Exercise 6B

1 The mean is taken from the sample so it is a statistic.

2 **i** and **ii** are statistics

3 **a** All the hairdressers who work for the chain of hairdressing shops.
The proportion p of the staff who are happy to wear an apron.

b This is a binomial distribution, since we are only interested in two options – whether the hairdressers are happy or not.

4 **a** Po(3)

b $P(X < 2) = \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!}$
$= 0.199$ (3 s.f.)

5 **a** $\mu = E(X) = 0.5 \times 50 + 0.25 \times 20 + 0.25 \times 10$
$= 32.5$
$Var(X) = 0.5 \times 50^2 + 0.25 \times 20^2 + 0.25 \times 10^2 - 32.5^2$
$= 318.75$

b (50, 50), (50, 20), (20, 50), (50, 10), (10, 50), (20, 20), (20, 10), (10, 20), (10, 10)

c

x	50	35	30	20	15	10
$P(X = x)$	0.25	0.25	0.25	0.0625	0.125	0.0625

6 **a** $\mu = E(X) = 0.4 \times 16 + 0.5 \times 20 + 0.1 \times 30$
$= 19.4$
$Var(X) = 0.4 \times 16^2 + 0.5 \times 20^2 + 0.1 \times 30^2 - 19.4^2$
$= 16.04$

b (16, 16), (16, 20), (20, 16), (16, 30), (30, 16), (30, 30), (30, 20) (20, 30) (20, 20)

c

x	16	18	20	23	25	30
$P(X = x)$	0.16	0.4	0.25	0.08	0.1	0.01

7 a $\mu = \mathrm{E}(X) = 3 \times 0.6 + 2 \times 0.4$
$= 2.6$
$\mathrm{Var}(X) = 3^2 \times 0.6 + 2^2 \times 0.4 - 2.6^2$
$= 0.24$

b (3, 3, 3), (3, 3, 2), (3, 2, 3), (2, 3, 3), (3, 2, 2), (2, 3, 2), (2, 2, 3), (2, 2, 2)

c The sampling distribution for $\overline{X}$ is shown in the table.

x	3	$\frac{8}{3}$	$\frac{7}{3}$	2
$\mathrm{P}(X = x)$	0.216	0.432	0.288	0.064

d The mode can only take the values 2 and 3.
Let $\mathrm{P}(2) = p = 0.4$
Let $\mathrm{P}(3) = q = 0.6$
$\mathrm{P}(M = 2) = \mathrm{P}(3, 2, 2) + \mathrm{P}(2, 3, 2) + \mathrm{P}(2, 2, 3) + \mathrm{P}(2, 2, 2)$
$= qpp + pqp + ppq + ppp$
$= 0.6 \times 0.4 \times 0.4 + 0.4 \times 0.6 \times 0.4 + 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.4 \times 0.4$
$= 0.352$
$\mathrm{P}(M = 10) = \mathrm{P}(3, 3, 3) + \mathrm{P}(3, 3, 2) + \mathrm{P}(3, 2, 3) + \mathrm{P}(2, 3, 3)$
$= qqq + qqp + qpq + pqq$
$= 0.6 \times 0.6 \times 0.6 + 0.6 \times 0.6 \times 0.4 + 0.6 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.6$
$= 0.352$

m	2	3
$\mathrm{P}(M = m)$	0.352	0.648

e The median can only take the values 2 and 3.
Let $\mathrm{P}(2) = p = 0.4$
Let $\mathrm{P}(3) = q = 0.6$
$\mathrm{P}(N = 2) = \mathrm{P}(3, 2, 2) + \mathrm{P}(2, 3, 2) + \mathrm{P}(2, 2, 3) + \mathrm{P}(2, 2, 2)$
$= qpp + pqp + ppq + ppp$
$= 0.6 \times 0.4 \times 0.4 + 0.4 \times 0.6 \times 0.4 + 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.4 \times 0.4$
$= 0.352$
$\mathrm{P}(N = 3) = \mathrm{P}(3, 3, 3) + \mathrm{P}(3, 3, 2) + \mathrm{P}(3, 2, 3) + \mathrm{P}(2, 3, 3)$
$= qqq + qqp + qpq + pqq$
$= 0.6 \times 0.6 \times 0.6 + 0.6 \times 0.6 \times 0.4 + 0.6 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.6$
$= 0.352$

n	2	3
$\mathrm{P}(N = n)$	0.352	0.648