Chapter Review 6

1 **a** A list of all the patients on the surgery books.

b A patient.

2 **a** Any two from:
It would take too long.
It would cost too much.
It could be difficult to get hold of all members.

b A list of all members of the gym.

c A member of the gym.

3 **a** A sampling frame has to be some sort of list – it may not be possible to list a population.

b A sample is usually easier to do, quicker to do and not as costly as a census.
(Also, a census is not appropriate if the testing process would destroy each sampling unit.)

4 **a** A statistic is a quantity calculated solely from the observations of a sample.

b **i** is a statistic

ii is not a statistic as it depends on the value μ

5 **a** The light bulbs would all be destroyed.

b A light bulb.

6 **a** Any two from:
It is quicker to do.
It is cheaper to do.
It is easier to do.

b It can be biased OR it is subject to natural variations.

c A numbered list of all 400 call centre operators.

d A call-centre operator.

e Yes, because he is using only the values from a sample. There are no parameters.

7 Any two from:
It gives everyone's views.
It is unbiased.
It is easy to conduct a census when the population has only 10 members.

8 **a** and **b** are statistics, **c** and **d** are not statistics since they involve a population parameter.

9 a $\text{E}(X)=\frac{1}{2}\times 5+\frac{1}{3}\times 10+\frac{1}{6}\times 20=\frac{55}{6}$

$\text{Var}(X)=\frac{1}{2}\times 5^2+\frac{1}{3}\times 10^2+\frac{1}{6}\times 20^2-\left(\frac{55}{6}\right)^2=\frac{1025}{36}$

b (5, 5), (10, 10), (20, 20), (5, 10), (10, 5), (5, 20), (20, 5), (10, 20), (20, 10)

c The sample space diagram shows the total of two randomly chosen coins.

		Coin 1					
		5	**5**	**5**	**10**	**10**	**20**
Coin 2	**5**	10	10	10	15	15	25
	5	10	10	10	15	15	25
	5	10	10	10	15	15	25
	10	15	15	15	20	20	30
	10	15	15	15	20	20	30
	20	25	25	25	30	30	40

The sample space diagram shows the mean of two randomly chosen coins.

		Coin 1					
		5	**5**	**5**	**10**	**10**	**20**
Coin 2	**5**	5	5	5	7.5	7.5	12.5
	5	5	5	5	7.5	7.5	12.5
	5	5	5	5	7.5	7.5	12.5
	10	7.5	7.5	7.5	10	10	15
	10	7.5	7.5	7.5	10	10	15
	20	12.5	12.5	12.5	15	15	20

The sampling distribution for $\overline{Y}$ is shown in the table.

y	5	7.5	10	12.5	15	20
$\text{P}(Y=y)$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{36}$

10 a (6, 6, 6), (6, 6, 10), (6, 10, 6), (10, 6, 6), (6, 10, 10), (10, 6, 10), (10, 10, 6), (10, 10, 10)

b The median can only take the values 6 and 10.
Let $\text{P}(6) = p = 0.6$
Let $\text{P}(10) = q = 0.4$
$\text{P}(N = 6) = \text{P}(6, 6, 6) + \text{P}(6, 6, 10) + \text{P}(6, 10, 6) + \text{P}(10, 6, 6)$
$= ppp + ppq + pqp + qpp$
$= 0.6 \times 0.6 \times 0.6 + 0.6 \times 0.6 \times 0.4 + 0.6 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.6$
$= 0.648$
$\text{P}(N = 10) = \text{P}(6, 10, 10) + \text{P}(10, 6, 10) + \text{P}(10, 10, 6) + \text{P}(10, 10, 10)$
$= pqq + qpq + qqp + qqq$
$= 0.6 \times 0.4 \times 0.4 + 0.4 \times 0.6 \times 0.4 + 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.4 \times 0.4$
$= 0.352$

n	6	10
$\text{P}(N = n)$	0.648	0.352

10 c The mode can only take the values 6 and 10.

Let P(6) = p = 0.6

Let P(10) = q = 0.4

$P(M = 6) = P(6, 6, 6) + P(6, 6, 10) + P(6, 10, 6) + P(10, 6, 6)$

$= ppp + ppq + pqp + qpp$

$= 0.6 \times 0.6 \times 0.6 + 0.6 \times 0.6 \times 0.4 + 0.6 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.6$

$= 0.648$

$P(M = 10) = P(6, 10, 10) + P(10, 6, 10) + P(10, 10, 6) + P(10, 10, 10)$

$= pqq + qpq + qqp + qqq$

$= 0.6 \times 0.4 \times 0.4 + 0.4 \times 0.6 \times 0.4 + 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.4 \times 0.4$

$= 0.352$

m	3	2
$P(M = m)$	0.648	0.352

Challenge

a $$E(\bar{X}) = E\left(\frac{X_1 + X_2 + \ldots + X_n}{n}\right)$$

$$= \frac{1}{n}E(X_1 + X_2 + \ldots + X_n)$$

$$= \frac{1}{n}(E(X_1) + E(X_2) + \ldots + E(X_n))$$

$$= \frac{1}{n}(\mu + \mu + \ldots + \mu)$$

$$= \frac{1}{n}(n\mu)$$

$$= \mu$$

b $$\mathrm{Var}(\bar{X}) = \mathrm{Var}\left(\frac{X_1 + X_2 + X_3}{3}\right)$$

$$= \frac{1}{9}\mathrm{Var}(X_1 + X_2 + X_3)$$

$$= \frac{1}{9}(\mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \mathrm{Var}(X_3))$$

$$= \frac{1}{9}(\sigma^2 + \sigma^2 + \sigma^2)$$

$$= \frac{\sigma^2}{3}$$