S2 Continuous Random Variables Past Qu.s

1) June 2004 no. 7

The random variable T represents the number of minutes that a train is late for a particular scheduled journey on a randomly chosen day.

(i) Give a reason why T could probably not be well modelled by a normal distribution. [1]

(ii) The following probability density function is proposed as a model for the distribution of T:

$$f(t) = \begin{cases} \frac{1}{67500}t(t-30)^2 & 0 \leqslant t \leqslant 30, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of $E(T)$. [3]

On a randomly chosen day I will allow for the train to be up to t_0 minutes late. I wish to choose the value of t_0 for which the probability that the train is less than t_0 minutes late is 0.95.

(b) Show that t_0 satisfies the equation

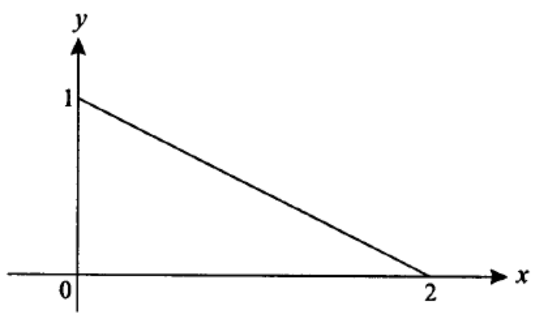
$$t_0^4 - 80t_0^3 + 1800t_0^2 = 256\,500.$$ [4]

(c) Show that the value of t_0 lies between 22 and 23. [2]

2) Jan 2005 no.6

Two models are proposed for the continuous random variable X. Model 1 has probability density function

$$f_1(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$



The diagram shows the graph of $y = f_1(x)$.

(i) Find the upper quartile of X (i.e., find the value q such that $P(X < q) = 0.75$) according to model 1. [4]

Model 2 has probability density function

$$f_2(x) = \begin{cases} k(4 - x^2) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) The graph of $y = f_2(x)$ intersects the y-axis at the point $(0, 4k)$. Copy the diagram showing the graph of $y = f_1(x)$. On your copy sketch the graph of $y = f_2(x)$, explaining how you can tell without doing any integration that $4k < 1$. [4]

(iii) State whether the value of q obtained from model 1 is greater than, equal to, or less than the value given by model 2. Use your diagram to justify your answer. [2]

3) June 2005 no.3

The lifetime, in years, of an electrical appliance may be modelled by the random variable T with probability density function

$$f(t) = \begin{cases} \frac{k}{t^2} & 1 \leqslant t \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{4}{3}$. [2]

(ii) Find the value of the mean of T, giving your answer in the form $a \ln b$. [3]

(iii) Find the time t_0 for which $P(T > t_0) = 0.1$. [3]

4) Jan 2006 no.8

A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where n and k are positive constants.

(i) Find k in terms of n. [3]

(ii) Show that $E(X) = \frac{n+1}{n+2}$. [3]

It is given that $n = 3$.

(iii) Find the variance of X. [3]

(iv) One hundred observations of X are taken, and the mean of the observations is denoted by $\bar{X}$. Write down the approximate distribution of $\bar{X}$, giving the values of any parameters. [3]

(v) Write down the mean and the variance of the random variable Y with probability density function given by

$$g(y) = \begin{cases} 4\left(y + \frac{4}{5}\right)^3 & -\frac{4}{5} \leqslant y \leqslant \frac{1}{5}, \\ 0 & \text{otherwise.} \end{cases}$$ [3]

5) June 2006 no. 1

Calculate the variance of the continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{3}{37}x^2 & 3 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$ [6]

6) Jan 2007 no.6

The continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} a + bx & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(i) Show that $2a + 2b = 1$. [3]

(ii) It is given that $E(X) = \frac{11}{9}$. Use this information to find a second equation connecting a and b, and hence find the values of a and b. [6]

(iii) Determine whether the median of X is greater than, less than, or equal to $E(X)$. [4]

7) June 2007 no.7

Two continuous random variables S and T have probability density functions as follows.

$$S: \quad f(x) = \begin{cases} \frac{1}{2} & -1 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

$$T: \quad g(x) = \begin{cases} \frac{3}{2}x^2 & -1 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) *Sketch* on the same axes the graphs of $y = f(x)$ and $y = g(x)$. [You should not use graph paper or attempt to plot points exactly.] [3]

(ii) Explain in everyday terms the difference between the two random variables. [2]

(iii) Find the value of t such that $P(T > t) = 0.2$. [5]

8) Jan 2008 no.7

A continuous random variable X_1 has probability density function given by

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find $E(X_1)$ and $Var(X_1)$. [5]

(iv) Sketch the graph of $y = f(x - 1)$. [2]

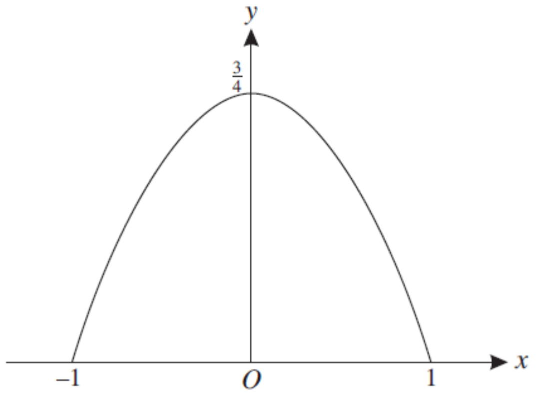
(v) The continuous random variable X_2 has probability density function $f(x - 1)$ for all x. Write down the values of $E(X_2)$ and $Var(X_2)$. [2]

9) June 2008 no.5

(i) A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{4}(1 - x^2) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

The graph of $y = f(x)$ is shown in the diagram.



Calculate the value of $Var(X)$. [4]

9) continued

(ii) A continuous random variable W has probability density function given by

$$g(x) = \begin{cases} k(9 - x^2) & -3 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Sketch the graph of $y = g(x)$. [1]

(b) By comparing the graphs of $y = f(x)$ and $y = g(x)$, explain how you can tell without calculation that $9k < \frac{3}{4}$. [2]

(c) State with a reason, but without calculation, whether the standard deviation of W is greater than, equal to, or less than that of X. [2]

10) Jan 2009 no.5

The continuous random variables S and T have probability density functions as follows.

$$S: \quad f(x) = \begin{cases} \frac{1}{4} & -2 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

$$T: \quad g(x) = \begin{cases} \frac{5}{64}x^4 & -2 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Sketch, on the same axes, the graphs of f and g. **[3]**

(ii) Describe in everyday terms the difference between the distributions of the random variables S and T. (Answers that comment only on the shapes of the graphs will receive no credit.) **[2]**

(iii) Calculate the variance of T. **[4]**

11) Jan 2010 no.7

The continuous random variable T is equally likely to take any value from 5.0 to 11.0 inclusive.

(i) Sketch the graph of the probability density function of T. **[2]**

(ii) Write down the value of $E(T)$ and find by integration the value of $\text{Var}(T)$. **[5]**

(iii) A random sample of 48 observations of T is obtained. Find the approximate probability that the mean of the sample is greater than 8.3, and explain why the answer is an approximation. **[6]**

12) June 2010 no.8

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^{-a} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants and a is greater than 1.

(i) Show that $k = a - 1$. **[3]**

(ii) Find the variance of X in the case $a = 4$. **[5]**

(iii) It is given that $P(X < 2) = 0.9$. Find the value of a, correct to 3 significant figures. **[4]**