Exercise 7B

1 a The critical value is the first value to fall inside of the critical region.

b A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.

c The acceptance region is the area in which we accept the null hypothesis.

2 Assume H_0 is true, then $X \sim B(10, 0.2)$

$P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.8791 = 0.1209 > 0.05$
$P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.9672 = 0.0328 < 0.05$

The critical value is $x = 5$ and the critical region is $X \geqslant 5$ since $P(X \geqslant 5) = 0.0328 < 0.05$

3 Assume H_0 is true, then $X \sim B(20, 0.15)$

$P(X \leqslant 1) = 0.1756 > 0.05$
$P(X = 0) = 0.0388 < 0.05$

The critical value is $x = 0$ and the critical region is $X = 0$

4 a Assume H_0 is true, then $X \sim B(20, 0.4)$

$P(X \leqslant 4) = 0.0510 > 0.025$
$P(X \leqslant 3) = 0.0160 < 0.025$

The critical value is $x = 3$

$P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9790 = 0.0210 < 0.025$
$P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.9435 = 0.0565 > 0.025$

The critical value is $x = 13$
The critical region is $X \geqslant 13$ and $X \leqslant 3$

b The actual significance level is 0.021 + 0.016 = 0.037 = 3.7%

5 Assume H_0 is true, then $X \sim B(20, 0.15)$

$B(X = 0) = 0.0388 < 0.05$
$B(X \leqslant 1) = 0.1756 > 0.05$

The critical value is $x = 0$ and the critical region is $X = 0$

6 a Assume H_0 is true, then $X \sim B(10, 0.63)$

$P(X = 10) = 0.63^{10} = 0.0098 < 0.05$

$P(X \geqslant 9) = 0.0098 + 10(0.63)^9(0.37) = 0.0675 > 0.05$

The critical value is $x = 10$ and the critical region is $X = 10$

b The actual significance level is 0.0098 = 0.98%

7 a The test statistic is the number of components in the sample that fail.

b H_0: $p = 0.3$, H_1: $p < 0.3$

c Assume H_0 is true, then $X \sim B(20, 0.3)$

$P(X \leqslant 2) = 0.0355$ (closer to 0.05)
$P(X \leqslant 3) = 0.1071$

The critical region is $X \leqslant 2$

d 0.0355 = 3.55%

8 a The test statistic is the number of seedlings that survive.
H_0: $p = \frac{1}{3}$, H_1: $p > \frac{1}{3}$

b Assume H_0 is true, then $X \sim B(36, \frac{1}{3})$

Using a calculator
$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 1 - 0.8906 = 0.1094 > 0.1$
$P(X \geqslant 17) = 1 - P(X \leqslant 16) = 1 - 0.9416 = 0.0584 < 0.1$

The critical region is $X \geqslant 17$

c 0.0584 = 5.84%

9 a In a given time, the number of customers choosing lasagne out of the total number.
H_0: $p = 0.2$, H_1: $p \neq 0.2$

b Assume H_0 is true, then $X \sim B(25, 0.2)$

Consider the lower tail:
$P(X \leqslant 0) = 0.0038$
$P(X \leqslant 1) = 0.0274$ (closer to 0.025)

Consider the upper tail:
$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.9532 = 0.0468$
$P(X \geqslant 10) = 1 - P(X \leqslant 9) = 1 - 0.9827 = 0.0173$ (closer to 0.025)

The critical region is $X \leqslant 1$ and $X \geqslant 10$.

9 c The probability of incorrectly rejecting H_o is 0.0274 + 0.0173 = 0.0447 = 4.47%

Challenge

a Assume H_0 is true then $X \sim B(50, 0.7)$

Consider the lower tail:
$P(X \leqslant 29) = 0.0478$ (closer to 0.05)
$P(X \leqslant 30) = 0.0848$

Consider the upper tail:
$P(X \geqslant 41) = 1 - P(X \leqslant 40) = 1 - 0.9598 = 0.0402$ (closer to 0.05)
$P(X \geqslant 40) = 1 - P(X \leqslant 39) = 1 - 0.9211 = 0.0789$

The critical region is $X \leqslant 29$ and $X \geqslant 41$

b The probability of one observation falling within the critical region is 0.0478 + 0.0402 = 8.8%
The probability of two observations falling within the critical region is $0.088^2 = 0.007744 = 0.77\%$
The probability that Chloe has incorrectly rejected H_0 is 0.77%