Chapter Review 7

1 $X \sim B(10, 0.20)$
$H_0: p = 0.20$ $H_1: p > 0.20$

$P(X \geqslant 3) = 1 - P(X \leqslant 2) = 1 - 0.6778 = 0.3222 > 0.05$
There is insufficient evidence to reject H_0. There is no evidence that the trains are late more often.

2 $X \sim B(5, 0.5)$
$H_0: p = 0.50$ $H_1: p > 0.50$

$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.8125 = 0.1875 > 0.05$
There is insufficient evidence to reject H_0
There is insufficient evidence that the company's claims are true.

3 **a** Fixed number; independent trials; two outcomes (pass or fail); p constant for each car.

b $X \sim B(5, 0.30)$
P(all pass) $= 0.70^5 = 0.16807$

c $X \sim B(10, 0.30)$
$H_0: p = 0.30$ $H_1: p < 0.30$

$P(X \leqslant 2) = 0.3828 > 0.05$
There is insufficient evidence to reject H_0.
There is no evidence that the garage fails fewer than the national average.

4 **a** $X \sim B(50, 0.1)$
$H_0: p = 0.10$ $H_1: p \neq 0.10$

$P(X \leqslant 1) = 0.0338$ (closer to 0.025)
$P(X = 0) = 0.0052$
Critical value = 1

$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.9421 = 0.0579$
$P(X \geqslant 10) = 1 - P(X \leqslant 9) = 1 - 0.9755 = 0.0245$ (closer to 0.025)
Critical value = 10
Critical region $X \leqslant 1$ and $X \geqslant 10$

b Actual significance level = 0.0338 + 0.0245 = 0.0583 = 5.83%

c $X \sim B(20, 0.1)$
$H_0: p = 0.1$ $H_1: p > 0.1$

$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.8670 = 0.133 > 0.1$
Accept H_0. There is no evidence that the proportion of faulty articles has increased.

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5 $X \sim B(20, 0.5)$
$H_0: p = 0.50$ $H_1: p \neq 0.50$
8 used Oriels powder.

$P(X \leqslant 8) = 0.2517 > 0.025$
There is insufficient evidence to reject H_0. There is no evidence that the claim is wrong.

6 $X \sim B(50, 0.2)$

a $P(X \leqslant 4) = 0.0185$ (closer to 0.025)
$P(X \leqslant 5) = 0.0480$
$c_1 = 4$

$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 1 - 0.9692 = 0.0308$ (closer to 0.025)
$P(X \geqslant 17) = 1 - P(X \leqslant 16) = 1 - 0.9856 = 0.0144$
$c_2 = 16$
Critical region $X \leqslant 4$ and $X \geqslant 16$

b Actual significance level = 0.0185 + 0.0308 = 0.0493 = 4.93%

c This is not in the critical region. Therefore, there is insufficient evidence to reject H_0. There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

7 a i A hypothesis is a statement made about the value of a population parameter. A hypothesis test uses a sample or an experiment to determine whether or not to reject the hypothesis.

ii The critical value is the first value to fall inside of the critical region.

iii The acceptance region is the region where we accept the null hypothesis.

b $H_0: p = 0.2$ $H_1: p \neq 0.2$
If H_0 is true $X \sim B(20, 0.2)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.05$ and $P(X \geqslant c_2) \leqslant 0.05$

For the lower tail:
$P(X = 0) = 0.0115 < 0.05$
$P(X \leqslant 1) = 0.0692 > 0.05$
So $c_1 = 0$

For the upper tail:
$P(X \geq 7) = 1 - P(X \leqslant 6) = 1 - 0.9133 = 0.0978 > 0.05$
$P(X \geqslant 8) = 1 - P(X \leqslant 7) = 1 - 0.9679 = 0.0321 < 0.05$
So $c_2 = 8$

So the critical region is $X = 0$ and $X \geqslant 8$

c Actual significance level = 0.0115 + 0.0321 = 0.0436 = 4.36%

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7 d As 7 does not lie in the critical region, H_0 is not rejected. Therefore, the proportion of times that Johan is late for school has not changed.

8 X is the number of days with zero or a trace of rain.
$X \sim B(30, 0.5)$
$H_0: p = 0.5$ $H_1: p > 0.5$

$P(X \geqslant 19) = 1 - P(X \leqslant 18) = 1 - 0.8998 = 0.1002 > 0.05$
$P(X \geqslant 20) = 1 - P(X \leqslant 19) = 1 - 0.9506 = 0.0494 < 0.05$
The critical region is $X \geqslant 20$

As 21 lies in the critical region, so we can reject the null hypothesis. There is evidence that the likelihood of a rain-free day in 2015 has increased.

9 a $H_0: p = 0.35$ $H_1: p \neq 0.35$
If H_0 is true $X \sim B(30, 0.35)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.025$ and $P(X \geqslant c_2) \leqslant 0.025$

For the lower tail:
$P(X \leqslant 5) = 0.0233 < 0.025$
$P(X \leqslant 6) = 0.0586 > 0.025$
So $c_1 = 5$

For the upper tail:
$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 1 - 0.9699 = 0.0301, 0.0301 - 0.025 = 0.0051$
$P(X \geqslant 17) = 1 - P(X \leqslant 16) = 1 - 0.9876 = 0.0124, 0.025 - 0.0124 = 0.0126$
So $c_2 = 16$

So the critical region is $X \leqslant 5$ and $X \geqslant 16$

b Actual significance test is $0.0233 + 0.0301 = 0.0534 = 5.34\%$

c $X = 4$ lies in the critical region so there is enough evidence to reject H_0.

10 a $X \sim B(20, 0.85)$

b $P(X = 16) = \binom{20}{16} 0.85^{16} 0.15^{4} = 0.18$ (2 d.p.)

c The test statistic is the proportion of patients who recover.
$H_0: p = 0.85$ $H_1: p < 0.85$

$P(X \leqslant 21) = 0.02778 < 0.05$
$P(X \leqslant 22) = 0.06978 < 0.05$
The critical region is $X \leqslant 21$

As 20 patients is in the critical region, there is enough evidence to reject H_0. The percentage of patients who recover after treatment with the new ointment is lower than 85%.

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11 a A hypothesis test about a population parameter p tests a null hypothesis H_0 which specifies a particular value for p, against an alternative hypothesis H_1 which is that p has increased, decreased or changed. H_1 will indicate whether the test is one- or two-tailed.

b Any one from:
- sales of coffee occur independently
- sales of coffee occur at a constant average rate (2 per minute)

c Let X be the number of cups of coffee sold in 30 minutes.
$X \sim \text{Po}(60)$
$P(X \geq 70) = 0.1118$
$P(X \geq 75) = 0.0341$
$P(X \geq 76) = 0.0260$
$P(X \geq 77) = 0.0196$

$P(X \leq 45) = 0.0266$
$P(X \leq 44) = 0.0190$

Therefore $\{X \leq 44 \text{ or } X \geq 77\}$ is the critical region.

d $0.0196 + 0.0190 = 0.0386$

12 Let X be the number of caravans hired out.

a $X \sim \text{Po}(6)$
$$P(X = 4) = \frac{e^{-6}6^4}{4!}$$
$= 0.134$ (3 s.f.)

b $H_0: \lambda = 6$, $H_1: \lambda > 6$
$P(X \geq 10) = 0.0839$
$P(X \geq 11) = 0.0426$
At the 5% significance level, the critical value is $X = 11$.
There is sufficient evidence to reject H_0.
There is evidence that the rate of hiring caravans has increased.

13 Let X be the number of particles of dirt in 20 litres of water after passing through the filter.
If filter is working properly, $X \sim \text{Po}(50)$
$H_0: \lambda = 50$, $H_1: \lambda > 50$
$Y \sim N(50, 50)$
$$P(Y \geq 64) = P\left(Z \geq \frac{63.5 - 50}{\sqrt{50}}\right) \text{ (apply continuity correction)}$$
$$= P(Z \geq 1.9091...)$$
$$= 1 - P(Z \leq 1.9091...)$$
$$= 1 - 0.9718...$$
$$= 0.02812$$
$0.02812 < 0.05$ therefore reject H_0.
There is evidence that the filter is failing to work properly.

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14 $H_0: \lambda = 10$, $H_1: \lambda > 10$
Let Y represent the number of sales in a six week period.
$Y \sim \text{Po}(60)$

$$\begin{aligned} P(Y \geq 84) &= 1 - P(Y \leq 83) \\ &= 1 - P(W \leq 83.5) \text{ where } W \sim N(60, 60) \\ &= 1 - P\left(Z \leq \frac{83.5 - 60}{\sqrt{60}}\right) \\ &= 1 - P(Z \leq 3.033...) \\ &= 1 - 0.9987\ldots \\ &= 0.00121 \end{aligned}$$

$0.00121 < 0.05$ therefore reject H_0.
There is evidence that the rate of sales of onion marmalade has increased after the program.

15 Let X be the number of plates that are rejected.
$X \sim B(150, 0.06)$
Use the approximation
$Y \sim \text{Po}(9)$
$H_0: \lambda = 9$, $H_1: \lambda > 9$
Critical region is $X \geq 15$
Therefore there is evidence that the process is getting worse.

16 $H_0: p = 0.45$, $H_1: p < 0.45$
Let X represent the number of undersized apples.
$X \sim B(200, 0.45)$

$$\begin{aligned} P(X \leq 60) &= P(W < 60.5) \text{ where } W \sim N(90, 49.5) \\ &= P\left(Z < \frac{60.5 - 90}{\sqrt{49.5}}\right) \\ &= P(Z < -4.192...) \\ &= 0.00001 \end{aligned}$$

$0.00001 < 0.05$ therefore reject H_0.
There is evidence that the new variety is better.

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