Exercise 7D

1 Using method 1
Distribution, $X \sim B(30, 0.50)$
$H_0: p = 0.50$ $H_1: p \neq 0.50$

$P(X \leqslant 10) = 0.0494 > 0.025$ (two-tailed)
There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.5$

2 Using method 2
$H_0: p = 0.3$ $H_1: p \neq 0.3$
If H_0 is true $X \sim B(25, 0.3)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.05$ and $P(X \geqslant c_2) \leqslant 0.05$ (two-tailed)

For the lower tail:
$P(X \leqslant 3) = 0.0332 < 0.05$
$P(X \leqslant 4) = 0.0905 > 0.05$
So $c_1 = 3$

For the upper tail:
$P(X \geqslant 11) = 1 - P(X \leqslant 10) = 1 - 0.9022 = 0.0978 > 0.05$
$P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.9558 = 0.0442 < 0.05$
So $c_2 = 12$

The observed value of 10 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt $p = 0.3$.

3 Using method 2
$H_0: p = 0.75$ $H_1: p \neq 0.75$
If H_0 is true $X \sim B(10, 0.75)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.025$ and $P(X \geqslant c_2) \leqslant 0.025$ (two-tailed)

For the lower tail:
$P(X \leqslant 4) = 0.0197 < 0.025$
$P(X \leqslant 5) = 0.0781 > 0.025$
So $c_1 = 4$

For the upper tail:
$P(X \geqslant 10) = 1 - P(X \leqslant 9) = 1 - 0.9437 = 0.0563 > 0.025$
So there is no upper tail.

The observed value of 9 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt p = 0.75.

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4 Using method 2

H_0: $p = 0.6$ H_1: $p \neq 0.6$

If H_0 is true $X \sim B(20, 0.6)$

Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.005$ and $P(X \geqslant c_2) \leqslant 0.005$ (two-tailed)

For the lower tail:

$P(X \leqslant 5) = 0.001612 < 0.005$

$P(X \leqslant 6) = 0.006466 > 0.005$

So $c_1 = 5$

For the upper tail:

$P(X \geqslant 17) = 1 - P(X \leqslant 16) = 1 - 0.984 = 0.016 > 0.005$

$P(X \geqslant 18) = 1 - P(X \leqslant 17) = 1 - 0.9964 = 0.0036 < 0.005$

So $c_2 = 18$

The observed value of 1 lies in the critical region so H_0 is rejected. Therefore, there is reason to doubt that $p = 0.6$.

5 Using method 2

H_0: $p = 0.02$ H_1: $p \neq 0.02$

If H_0 is true $X \sim B(50, 0.02)$

Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.01$ and $P(X \geqslant c_2) \leqslant 0.01$ (two-tailed)

For the lower tail:

$P(X = 0) = 0.3642 > 0.01$

So there is no lower tail.

For the upper tail:

$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.9822 = 0.0178 > 0.01$

$P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.9968 = 0.0033 < 0.01$

So $c_2 = 5$

The observed value of 4 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt $p = 0.02$.

6 Using method 1

Distribution, $X \sim B(20, 0.50)$

H_0: $p = 0.50$ H_1: $p \neq 0.50$

$P(X \leqslant 6) = 0.0577 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so we conclude there is no evidence the coin is biased.

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7 $X \sim B(20, 0.20)$

a Using method 2
$H_0: p = 0.20 \quad H_1: p \neq 0.20$

$P(X \leqslant 1) = 0.0692$
$P(X = 0) = 0.0115$ (closer to 0.025)
Critical value $= 0$

$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.9900 = 0.0100$
$P(X \geqslant 8) = 1 - P(X \leqslant 7) = 1 - 0.9679 = 0.0321$ (closer to 0.025)
Critical region $X = 0$ and $X \geq 8$

b Actual significance level is $0.0115 + 0.0321 = 0.0436 = 4.36\%$

c $X = 8$ is in the critical region. There is enough evidence to reject H_0. The hospital's proportion of complications differs from the national figure.

8 Using method 2
The test statistic is the number of cracked bowls.
$H_0: p = 0.1 \quad H_1: p \neq 0.1$
If H_0 is true $X \sim B(20, 0.1)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.05$ and $P(X \geqslant c_2) \leqslant 0.05$ (two-tailed)

For the lower tail:
$P(X = 0) = 0.1216 > 0.05$
So there is no lower tail.

For the upper tail:
$P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.8670 = 0.133 > 0.05$
$P(X \geqslant 6) = 1 - P(X \leqslant 5) = 1 - 0.9568 = 0.0432 < 0.05$
So $c_2 = 6$

The observed value of 1 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to doubt $p = 0.1$. So the proportion of cracked bowls has not changed.

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9 Using method 2
The test statistic is the number of carrots longer than 7 cm.
H_0: $p = 0.25$ H_1: $p \neq 0.25$
If H_0 is true $X \sim B(30, 0.25)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.025$ and $P(X \geqslant c_2) \leqslant 0.025$ (two-tailed)

For the lower tail:
$P(X \leqslant 2) = 0.0106 < 0.025$
$P(X \leqslant 3) = 0.0374 > 0.025$
So $c_1 = 2$

For the upper tail:
$P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.9493 = 0.0507 > 0.025$
$P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9784 = 0.0216 < 0.025$
So $c_2 = 13$

The observed value of 13 lies in the critical region so H_0 is rejected. Therefore, there is reason to doubt $p = 0.25$. So the probability of a carrot being longer than 7 cm has increased.

10 Using method 2
The test statistic is the number of patients correctly identified.
H_0: $p = 0.96$ H_1: $p \neq 0.96$
If H_0 is true $X \sim B(75, 0.96)$
Let c_1 and c_2 be the two critical values so $P(X \leqslant c_1) \leqslant 0.05$ and $P(X \geqslant c_2) \leqslant 0.05$

For the lower tail:
$P(X \leqslant 68) = 0.03046 < 0.05$
$P(X \leqslant 69) = 0.07978 > 0.05$
So $c_1 = 68$

For the upper tail:
$P(X \geqslant 74) = 1 - P(X \leqslant 73) = 1 - 0.8069 = 0.1931 > 0.05$
$P(X \geqslant 75) = 1 - P(X \leqslant 74) = 1 - 0.9532 = 0.0468 < 0.05$
So $c_2 = 75$

The observed value of 63 lies in the critical region so H_0 is rejected. Therefore, there is reason to doubt $p = 0.96$. So the new test does not have the same probability of success as the old test.

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