>**	M1 A1		OE; allow (+0.5) AWFW
	Evidence, at 5% level, that $\boldsymbol{\lambda} > \mathbf{10}$	AF1	4	OE; F on z-value & CV Definitive conclusion ⇒ AF0
(B)	$\frac{\text{CV}(-0.5) - 200}{\sqrt{200 \text{ or } 241}} = 1.6449$	M1 AF1		OE; allow (+0.5) but must be for **total** number of faults F on {(CV for z) & (z-statistic)} in (A)
	CV for X = **<u>223 to 224</u>**	A1	3	AWFW
(C)	P(Type II error) = P(accept H_0 | H_0 false)	B1		OE; stated or used
	$P(X < \text{CV} \mid \lambda = 12) =$ $P\left(Z < \frac{(222 \text{ to } 224) - 240}{\sqrt{240 \text{ or } 200}}\right) =$	M1		OE; FT on CV from (B)
	$P(Z < -1.1 \text{ to } -1.03) = 1 - P(Z < 1.03 \text{ to } 1.1)$	m1		Area change
	= 1 – (0.848 to 0.865) = **<u>0.13 to 0.16</u>**	A1	4	AWFW
		Total	**20**	

A-LEVEL

Mathematics

Statistics 3 – MS03

Mark scheme

6360

June 2015

Version/Stage: 1.0: Final

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Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS03

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN4 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage is shown

GN5 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds

GN6 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c\times10^{-n}$

Q	Solution	Marks	Total	Comments
1(a)	$r = \dfrac{3095}{\sqrt{7410 \times 1642}} = \underline{\mathbf{0.887}}$	M1 A1		Numerical expression AWRT (0.88729)
	or $r = \underline{\mathbf{0.887}}$ $r = \underline{\mathbf{0.88 \text{ to } 0.89}}$	(B2) (B1)	**2**	AWRT AWFW
Note	**1** $\sum x = 3036$ $\sum x^2 = 775518$ $\sum xy = 561719$ $\sum y = 2208$ $\sum y^2 = 407914$ $\bar{x} = 253$ $\bar{y} = 184$			
(b)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1		Both; do not accept in terms or r but accept in words providing clear indication of population pmcc
	SL $\alpha = 0.01$ (1%)			
	CV $r = \underline{\mathbf{(+)0.658 \text{ to } (+)0.6581}}$	B1		AWFW (0.6581)
	Calculated r > Tabulated r	M1		Comparison; can be implied by conclusion
	Evidence, at 1% level, of **a positive correlation** between the right foot length and right hand length of males aged between 19 years and 25 years	AF1	**4**	F on r and CV OE in context
Note	**1** For H_1: $\rho \neq 0$ then CV $r = (\pm)0.7079$ so same conclusion $\Rightarrow$ B0 B0 M1 AF1			
		Total	**6**	

Q	Solution	Marks	Total	Comments
2(a)	99% $\Rightarrow$ z = **2.57 to 2.58**	B1		AWFW (2.5758)
	CI for 26 weeks is:			
		M1		$(507-416) \pm z\sqrt{a}$
	$(507-416) \pm 2.5758\sqrt{507+416}$	m1		$z\sqrt{507+416}$
		A1		Correct expression; $2.32 \le z \le 2.58$
	ie 91 ± (78 to 78.5) or (12.5 to 13, 169 to 169.5)			
	Dividing by 26 gives: **3.5 ± 3.0 or (0.5, 6.5)**	M1 A1		CAO/AWRT or AWRT
	OR			
	99% $\Rightarrow$ z = **2.57 to 2.58**	(B1)		AWFW (2.5758)
	CI for 1 week is:	(B1)		19.5 & 16
	$\left(\frac{507}{26}-\frac{416}{26}\right) \pm 2.5758\sqrt{\frac{507}{26^2}+\frac{416}{26^2}} =$	(M1)		$(19.5-16.0) \pm z\sqrt{b}$
	$(19.5-16.0) \pm 2.5758\sqrt{\frac{19.5}{26}+\frac{16.0}{26}}$	(m1)		$z\sqrt{\frac{35.5}{26}}$ or $z\sqrt{35.5}$
		(A1)		Correct expression; $2.32 \le z \le 2.58$
	ie **3.5 ± 3.0 or (0.5, 6.5)**	(A1)	6	CAO/AWRT or AWRT
(b)	Since CI **is above 0**	B1		OE; providing CI > 0
	Emilia's belief is **justified**	Bdep1	2	Dependent on B1; OE in context
		Total	**8**	

Q	Solution	Marks	Total	Comments
3	S: 0.55 L: 30 VL: 0.15			**In (a)(i) & (iv), accept any equivalent fractional answer with den ≤ 100 or the equivalent percentage answer with %- sign (see GN4)**
(a) (i)	$P(S \cap £1) = 0.55 \times 0.20 = \underline{\mathbf{0.11}}$	B1	**(1)**	CAO
(ii)	$P(£0) =$ $(0.55 \times 0.70) + (0.30 \times 0.65) + (0.15 \times 0.55)$ $= 0.385 + 0.195 + 0.0825 = \underline{\mathbf{0.662 \text{ to } 0.663}}$	M1 A1	**(2)**	>1 term correct; may be implied AWFW (0.6625)
(iii)	$P(L \mid £0) = \dfrac{P(L \cap £0)}{P(£0)} = \dfrac{0.30 \times 0.65}{(ii)}$ $= \dfrac{0.195}{0.6625} = \underline{\mathbf{0.294 \text{ to } 0.295}}$	M1 A1	**(2)**	May be implied AWFW (0.29434)
(iv)	$P(VL \mid >£0) = \dfrac{P(VL \cap >£0)}{P(>£0)} = \dfrac{0.15 \times 0.45}{1-(ii)}$ $= \dfrac{0.0675}{0.3375} = \underline{\mathbf{0.2}}$	M1 M1 A1	**(3)**	Numerator Denominator CAO
			8	
(b)	$P((S \cap L \cap VL) \mid >£0) =$ $\dfrac{0.55 \times 0.30}{0.3375} \times \dfrac{0.30 \times 0.35}{0.3375} \times \dfrac{0.15 \times 0.45}{0.3375} \times 6 =$ $\dfrac{0.165 \times 0.105 \times 0.0675 \times 6}{0.3375^3} = \dfrac{0.0011694375 \times 6}{0.3375^3}$ **or** $= \dfrac{22}{45} \times \dfrac{14}{45} \times \dfrac{9}{45} \times 6 = \dfrac{16632}{91125} = \dfrac{616}{3375}$ $= \underline{\mathbf{0.182 \text{ to } 0.183}}$	M1 M1 m1 A1	**4**	>1 term correct in numerator (1 – (ii)) in denominator 6 or 3!; must have at least one M1 AWRT (0.18252)
		Total	**12**	

Q	Solution	Marks	Total	Comments
4(a)	H_0: p = 0.60 (60%) H_1: $p \neq$ 0.60 (60%)	B1		Both
	5% ⇒ z = **1.96**	B1		AWRT
	$\hat{p} = \dfrac{164}{250}$ = **0.656**	B1		CAO
	$z = \dfrac{0.656 - 0.6}{\sqrt{\dfrac{0.6 \times 0.4}{250}}}$	M1		Allow use of 0.656 in denominator
		m1		Correct denominator
	= **1.8 to 1.81**	A1		AWFW (1.80739) (p-value = 0.07070 > 0.05)
	No evidence, at 5% level, to suggest percentage **is not 60%** or is different	AF1	7	F on z and CV OE in context
Notes	1 (0.656 – 0.6)/√((0.656 × 0.344)/250) = 1.86392 ⇒ (B1) (B1) (B1) M1 m0 A0 (AF1) **2** ((163.5 or 164) – 150)/√60 = 1.74284 or 1.80739 ⇒ (B1) (B1) (B1) M1 m1 A1 (AF1) **3** (164.5 – 150)/√60 = 1.87194 ⇒ (B1) (B1) (B1) M1 m0 A0 (AF1) **4** ((163.5 or 164 or 164.5) – 150)/√56.416 = 1.79735 or 1.86392 or 1.93049 ⇒ (B1) (B1) (B1) M1 m0 A0 (AF1) **5** P($X \geq$ 164 | B(250, 0.6)) = 0.039794 > 0.025 ⇒ (B1) B4 M1 (AF1)			
(b)	H_0: p = 0.25 (25%) H_1: p < 0.25 (25%)	B1		Both
	Use of B(40, 0.25)	M1		May be implied
	P($X \leq$ 5) = **0.043**	A1		AWRT (0.0433)
	Calculated p-value < 0.05 (5%)	M1		Comparison of p-value and 0.05
	Evidence, at 5% level, to suggest percentage **is less than 25%**	AF1	5	F on p-value and 0.05 OE in context
Notes	**1** P($X \leq 4$) = 0.0160 and P($X \leq 6$) = 0.0962 **2** Use of normal approximation ⇒ B1 max			
(c)	98% ⇒ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	$z\sqrt{\dfrac{p(1-p)}{n}} = 2.3263\sqrt{\dfrac{0.3 \times 0.7}{n}} < 0.05$	M1		Use of $z \times \text{SD}(\hat{p})$
		A1		Allow use of p = 0.5, (×2) & z = 2.05 to 2.33
	$n > \dfrac{2.3263^2 \times 0.21}{0.05^2}$ = **450 to 460**	m1 A1	5	Attempt at solution for n AWFW; must be an integer (455)
Note	**1** Use of p = 0.5 gives n = 541.2 so 535 to 545 (AWRT) ⇒ B1 M1 AF1 M1 A0			
		Total	**17**	

Q	Solution	Marks	Total	Comments
5 (a)(i)	$\text{E}(X)=\sum_{x=0}^{n}x\binom{n}{x}p^x(1-p)^{n-x}\ =$	M1		Used; ignore limits until A1
	$np\sum_{x=1}^{n-1}\frac{(n-1)!}{(x-1)!(n-x)!}p^{x-1}(1-p)^{n-x}\ =$	M1		**≥ 2** of: factor of np plus p^x to p^{x-1}, $n!$ to $(n–1)!$ and $x!$ to $(x–1)!$
	$np\sum_{x=1}^{n-1}\text{B}(n-1,\ p)\ =\ np$	A1	3	Fully complete and correct derivation **AG**
(ii)	$\text{Var}(X)=\text{E}(X^2)-n^2p^2$ $\text{E}(X(X-1))=\text{E}(X^2)-np\ =\ n(n-1)p^2$	M1		Both used; OE
	so $\text{Var}(X)=n(n-1)p^2+np-n^2p^2\ =\ \underline{\mathbf{np(1-p)}}$	A1	2	Fully complete and correct derivation
Notes	**1** $\text{E}(X(X-1))\ =\ \text{E}(X^2)-np\ =\ \text{V}(X)+n^2p^2-np\ =\ n(n-1)p^2\ \Rightarrow\ \text{V}(X)\ =\ np(1-p)\ \Rightarrow$ M1 A1 **2** $\text{E}(X^2)\ =\ n^2p^2-np^2\ +np\ \Rightarrow\ \text{V}(X)\ =\ n^2p^2-np^2\ +np\ -n^2p^2\ =\ np(1-p)\ \Rightarrow$ M1 A1			
(b)(i)	$\frac{\text{Var}(Y)}{\text{E}(Y)}=\frac{np(1-p)}{np}=1-p=\frac{2.985}{3}=0.995$	M1		OE
	so $p\ =\ \underline{\mathbf{0.005}}$ and so $n\ =\ \frac{3}{0.005}\ =\ \underline{\mathbf{600}}$	A1 A1	3	CAO both
(ii)	$\frac{\text{Var}(U)}{\text{E}(U)}=\frac{np(1-p)}{np}=1-p=\frac{6.25}{5}=1.25$	M1		OE
	$\Rightarrow$ $\mathbf{p<0}$ **or** $\mathbf{(1-p)>1}$ which is impossible	A1	2	Indication that $p<0$ or $(1-p)>1$
(c)	$\text{E}(W)=2\times5+10\ =\ \underline{\mathbf{20}}$	B1		CAO; must be justified
	$\text{Var}(W)=2^2\times5\ =\ \underline{\mathbf{20}}$	B1		CAO; must be justified
	No odd values **or** no values < 10	B1	3	Either
(d)	$n\ =\ 5000\ \&\ p\ =\ 0.002\ \Rightarrow\ \underline{\mathbf{Po(10)}}$	B1		
	$\text{P}(6\ \le\ AB–\ \le\ 12)\ =\ \mathbf{0.7916}$ $-\ \mathbf{(0.0671\ \ or\ \ 0.1301)}$	M1		
	$=\ \underline{\mathbf{0.724\ \ to\ \ 0.725}}$	A1	3	AWFW (0.7245)
Note	**1** Use of normal approximation $\Rightarrow$ B0 M0 A0			
		Total	**16**	

Q	Solution	Marks	Total	Comments
6 (a)	$\text{Var}(\bar{L}-2\bar{S}) = \text{Var}(\bar{L}) + 2^2\,\text{Var}(\bar{S})$	M1		Use of + and 2^2
	but $\text{Var}(S) = \text{Var}(L) = \sigma^2$			
	so $\text{Var}(\bar{S}) = \text{Var}(\bar{L}) = \dfrac{\sigma^2}{n}$	M1		Use of $\dfrac{\sigma^2}{n}$
	giving $\text{Var}(\bar{L}-2\bar{S}) = \underline{\mathbf{5\sigma^2/n}}$	A1	3	CAO
Note	1 Answer of $3\sigma^2/n \Rightarrow$ M0 M1 A0			
(b) (i)	H_0: $\mu_L = 2\mu_S$	B1		Award B1 B0 for $\mu_L = \mu_S$
	H_1: $\mu_L > 2\mu_S$	B1		
	$10\% \Rightarrow z = \underline{\mathbf{1.28}}$	B1		AWRT (1.2816)
	$z = \dfrac{522-(2\times 258)}{\sqrt{\dfrac{5\times 8^2}{25}}}$	M1		Numerator; allow $(522-258)$
		M1		Denominator; allow $\sqrt{2\times 8^2/25}$ OE or $\sqrt{3\times 8^2/25}$ OE
	$= \underline{\mathbf{1.68}}$	A1		AWRT (1.67705)
	Evidence, at 10% level, to suggest that $\mu_L > 2\mu_S$	Adep1	7	Dep on A1 OE in context
(ii)	CV is given by $\dfrac{\bar{l}-2\bar{s}}{\sqrt{\dfrac{5\times 8^2}{25}}}$ or $\dfrac{\bar{l}-2\bar{s}}{\sqrt{12.8}} = 1.28(16)$	M1		Completely correct equality
	ie CV = $\underline{\mathbf{4.585}}$	A1	2	AWRT; **AG** (4.58519)
(iii)	P(Type II error) = P(accept H_0 | H_0 false)	B1		OE; stated or used
	$= P(\bar{L}-2\bar{S} < 4.585 \mid \mu_L - 2\mu_S = 10) =$			
	$P\left(Z < \dfrac{4.585-10}{\sqrt{\dfrac{5\times 8^2}{25}}}\right) = P(Z < \underline{\mathbf{\pm 1.51}})$	M1		Must have correct numerator Denominator; allow $\sqrt{2\times 8^2/25}$ OE or $\sqrt{3\times 8^2/25}$ OE
		A1		AWRT (–1.51354)
	$= \underline{\mathbf{0.064\ \text{to}\ 0.066}}$	A1	4	AWFW (0.06504)
		Total	**16**	

A-level

Mathematics

MS03 – Statistics 3
Mark scheme

6360
June 2016

Version 1.0: Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS03

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

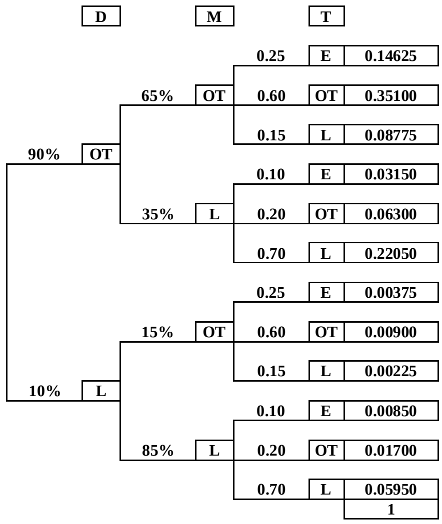
GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

Q	Solution	Marks	Total	Comments
1 (a)	$\hat{p}_M = \frac{264}{480} = \frac{11}{20}$ **or** **0.55** and $\hat{p}_W = \frac{220}{500} = \frac{11}{25}$ **or** **0.44**	B1		Both CAO $\left(\hat{p}_p = 0.49388\right)$
	95% $\Rightarrow$ z = **1.96**	B1		AWRT (1.95996)
	CI for $p_M - p_W$ is			
	$(0.55-0.44) \pm 1.96\sqrt{\frac{0.55\times0.45}{480}+\frac{0.44\times0.56}{500}}$	M1 M1 AF1		$\left(\hat{p}_M - \hat{p}_W\right) \pm (1.96 \text{ or } 1.64 \text{ to } 1.65)\sqrt{a}$ Expression for $\sqrt{a}$ F on $\hat{p}_M$ and $\hat{p}_W$ and z
	ie **0.11 ± 0.06** or **(0.05, 0.17)**	A1	6	CAO/AWRT (0.06224) AWRT
Note	**1** A pooled estimate of variance (0.11 ± 0.06062) $\Rightarrow$ B1 B1 M1 M0 AF0 A1 (a maximum of 4 marks)			
(b)	CI > 0.025 or LCL > 0.025	BF1		F on CI providing CI > 0.025
	Evidence to **support the claim**	Bdep1	2	Dep on BF1
Notes	**1** There must be a reference to 0.025 (OE) **and** a clear comparison with the answer to (a) **2** Accept answers suggesting that selections may not be random and/or independent **or** that based on 480 & 500 may not be representative **or** changes of opinions between opinion poll and referendum			
		Total	**8**	

Q	Solution	Marks	Total	Comments
2 (a)		M1 M1 M1	3	Shape; $2 \times 2 \times 3 = 12$ branches Labels; OT & L **and** E & OT & L Attempt at percentages or probabilities for D **and** M **and** T
(b)(i)	$P(T_{OT}) = 0.351 + 0.063 + 0.009 + 0.017 = \underline{\mathbf{0.44}}$	B1	**(1)**	CAO
(ii)	$P(T_{OT} \mid D_{OT}) = \frac{0.351+0.063}{0.9} = \frac{0.414}{0.9}$ $= \underline{\mathbf{0.46}}$	M1 A1	**(2)**	Correct numerator; PI CAO
(iii)	$P(T_{E \text{ or } OT} \mid D_{OT}) = 0.46 + \frac{0.14625+0.0315}{0.9} =$ $0.46 + \frac{0.17775}{0.9} = 0.46 + \underline{\mathbf{0.197 \text{ to } 0.20}}$ $= \underline{\mathbf{0.657 \text{ to } 0.66}}$	M1 A1 A1	**(3)**	(ii) + p AWFW; PI (0.1975) AWFW (0.6575)
(iv)	$P(T_{E \text{ or } OT} \mid M_{OT}) =$ $\frac{0.14625+0.351+0.00375+0.009}{0.9 \times 0.65 + 0.1 \times 0.15} = \frac{0.51}{0.6}$ $= \underline{\mathbf{0.85}}$	M1 A1	**(2)**	Correct numerator; PI CAO
SCs	**1** $0.25 + 0.60 = 0.85 \Rightarrow$ B2 **2** $1 - 0.15 = 0.85 \Rightarrow$ B2			
			8	
(c)	$P(T_{OT} \mid D_{OT}) = 0.46$ $P(T_E \mid D_{OT}) = 0.6575 - 0.46 = \underline{\mathbf{0.197 \text{ to } 0.20}}$ $P(T_{OT} \cap T_{OT} \cap T_E) = 0.46^2 \times 0.1975$ $\times 3$ $= \underline{\mathbf{0.125 \text{ to } 0.126}}$	 B1 M1 m1 A1	4	 AWFW; PI (0.1975) $p_1^2 \times p_2$ CAO AWFW (0.12537)
		Total	**15**	

Q	Solution	Marks	Total	Comments
3	H_0: $\lambda_B = \lambda_A$ H_1: $\lambda_B > \lambda_A$	B1		Both
	CV(1%) $\Rightarrow$ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	$\hat{\lambda}_A = \frac{315}{30}$ = **10.5** and $\hat{\lambda}_B = \frac{747}{60}$ = **12.45**	B1		Both CAO $\hat{\lambda} = \frac{1062}{90}$ = **11.8**
	$z = \frac{12.45 - 10.5}{\sqrt{\frac{12.45}{60} + \frac{10.5}{30}}}$ = **2.61**	M1 M1 Adep1		Correct numerator Correct denominator AWRT; dep on M1 M1 (2.61163)
	or			
	$z = \frac{12.45 - 10.5}{\sqrt{11.8\left(\frac{1}{60} + \frac{1}{30}\right)}}$ = **2.54**	(M1) (M1) (A1)		Correct numerator Correct denominator AWRT; dep on M1 M1 (2.53868)
	Thus evidence, at 1% level, to **support the claim** that $\lambda_B > \lambda_A$	Adep1	7	Dep on z-value and CV
		Total	7	

Q	Solution	Marks	Total	Comments
4(a) (i)	R: mean = **35** variance = **125**	B1	(1)	Both CAO
(ii)	F: mean = **115**	B1		CAO
	variance = $15^2 + 20^2 + (2 \times 15 \times 20 \times 0.25)$	M1		Attempt at $a^2 + b^2 \pm (2) \times a \times b \times 0.25$
	= **775**	A1	(3)	CAO
(iii)	T: mean = **150** variance = **900**	B1 A1	(2)	CAO CAO
(iv)	D: mean = **35**	B1		CAO
	variance = $20^2 + 15^2 - (2 \times 20 \times 15 \times 0.25)$ **or** = (ii) $- 4 \times 15 \times 20 \times 0.25$	(M1)		Only if M1 not scored in (ii)
	= **475**	B1	(2)	CAO
			8	
(b) (i)	$P(T < 180) = P\left(Z < \frac{180-150}{\sqrt{900}}\right)$	M1		Standardising 180 with values from (a)(iii) but must involve $\sqrt{\ }$
	$= P(Z < 1)$ = **0.841**	A1	(2)	AWRT (0.84134)
(ii)	$P(W - V > 60) =$ $P(D > 60) = P\left(Z > \frac{60-35}{\sqrt{475}}\right)$	M1		Standardising 60 with values from (a)(iv) but must involve $\sqrt{\ }$
	$= P(Z > 1.147) = 1 - P(Z < 1.147)$	M1		Area change; can be implied by **any final answer < 0.5**
	$= 1 - (0.873$ to $0.875)$ = **0.125 to 0.127**	A1	(3)	AWFW (0.12567)
			5	
		Total	13	

Q	Solution	Marks	Total	Comments
5 (a)	$\bar{D}$ has a **normal** distribution with mean = **0** and variance $= \dfrac{\sigma^2}{n} + 1.5^2 \times \dfrac{\sigma^2}{n}$ $= \dfrac{\mathbf{3.25\sigma^2}}{\boldsymbol{n}}$	B1 B1 M1 A1	4	Normal CAO Must have (+ sign) & (1.5 or 1.5^2) but allow no ($\div n$) OE single expression
(b)	H_0: $\mu_{XL} = 1.5\mu_L$ H_1: $\mu_{XL} \neq 1.5\mu_L$ 5% $\Rightarrow$ $z =$ **(±)1.96** $z = \dfrac{\lvert 2261 - 1.5 \times 1509 \rvert}{\sqrt{\dfrac{3.25 \times 4.5^2}{50}}} = \dfrac{\pm 2.5}{\sqrt{1.31625}}$ $=$ **(±)2.18** Evidence, at 5% level, that **claim is not supported**	B1 B1 M1 M1 A1 Adep1	6	B1 both; allow any valid notation AWRT (1.95996) Numerator; allow (2261 – 1509) Denominator; allow $\sqrt{2 \times 4.5^2 / 50}$ OE AWRT (2.17907) Dep on z-value and CV Must have consistent signs
		Total	**10**	

Q	Solution	Marks	Total	Comments
6 (a)	$\text{E}(X)=\sum_{x=0}^{\infty} x\frac{\text{e}^{-\lambda}\lambda^x}{x!}$ =	M1		Used; ignore limits until A1
	$\lambda\sum_{x=1}^{\infty}\frac{\text{e}^{-\lambda}\lambda^{x-1}}{(x-1)!}$ =	M1		Factor of λ plus $x!$ to $(x-1)!$
	with $y = x - 1$ $\lambda\sum_{y=0}^{\infty}\frac{\text{e}^{-\lambda}\lambda^{y}}{y!} = \lambda\times 1 = \lambda$	A1	(3)	Fully complete and correct derivation AG
	$\text{E}(X(X-1))=\sum_{x=0}^{\infty} x(x-1)\frac{\text{e}^{-\lambda}\lambda^x}{x!}$ =	M1		Used; ignore limits until A1
	$\lambda^2\sum_{x=2}^{\infty}\frac{\text{e}^{-\lambda}\lambda^{x-2}}{(x-2)!} = \lambda^2$	A1	(2)	Factor of λ^2 plus $x!$ to $(x-2)!$ and fully complete and correct derivation
	$\text{Var}(X)=\text{E}(X^2)-(\text{E}(X))^2$ =	M1		Used
	$\text{E}(X(X-1))+\lambda-\lambda^2=\lambda$	A1	(2)	Fully complete and correct derivation
Note	1 Other derivations are possible throughout (a)			
			7	
(b)(i)	<u>**Po(0.75)**</u>	B1		PI
	P(0 faults) = $\text{e}^{-0.75}$ = <u>**0.472**</u>	B1	2	AWRT (0.47237)
(ii)	Po(37.5) ⇒ <u>**N(37.5, 37.5)**</u>	B1		Normal with mean = variance = 37.5 in (A) or (B)
(A)	P(F < 30) = $\text{P}\left(Z<\frac{29.5-37.5}{\sqrt{37.5}}\right)$	M1		Standardising (29.5 or 30 or 30.5) with C's mean = variance
	= P(Z < –1.30639) = 1 – P(Z < 1.30639)	m1		Area change; can be implied by **any final answer < 0.5**
	= <u>**0.095 to 0.097**</u>	A1	(4)	AWFW (0.09571)
(B)	P(35 ≤ F ≤ 45) = P(F ≤ 45.5 or 45) – P(F ≤ 34.5 or 35) =	M1		Area difference
	P(Z < <u>**1.31**</u>) – P(Z < –<u>**0.49**</u>)	A1		Both AWRT (1.30639 & 0.48990)
	= <u>**0.591 to 0.597**</u>	A1	(3)	AWFW (0.59219)
SC	1 Use of Poisson: **(A)** 0.092 (AWRT) ⇒ B2 **(B)** 0.582 (AWRT) ⇒ B1 (max of 3 marks)			
			7	
	Total for (a) & (b)		**16**	

Q	Solution	Marks	Total	Comments
6	**Total for (a) & (b)**		**16**	
(c)	98% $\Rightarrow$ $z =$ **<u>2.32 to 2.33</u>**	B1		AWFW (2.3263)
	CI: $\begin{pmatrix} 49 \\ 4.9 \\ 0.98 \\ 0.098 \end{pmatrix} \pm \begin{pmatrix} 2.32 \text{ to } 2.33 \\ 2.05 \text{ to } 2.06 \end{pmatrix} \begin{pmatrix} \sqrt{49} = 7 \\ \sqrt{4.9/10} = 0.7 \\ \sqrt{0.98/50} = 0.14 \\ \sqrt{0.098/500} = 0.014 \end{pmatrix}$	M1 A1 A1		$\lambda \pm z\sqrt{a}$ Any correct value for λ Correct expression for a given λ
	or 49 ± (16.2 to 16.4) = (32.6 to 32.8, 65.2 to 65.4) 4.9 ± (1.62 to 1.64) = (3.26 to 3.28, 6.52 to 6.54) 0.98 ± (0.32 to 0.34) = (0.64 to 0.66, 1.30 to 1.32) 0.098 ± (0.032 to 0.034) = (0.064 to 0.066, 0.130 to 0.132)			
	Dividing by 500, 50, 10 or 1 as appropriate	B1		CAO
	ie **<u>0.098 ± (0.032 to 0.034)</u>** **or** **<u>(0.064 to 0.066, 0.130 to 0.132)</u>**	A1	6	CAO ± AWFW (0.03257) AWFW
		Total	**22**	