Stats 2 Poisson Distribution Questions

1 A study undertaken by Goodhealth Hospital found that the number of patients each month, X, contracting a particular superbug can be modelled by a Poisson distribution with a mean of 1.5 .

(a) (i) Calculate $\mathrm{P}(X = 2)$. *(2 marks)*

(ii) Hence determine the probability that exactly 2 patients will contract this superbug in each of three consecutive months. *(2 marks)*

(b) (i) Write down the distribution of Y, the number of patients contracting this superbug in a given 6-month period. *(1 mark)*

(ii) Find the probability that at least 12 patients will contract this superbug during a given 6-month period. *(2 marks)*

(c) State **two** assumptions implied by the use of a Poisson model for the number of patients contracting this superbug. *(2 marks)*

---

1 The number of A-grades, X, achieved in total by students at Lowkey School in their Mathematics examinations each year can be modelled by a Poisson distribution with a mean of 3.

(a) Determine the probability that, during a 5-year period, students at Lowkey School achieve a total of more than 18 A-grades in their Mathematics examinations. *(3 marks)*

(b) The number of A-grades, Y, achieved in total by students at Lowkey School in their English examinations each year can be modelled by a Poisson distribution with a mean of 7.

(i) Determine the probability that, during a year, students at Lowkey School achieve a total of fewer than 15 A-grades in their Mathematics and English examinations. *(3 marks)*

(ii) What assumption did you make in answering part (b)(i)? *(1 mark)*

---

2 The number of computers, A, bought during one day from the Amplebuy computer store can be modelled by a Poisson distribution with a mean of 3.5.

The number of computers, B, bought during one day from the Bestbuy computer store can be modelled by a Poisson distribution with a mean of 5.0.

(a) (i) Calculate $P(A = 4)$. *(2 marks)*

(ii) Determine $P(B \leqslant 6)$. *(1 mark)*

(iii) Find the probability that a total of fewer than 10 computers is bought from these two stores on one particular day. *(3 marks)*

(b) Calculate the probability that a total of fewer than 10 computers is bought from these two stores on at least 4 out of 5 consecutive days. *(3 marks)*

(c) The numbers of computers bought from the Choicebuy computer store over a 10-day period are recorded as

8	12	6	6	9	15	10	8	6	12

(i) Calculate the mean and variance of these data. *(2 marks)*

(ii) State, giving a reason based on your results in part (c)(i), whether or not a Poisson distribution provides a suitable model for these data. *(2 marks)*

---

2 The number of telephone calls per day, X, received by Candice may be modelled by a Poisson distribution with mean 3.5.

The number of e-mails per day, Y, received by Candice may be modelled by a Poisson distribution with mean 6.0.

(a) For any particular day, find:

(i) $P(X = 3)$; *(2 marks)*

(ii) $P(Y \geqslant 5)$. *(2 marks)*

(b) (i) Write down the distribution of T, the total number of telephone calls and e-mails per day received by Candice. *(1 mark)*

(ii) Determine $P(7 \leqslant T \leqslant 10)$. *(3 marks)*

(iii) Hence calculate the probability that, on each of three consecutive days, Candice will receive a total of at least 7 but at most 10 telephone calls and e-mails. *(2 marks)*

Stats 2 Poisson Distribution Answers

1(a)(i)	$P(X=2)=\dfrac{e^{-1.5}\times(1.5)^2}{2!}=0.251$	M1A1	2	
(ii)	$p=(0.251)^3=0.0158$	M1A1√	2	on their p from (i)
(b)(i)	$Y\sim P_o(9.0)$	B1	1	
(ii)	$P(Y\geq 12)=1-P(Y\leq 11)$ $=1-0.8030$ $=0.197$	M1 A1	2	
(c)	attacks patients: randomly (p constant)	B1		mean of 1.5 $\Rightarrow p$ small (B1) (unless very few patients)
	independently	B1	2	
	Total		**9**	

1(a)	For a 1-year period The number of A grades ~ $\text{Po}(3)$			
	For a 5-year period Number of A grades ~ $\text{Po}(15)$	B1		
	$P(\text{Total A-grades}>18)$ $=1-(\text{Total}\leq 18)$	M1		
	$=1-0.8195$ $=0.1805$ $=0.181$	A1	3	AWFW 0.180 to 0.181
(b)(i)	$X+Y\sim\text{Po}(10)$	B1		
	$P(X+Y\leq 14)=0.917$	M1A1	3	AWFW 0.916 to 0.917 incl
(ii)	X and Y are independent variables.	E1	1	
	Total		**7**	

2(a)(i)	$P(A=4)=\dfrac{e^{-3.5}\times(3.5)^4}{4!}=0.189$	M1A1	2	
(ii)	$P(B\le 6)=0.762$	B1	1	
(iii)	$T=A+B\sim Po(8.5)$			
	$P(T \text{ fewer than } 10)=P(T<10)$	M1		Use of Po (8.5)
	$=P(T\le 9)$	M1		$T\le 9$ attempted
	$=0.653$	A1	3	CAO
(b)	$X\sim B(5,0.653)$	B1		$X\sim B(5, \text{their } p)$
	$P(X\ge 4)=\binom{5}{4}(0.653)^4(0.347)$			
	$+(0.653)^5$	M1		
	$=0.31547+0.11873$			
	$=0.434$	A1√	3	On their p from (a)(iii)
(c)(i)	$\bar{x}=9.2$	B1		
	$s^2=9.29$	B1	2	$\sigma^2=8.36$
(ii)	Mean and variance have similar values which suggests that Poisson distribution may be appropriate	B1√ B1√	2	
	Total		**13**	

2(a)(i)	$P(X=3)=\dfrac{e^{-3.5}\times(3.5)^3}{3!}=0.216$	M1 A1	2	
(ii)	$P(Y\ge 5)=1-P(Y\le 4)$	M1		used
	$=1-0.2851$			
	$=0.715$	A1	2	
(b)(i)	$T\sim Po(9.5)$	B1	1	
(ii)	$P(7\le T\le 10)=P(T\le 10)-P(T\le 6)$	M1		
	$=0.6453-0.1649$	A1		
	$=0.480$	A1	3	Accept 0.48
(iii)	$p=(0.4804)^3=0.111$	M1 A1√	2	
	Total		**10**	
