Solutionbank S2
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 1

Question:

It is estimated that 4% of people have green eyes. In a random sample of size n, the expected number of people with green eyes is 5.

a Calculate the value of n.

The expected number of people with green eyes in a second random sample is 3.

b Find the standard deviation of the number of people with green eyes in this second sample. ***E***

Solution:

a $X \sim B(n, 0.04)$

X is the random variable 'number of people who have green eyes'.

X is binomial, as there is clear 'success' $(p = 0.04)$ i.e. a person has green eyes or not.

Take sample size n from the question.

$$E(X) = np$$
$$5 = 0.04n$$
$$n = 125$$

Use the expectation of a binomial distribution to form an equation.

Solve the equation.

b

$$E(X) = 3$$
$$np = 3$$
$$\text{Standard deviation} = \sqrt{npq}$$
$$= \sqrt{3(1-0.04)}$$
$$= \sqrt{2.88}$$
$$= 1.70$$

This is a second random sample so np is different from part **a**.

The binomial distribution has a formula for the variance, npq, so square root this.

$q = 1 - p$ and $np = 3$

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Review Exercise
Exercise A, Question 2

Question:

In a manufacturing process, 2% of the articles produced are defective. A batch of 200 articles is selected.

a Giving a justification for your choice, use a suitable approximation to estimate the probability that there are exactly 5 defective articles.

b Estimate the probability there are less than 5 defective articles. ***E***

Solution:

a $X \sim B(200, 0.02)$

X is random variable number of defective articles.

X is binomial as there is 'success' (article is defective) and failure (article is not defective).

n is large, p is small, and $np = 4$ so use Poisson approximation.

X is approximately poisson, $\lambda = 4$

$$P(X = 5) = \frac{e^{-4}4^5}{5!} = 0.1563$$

Use the formula for Poisson probability. You can also use tables: $P(X \le 5) - P(X \le 4)$ which gives the same answer.

b

$$P(X < 5) = P(X \le 4) = 0.6288$$

'Less than 5' does *not* include 5. X can only take discrete values and tables *include* the value you look up.

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Review Exercise
Exercise A, Question 3

Question:

A continuous random variable X has probability density function

$$f(x) = \begin{cases} k(4x - x^3), & 0 \le x \le 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{1}{4}$.

Find

b $E(X)$,

c the mode of X,

d the median of X.

e Comment on the skewness of the distribution.

f Sketch $f(x)$. ***E***

Solution:

a

$$\int_0^2 k(4x - x^3)\,dx = 1$$

$$k\left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 1$$

Integrating a p.d.f. over an possible values of x gives 1. Here $x = 0$ to $x = 2$.

Make sure you integrate properly!

$$k(8-4) = 1$$

Substitute the limits and take care with the arithmetic here too.

$$k = \frac{1}{4}$$

b

$$E(X) = \int_0^2 x \times \frac{1}{4}(4x - x^3)\,dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]_0^2$$

$$= \frac{16}{15}$$

The expectation is $\int_0^2 x\mathrm{f}(x)\,dx$.

Expanding gives $\int_0^2 (x^2 - \frac{1}{4}x^4)\,dx$.

Integrate carefully and substitute the limits.

c

At mode, $\mathrm{f}'(x) = 0$

$$4 - 3x^2 = 0$$

$$x = \frac{2}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= 1.15 \text{ (3 s.f.)}$$

The mode is the value of x at the maximum of $\mathrm{f}(x)$, i.e. the highest point of the graph.

$$\mathrm{f}(x) = \frac{1}{4}(4x - x^3)$$

$$\mathrm{f}'(x) = \frac{1}{4}(4 - 3x^2)$$

Solving gives a surd which is 1.15 (3 s.f.).

d At median m

$$\int_0^m \frac{1}{4}(4x - x^3)\,dx = \frac{1}{2}$$

$$\frac{1}{4}(2m^2 - \frac{1}{4}m^4) = \frac{1}{2}$$

$$m^4 - 8m^2 + 8 = 0$$

$$m^2 = 4 \pm 2\sqrt{2}$$

$$m = 1.08$$

$\mathrm{F}(m) = \frac{1}{2}$ so integrate $\mathrm{f}(x)$.

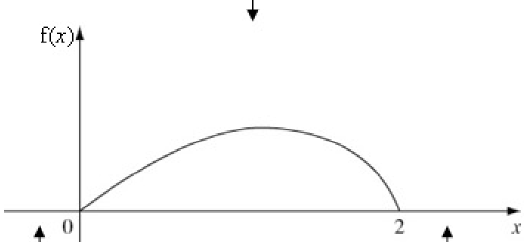
Integrate carefully and substitute m. Bottom limit is 0.

Form a quadratic in m^2 and solve using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

e mean(1.07) < median(1.08) < mode(1.15)

$\Rightarrow$ negative skew

f



Be careful as the graph is negatively skewed, but the mean, median and mode are close to each other.

Remember to add the regions $x < 0$ and $x > 2$ to your sketch.

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Review Exercise
Exercise A, Question 4

Question:

A fair coin is tossed 4 times.
Find the probability that

a an equal number of heads and tails occur,

b all the outcomes are the same,

c the first tail occurs on the third throw. ***E***

Solution:

a Let X be the random variable 'the number of heads'.
$X \sim \mathrm{B}(4, 0.5)$

Binomial distribution with number of tosses, $n = 4$ and 'success' is head, 'failure' is tail.

$$\mathrm{P}(X=2) = {}^4C_2 0.5^2 \times 0.5^2 = \frac{4!}{2!2!} 0.5^2 \times 0.5^2 = 0.375$$

Use the formula for the probability of a binomial distribution.

$$\frac{4!}{2!2!} = \frac{4\times3\times2\times1}{(2\times1)\times(2\times1)} = 6$$

b $\mathrm{P}(X=4)$ or $\mathrm{P}(X=0)$

'Outcomes are the same'
HHHH i.e. $X = 4$
TTTT i.e. $X = 0$

$$= 2 \times 0.5^4 = 0.125$$

$\mathrm{P}(\mathrm{HHHH}) = 0.5^4$
$\mathrm{P}(\mathrm{TTTT}) = 0.5^4$
'of' means add them.

c

$$\mathrm{P}(\mathrm{HHT}) = 0.5^3 = 0.125$$

'The first tail occurs on the third throw' means the first two outcomes must be heads so no ${}^nC_r$ required.

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Review Exercise
Exercise A, Question 5

Question:

Accidents on a particular stretch of motorway occur at an average rate of 1.5 per week.

a Write down a suitable model to represent the number of accidents per week on this stretch of motorway.

Find the probability that

b there will be 2 accidents in the same week,

c there is at least one accident per week for 3 consecutive weeks,

d there are more than 4 accidents in a two-week period. ***E***

Solution:

a Let X be the random variable, 'the number of accidents per week'.
$X \sim \text{Po}(1.5)$

'Rate' used in the question indicates this is a Poisson model.

b

$$P(X=2) = \frac{e^{-1.5}1.5^2}{2} = 0.2510 = 0.251 \text{ (3 s.f.)}$$

This is the formula for a Poisson probability. You can also use tables to calculate $P(X \le 2) - P(X \le 1)$.

c

$$P(X \ge 1) = 1 - P(X=0) = 1 - e^{-1.5} = 0.7769$$

'At least one' so we want 'greater than or equal to 1'.

$X = 0$ is the only unknown not required.

P (at least one accident per week for 3 weeks)

$$= 0.7769^3 = 0.4689 = 0.469 \text{ (3 s.f.)}$$

We want first week *and* second week *and* third week.

d $X \sim \text{Po}(3)$

$$P(X > 4) = 1 - P(X \le 4) = 1 - 0.8153 = 0.1847 = 0.185 \text{ (3 s.f.)}$$

'More than 4' so 4 not included in the answer.

Use tables to find $P(X \le 4)$ and subtract from 1.

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Review Exercise
Exercise A, Question 6

Question:

The random variable $X \sim B(150, 0.02)$. Use a suitable approximation to estimate $P(X > 7)$. ***E***

Solution:

$X \sim Po(3)$

n is large, p is small.
$np = 150 \times 0.02 = 3$
so use Poisson approximation.

$$\begin{aligned} P(X > 7) &= 1 - P(X \le 7) \\ &= 1 - 0.9881 \\ &= 0.0119 (3 \text{ s.f.}) \end{aligned}$$

Use tables to look up $P(X \le 7)$ and subtract from 1.

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Review Exercise
Exercise A, Question 7

Question:

A continuous random variable X has probability density function $f(x)$ where,

$$f(x) = \begin{cases} kx(x-2), & 2 \le x \le 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

a Show that $k = \frac{3}{4}$.

Find

b $E(X)$,

c the cumulative distribution function $F(x)$.

d Show that the median value of X lies between 2.70 and 2.75. ***E***

Solution:

a

$$\int_2^3 kx(x-2)\,dx = 1$$

> $\int_2^3 f(x) = 1$

$$k\left[\frac{1}{3}x^3 - x^2\right]_2^3 = 1$$

> $f(x) = k(x^2 - 2x)$ then integrate.

$$k\left[(9-9) - \left(\frac{8}{3} - 4\right)\right] = 1$$

$$k\left(\frac{4}{3}\right) = 1$$

> Don't forget to substitute the bottom limit and subtract.

$$k = \frac{3}{4}$$

b

$$\mathrm{E}(x) = \int_2^3 \frac{3}{4}x^2(x-2)\,dx$$

> Use $\mathrm{E}(X) = \int x f(x)\,dx$.

$$= \left[\frac{3}{16}x^4 - \frac{1}{2}x^3\right]_2^3$$

> $xf(x) = \frac{3}{4}x^3 - \frac{3}{2}x^2$

$$= \left(\frac{3}{16}\times 3^4 - \frac{1}{2}\times 3^3\right) - \left(\frac{3}{16}\times 2^4 - \frac{1}{2}\times 2^3\right)$$

> Substitute the upper limit, 3, and subtract the substitution of the lower limit, 2.

$$= 2\frac{11}{16} \text{ or } 2.6875$$

> This is the *exact* answer.

c

$$\mathrm{F}(x) = \int_2^x \frac{3}{4}(t^2 - 2t)\,dt$$

> Use a variable upper limit with $\int f(t)\,dt$.

$$= \left[\frac{3}{4}\left(\frac{1}{3}t^3 - t^2\right)\right]_2^x$$

> Don't forget lower limit of 2.

$$= \left(\frac{3}{4}\left(\frac{1}{3}x^3 - x^2\right) - \frac{3}{4}\left(\frac{1}{3}\times 2^3 - 2^2\right)\right)$$

$$= \frac{1}{4}(x^3 - 3x^2 + 4)$$

$$\mathrm{F}(x) = \begin{cases} 0 & x \le 2 \\ \frac{1}{4}(x^3 - 3x^2 + 4) & 2 < x < 3 \\ 1 & x \ge 3 \end{cases}$$

> Display your answer carefully and don't forget $\mathrm{F}(x) = 0$ and $f(x) = 1$.

d Look at $\mathrm{F}(x)$.

$$\mathrm{F}(2.70) = 0.453$$
$$\mathrm{F}(2.75) = 0.527$$

> $\mathrm{F}(m) = 0.5$ is in between these.

0.5 lies between F(2.70) and F(2.75)
so m lies between 2.70 and 2.75

> Be careful to write your answer clearly and do not get confused between 2.70 and F(2.70) or 2.75 and F(2.75).

Alternative method

$$\frac{1}{4}(m^3 - 3m^2 + 4) = \frac{1}{2}$$

> Use your answer to *c*. The median, m, is where $\mathrm{F}(m) = \frac{1}{2}$.

$$m^3 - 3m^2 + 2 = 0$$

> This is a cubic, so it will be difficult to solve. You use the values given in the question and show that the left hand side changes sign.

$x = 2.75, x^3 - 3x^2 + 2 = 0.109315 > 0$

$x = 2.70, x^3 - 3x^2 + 2 = -0.187 < 0$

Root between 2.70 and 2.75 $\Rightarrow m$
between 2.70 and 2.75 since the cubic changes sign.

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Review Exercise
Exercise A, Question 8

Question:

The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are

a exactly 2 faulty bolts,

b more than 3 faulty bolts.

These bolts are sold in bags of 20. John buys 10 bags.

c Find the probability that exactly 6 of these bags contain more than 3 faulty bolts. ***E***

Solution:

a Let X be the random variable 'the number of faulty bolts'.

$X \sim B(20, 0.3)$

'Success' is 'faulty'. X is binomial with $n = 20$ bolts and probability of a faulty bolt, $p = 0.3$.

$$P(X = 2) = \frac{20!}{18!2!}(0.3)^2(0.7)^{18} = 0.0278$$

Substitute into the formula for binomial probability, don't forget $C_2^{20} = \frac{20!}{18!2!}$

You can use tables instead:

$$P(X \le 2) - P(X \le 1) = 0.0355 - 0.0076 = 0.0279$$

b

$$P(X > 3) = 1 - P(X \le 3) = 1 - 0.1071 = 0.8929$$

Use tables for this as you would need to use the formula 4 times to work out $1 - (P(X = 3) + P(X = 2) + P(X = 1) + P(X = 0))$ and you are more likely to make a mistake.

c P (exactly 6 of these bags contain more than 3 faulty bolts)

More than 3 faulty bolts in a bag of 20 is the answer to **b**.

$$= \frac{10!}{4!6!}(0.8929)^6(0.1071)^4 = 0.0140$$

10 bags bought so $n = 10$.
Answer to b is p.
So we are finding $P(X = 6)$ where $X \sim B(10, p)$.

$$\frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2) \times (6 \times 5 \times 4 \times 3 \times 2 \times 1)}$$

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Review Exercise
Exercise A, Question 9

Question:

a State two conditions under which a Poisson distribution is a suitable model to use in statistical work.

The number of cars passing an observation point in a 10-minute interval is modelled by a Poisson distribution with mean 1.

b Find the probability that in a randomly chosen 60-minute period there will be

- **i** exactly 4 cars passing the observation point,
- **ii** at least 5 cars passing the observation point.

The number of other vehicles, (i.e. other than cars), passing the observation point in a 60-minute interval is modelled by a Poisson distribution with mean 12.

c Find the probability that exactly 1 vehicle, of **any type**, passes the observation point in a 10-minute period. ***E***

Solution:

a Events occur at a constant rate.
Events occur independently or randomly.
Events occur singly.

> There is no context stated in **a**, but Poisson requires an event to occur.

b Let X be the random variable 'the number of cars passing the point'

i $X \sim \text{Po}(6)$

> For 10 minutes, $\lambda = 1$
> For 60 minutes, $\lambda = 6$
> **a** suggests this is Poisson with $\lambda = 6$.

$$P(X=4) = \frac{e^{-6}6^4}{4!} = 0.1339 = 0.134 \text{ (3 s.f.)}$$

> This is solved using the formula, but you can use tables and find $P(X \le 4) - P(X \le 3) = 0.2851 - 0.1512$.

ii

$$P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.2851 = 0.7149 = 0.715 \text{ (3 s.f.)}$$

> At least 5 means include 5 in your probability.

> Use tables here as otherwise the formula needs to be used 5 times.

c

$$\lambda = 1 + 2 = 3$$
$$P(X=1) = 3e^{-3} = 0.149$$

> For car, $\lambda = 1$
> For others, $\lambda = 2$ in 10 minutes

> $X = 1$ is '1 vehicle of any type'.

Alternative method

P(1 car and 0 other) + P(0 car and 1 other)

$$= 1e^{-1} \times e^{-2} + e^{-1} \times 2e^{-2}$$

> For 'other'
> 60-minute interval $\lambda = 12$
> 10-minute interval $\lambda = 2$
> For car, $\lambda = 1$

$$= 0.3679 \times 0.1353 + 0.3679 \times 0.2707 = 0.149$$

> 'and' means 'multiply'

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Review Exercise
Exercise A, Question 10

Question:

The continuous random variable Y has cumulative distribution function $F(y)$ given by

$$F(y)=\begin{cases}0, & y<1,\\ k(y^4+y^2-2), & 1\le y\le 2,\\ 1, & y>2.\end{cases}$$

a Show that $k=\frac{1}{18}$.

b Find $P(Y>1.5)$.

c Specify fully the probability density function $f(y)$. ***E***

Solution:

a

$$\begin{aligned}F(2) &= 1\\ k(2^4+2^2-2) &= 1\\ 18k &= 1\\ k &= \frac{1}{18}\end{aligned}$$

F(y) is the cumulative distribution function, so F(2) is found and equated to 1, the total probability.

b

$$\begin{aligned}P(Y>1.5) &= 1-P(Y\le 1.5)\\ &= 1-F(1.5)\\ &= 1-\frac{1}{18}(1.5^4+1.5^2-2)\\ &= 0.705\left[\text{or }\frac{203}{288}\right]\end{aligned}$$

c

$$\begin{aligned}f(y) &= \frac{dF(y)}{dy}\\ &= \frac{d}{dy}\left[\frac{1}{18}(y^4+y^2-2)\right]\\ &= \frac{1}{18}(4y^3+2y)\\ &= \frac{1}{9}(2y^3+y), 1\le y\le 2\end{aligned}$$

Differentiate the c.d.f. to get the p.d.f.

$$f(y)=\begin{cases}0, \text{ otherwise}\\ \frac{1}{9}(2y^3+y), & 1\le y\le 2\end{cases}$$

Set out f(y) clearly and don't forget $f(y)=0$.

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Review Exercise
Exercise A, Question 11

Question:

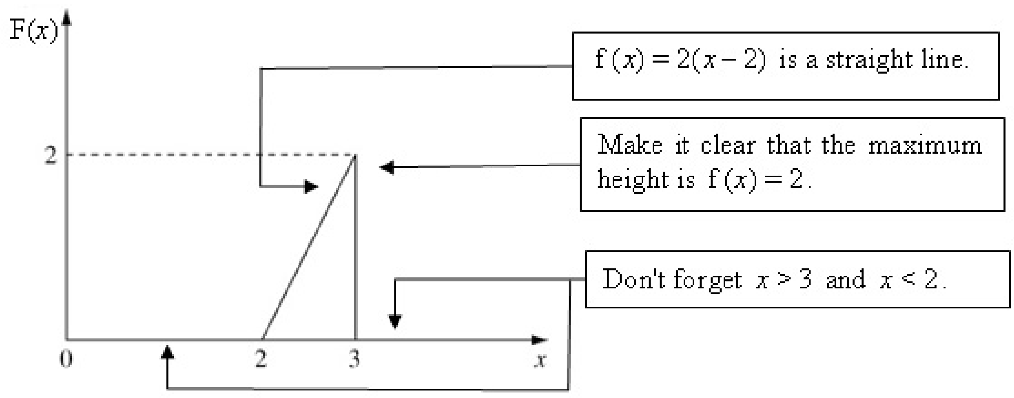
The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} 2(x-2), & 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch f(x) for all values of x.
b Write down the mode of X.
Find
c E(X),
d the median of X.
e Comment on the skewness of this distribution. Give a reason for your answer. ***E***

Solution:

a



b Mode of X is 3.

This is the value of x where $f(x)$ is at its greatest value.

c

$$\begin{aligned} E(x) &= \int_2^3 2x(x-2)\,dx \\ &= \left[\frac{2x^3}{3} - 2x^2\right]_2^3 \\ &= 2\tfrac{2}{3} \end{aligned}$$

Use $\int_2^3 x f(x)\,dx$.

Integrate after expanding to $2x^2 - 4x$.

d

$$\begin{aligned} \int_2^m 2(x-2)\,dx &= 0.5 \\ (x^2 - 4x)_2^m &= 0.5 \\ m^2 - 4m + 4 &= 0.5 \\ 2m^2 - 8m + 7 &= 0 \end{aligned}$$

$F(m) = 0.5$ for median.

$$\begin{aligned} m &= \frac{8 \pm \sqrt{64-56}}{4} \\ m &= \frac{4 \pm \sqrt{2}}{2} \\ m &= 2.71 \end{aligned}$$

Solve using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and cancel by 2.

Ignore $m = 1.29$ as outs the $2 \le x \le 3$.

e Negative skew

mean(2.6) < median(2.71) < mode(3)

$E(x)$ is the mean, $2\frac{2}{3}$.

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Review Exercise
Exercise A, Question 12

Question:

An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty.
Faulty components are detected at a rate of 1.5 per hour.

a Suggest a suitable model for the number of faulty components detected per hour.
b Describe, in the context of this question, two assumptions you have made in part **a** for this model to be suitable.
c Find the probability of 2 faulty components being detected in a 1-hour period.
d Find the probability of at least one faulty component being detected in a 3–hour period. ***E***

Solution:

a Let x be the random variable 'number of faulty components detected'
$X \sim \text{Po}(1.5)$

b *Faulty* components occur at a constant rate.
Faulty components occur independently and randomly.
Faulty components occur singly.

Make sure you write about the context of *faulty* components.

c $\text{P}(X=2) = \dfrac{e^{-1.5}(1.5)^2}{2!}$
$= 0.251$

Use the formula for the probability of a Poisson distribution with $\lambda = 1.5$. You could also use tables and $\text{P}(X \le 2) - \text{P}(X \le 1)$.

d $X \sim \text{Po}(4.5)$

Three-hour period, so $\lambda = 3 \times 1.5 = 4.5$

$\text{P}(X \ge 1) = 1 - \text{P}(X=0)$
$= 1 - e^{-4.5}$
$= 1 - 0.0111$
$= 0.9889$
$= 0.989$ (3 s.f.)

'At least 1' so 1 is included in the probability.

Use formula for Poisson.

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Review Exercise
Exercise A, Question 13

Question:

a Write down the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01.

b Find the probability that 2 consecutive calls will be connected to the wrong agent.

c Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent.

The call centre receives 1000 calls each day.

d Find the mean and variance of the number of wrongly connected calls.

e Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent. ***E***

Solution:

a If $X \sim B(n, p)$ and
n is large
p is small
then X can be approximated by $Po(np)$.

b

$$P(2 \text{ consecutive calls}) = 0.01^2 = 0.0001$$

c $X \sim B(5, 0.01)$

'Success' is 'connected to wrong agent' number of trials, $n = 5$.

$$P(X > 1) = 1 - P(X = 1) - P(X = 0) = 1 - 5(0.01)(0.99)^4 - (0.99)^5 = 0.00098$$

'More than 1' means 1 is not included in the probability.

d $X \sim B(1000, 0.01)$

$n = 1000$ calls per day $p = 0.01$ probability of a wrongly connected call

mean $= np = 10$
variance $= np(1-p) = 9.9$

Use formulae for mean and variance of binomial distribution.

e $X \sim Po(10)$

$np = 10$ from **d**.

$$P(X > 6) = 1 - P(X \le 6) = 1 - 0.1301 = 0.8699 = 0.870 \text{ (3 s.f.)}$$

'More than 6' means 6 is not included.

Look up 6 in Poisson tables with $\lambda = 10$ and subtract from 1.

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Review Exercise
Exercise A, Question 14

Question:

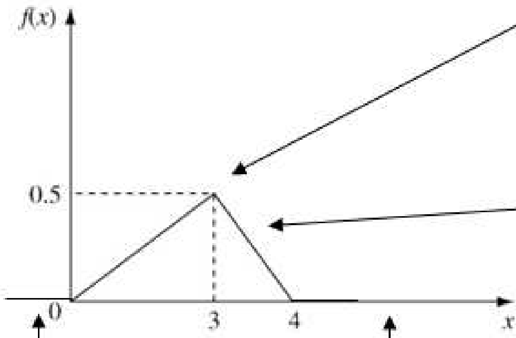
The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x, & 0 < x < 3, \\ 2 - \frac{1}{2}x, & 3 \le x < 4, \\ 0, & \text{otherwise.} \end{cases}$$

a Sketch the probability density function of X.
b Find the mode of X.
c Specify fully the cumulative distribution function of X.
d Using your answer to part **c**, find the median of X. ***E***

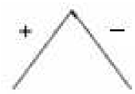
Solution:

a



Make sure you draw each part as a straight line.

$\frac{1}{6}x$ has positive gradient



$2-\frac{1}{2}x$ has negative gradient.

Don't forget $x \le 0$
$x \ge 4$

b Mode is $x = 3$.

Mode is the value of x at the highest point of f(x) on the sketch.

c

$$F(x) = \int_0^x \frac{1}{6}t\,dt = \frac{1}{12}x^2,\ 0 < x \le 3$$

$\int f(t)\,dt$ up to $x = 3$

$$\begin{aligned} F(x) &= \int_3^x \left(2-\tfrac{1}{2}t\right)dt + \int_0^3 \tfrac{1}{6}t\,dt \\ &= \left(2x-\tfrac{1}{4}x^2\right) - \left(2\times 3-\tfrac{1}{4}\times 3^2\right) + \tfrac{1}{12}\times 3^2 \\ &= 2x-\tfrac{1}{4}x^2-3,\quad 3 < x < 4 \end{aligned}$$

Don't forget the area between $x = 0$ and $x = 3$.

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{12}x & 0 < x \le 3 \\ 2x-\frac{1}{4}x^2-3 & 3 < x < 4 \\ 1 & x \ge 4 \end{cases}$$

Don't forget the ends of the c.d.f.

d

$$\begin{aligned} F(m) &= 0.5 \\ \frac{1}{12}m^2 &= 0.5 \\ m &= \sqrt{6} = 2.45 \end{aligned}$$

Median is 'half-way'.

$F(3) = \frac{3}{4}$, so m is between 0 and 3.

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Review Exercise
Exercise A, Question 15

Question:

The random variable J has a Poisson distribution with mean 4.

a Find $P(J \geq 10)$

The random variable K has a binomial distribution with parameters $n = 25,\ p = 0.27$.

b Find $P(K \leq 1)$

Solution:

a

$$\begin{aligned} P(J \geq 10) &= 1 - P(J \leq 9) \\ &= 1 - 0.9919 \\ &= 0.0081 \end{aligned}$$

$J \sim Po(4)$

Value from tables $n = 10, \lambda = 4$

b

$$\begin{aligned} P(K \leq 1) &= P(K = 0) + P(K = 1) \\ &= (0.73)^{25} + 25(0.73)^{24}(0.27) \\ &= 0.00392 \end{aligned}$$

$K \sim B(25, 0.27)$

Use formula for binomial probability.

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Review Exercise
Exercise A, Question 16

Question:

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 2x^2 - x^3, & 0 \le x \le 1, \\ 1, & x > 1. \end{cases}$$

a Find $P(X > 0.3)$.

b Verify that the median value of X lies between $x = 0.59$ and $x = 0.60$.

c Find the probability density function f(x).

d Evaluate E(X).

e Find the mode of X.

f Comment on the skewness of X.
Justify your answer.

Solution:

a

$$\begin{aligned} P(X > 0.3) &= 1 - F(0.3) \\ &= 1 - (2 \times 0.3^2 - 0.3^3) \\ &= 0.847 \end{aligned}$$

Remember to 'one minus' as we want $X > 0.3$.

b

$$\begin{aligned} F(0.59) &= 0.4908 < 0.5 \\ F(0.60) &= 0.5040 > 0.5 \end{aligned}$$

0.5 lies between F(0.59) and F (0.60)
so median lies between 0.59 and 0.60

'Verify' so write your answer clearly.

c

$$\begin{aligned} f(x) &= \frac{dF(x)}{dx} \\ &= \frac{d}{dx}(2x^2 - x^3) \\ f(x) &= 4x - 3x^2, 0 \le x \le 1 \\ f(x) &= 0, \text{otherwise} \end{aligned}$$

Differentiate c.d.f. to find p.d.f.

Remember $x < 0$ and $x > 1$.

$$f(x) = \begin{cases} 4x - 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

d

$$\begin{aligned} E(X) &= \int_0^1 x f(x)\, dx \\ &= \int_0^1 (4x^2 - 3x^3)\, dx \\ &= \left[4\frac{x^3}{3} - 3\frac{x^4}{4}\right]_0^1 \\ &= \frac{7}{12} \text{ or } 0.58\dot{3} \end{aligned}$$

Bottom limit substitutes to give 0.

$$\frac{4}{3} - \frac{3}{4} = \frac{7}{12}$$

e $\frac{df(x)}{dx} = -6x + 4$

For mode,

$$\begin{aligned} -6x + 4 &= 0 \\ x &= \frac{2}{3} \text{ or } 0.\dot{6} \end{aligned}$$

Mode occurs at maximum value of $f(x)$ where $\frac{df(x)}{dx} = 0$.

f $\text{mean}(0.58\dot{3}) < \text{median}(0.59 - 0.6) < \text{mode}(0.\dot{6})$
so negative skew