

Exercise 5B

$$1 \text{ a } E(Y) = E(X^2)$$

$$E(X) = \frac{4.5 + 5.5}{2} = 5$$

$$\text{Var}(X) = \frac{(5.5 - 4.5)^2}{12} = \frac{1}{12}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\Rightarrow E(X^2) = \text{Var}(X) + (E(X))^2 = \frac{1}{12} + 25 = 25\frac{1}{12} = \frac{301}{12} = 25.083 \text{ (3 d.p.)}$$

Alternatively use $E(g(X)) = \int g(x)f(x)dx$ with

$$f(x) = \begin{cases} 1 & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \quad \text{and } g(x) = x^2$$

$$E(Y) = E(X^2) = \int_{4.5}^{5.5} x^2 dx = \left[\frac{x^3}{3} \right]_{4.5}^{5.5} = \left(\frac{5.5^3}{3} - \frac{4.5^3}{3} \right) = 25.083 \text{ (3 d.p.)}$$

$$2 \text{ a } f(x) = \begin{cases} \frac{1}{6} & 5 \leq x \leq 11 \\ 0 & \text{otherwise} \end{cases}$$

$$b \text{ } P(7 < R < 10) = \frac{10 - 7}{11 - 5} = \frac{3}{6} = \frac{1}{2} = 0.5$$

$$c \text{ } E(A) = E(\pi R^2) = \pi(\text{Var}(R) + (E(R))^2)$$

$$E(A) = \pi \left(\frac{36}{12} + \left(\frac{5 + 11}{2} \right)^2 \right) = \pi(3 + 64) = 67\pi \text{ cm}^2$$

$$3 \text{ a } \frac{1}{b-a} = \frac{1}{1-0} = 1$$

$$P(T < 0.2) = (0.2 - 0) \times 1 = 0.2$$

$$b \text{ } E(T) = 0.5$$

$$c \text{ } \text{Var}(T) = \int_0^1 t^2 f(t) dt - \mu^2 = \int_0^1 t^2 dt - 0.5^2 = \left[\frac{t^3}{3} \right]_0^1 - 0.25 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$4 \text{ a } \frac{1}{b-a} = \frac{1}{10-2} = \frac{1}{8}$$

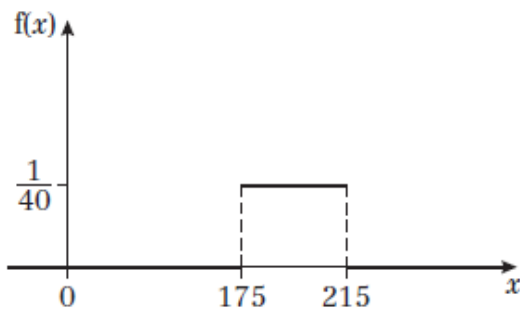
$$P(T > 7) = (10 - 7) \times \frac{1}{8} = \frac{3}{8}$$

$$4 \text{ b } P(T < 5) = (5 - 2) \times \frac{1}{8} = \frac{3}{8}$$

So probability that Priya will take less than 5 minutes on three successive visits = $\left(\frac{3}{8}\right)^3 = \frac{27}{512}$

$$c \quad P(T < 8 | T > 5) = \frac{P(T < 8 \cap T > 5)}{P(T > 5)} = \frac{P(5 < T < 8)}{P(T > 5)} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

$$5 \text{ a } f(x) = \begin{cases} \frac{1}{40} & 175 \leq x \leq 215 \\ 0 & \text{otherwise} \end{cases}$$



$$b \text{ i } P(X < 187) = \frac{187 - 175}{40} = \frac{12}{40} = \frac{3}{10} = 0.3$$

$$ii \quad P(X = 187) = 0$$

(the probability of a continuous random variable taking a specific value is always 0)

$$c \quad Q_1 = 185, \text{ as } P(X < 185) = \frac{10}{40} = 0.25 \text{ and } Q_3 = 205, \text{ as } P(X < 205) = \frac{30}{40} = 0.75$$

$$\text{Interquartile range} = 205 - 185 = 20$$

$$d \quad P(X \geq x) = \frac{215 - x}{40} = 0.65 \Rightarrow 215 - x = 26 \Rightarrow x = 189$$

e Let the random variable Y represent the number of cups that Rashmi buys contains less than 187 ml, then using the result from part b i, $Y \sim B(5, 0.3)$

$$P(Y = 3) = \binom{5}{3} 0.3^3 (1 - 0.3)^2 = \frac{5 \times 4}{2} \times 0.3^3 \times 0.7^2 = 0.1323 \text{ (4 d.p.)}$$

$$6 \text{ a } P(X < -2.3) = \frac{-2.3 - (-3.0)}{3.0 - (-3.0)} = \frac{0.7}{6} = \frac{7}{60}$$

$$b \quad P(|X| > 2.0) = P(X > 2.0) + P(X < -2.0) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

- 6 c Let the random variable Y represent the number of the 10 rods that are cut within 2 mm of the target length, then using the result from part b i, $Y \sim B\left(10, \frac{2}{3}\right)$

$$P(Y = 6) = \binom{10}{6} \times \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{3}\right)^4 = 0.2276 \text{ (4 d.p.)}$$

7 a $P(Y > 26) = \frac{28-26}{28-20} = \frac{2}{8} = \frac{1}{4} = 0.25$

- b Let the random variable Z represent the number of sweets in a bag of 20 sweets that have a length greater than 26 mm, then using the result from part a, $Z \sim B(20, 0.25)$.

$$P(Z \geq 7) = 1 - P(Z \leq 6) = 1 - 0.7858 = 0.2142 \text{ (4 d.p.)}$$

(Obtaining the value for $P(Z \leq 6)$ from a binomial cumulative distribution function table)

8 a $P(X < 5) = \frac{5-2}{7-2} = \frac{3}{5} = 0.6$

b $P(X > 6) = \frac{7-6}{7-2} = \frac{1}{5} = 0.2$

Let the random variable Y represent the number of flights in the next 10 flights that have a waiting time of more than 6 minutes, then $Y \sim B(10, 0.2)$

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - 0.6778 = 0.3222 \text{ (4 d.p.)}$$

(Obtaining the value for $P(Y \leq 2)$ from a binomial cumulative distribution function table)

- 9 The length of the shorter side is $(20 - X)$ cm, so

$$E(A) = E(X(20 - X)) = E(20X - X^2) = 20E(X) - E(X^2)$$

$$E(X) = \frac{20+10}{2} = 15$$

$$\text{Var}(X) = \frac{(20-10)^2}{12} = \frac{100}{12} = \frac{25}{3}$$

$$E(X^2) = \text{Var}(X) + (E(X))^2 = 225 + \frac{25}{3}$$

$$\begin{aligned} \text{Hence } E(A) &= 20E(X) - E(X^2) = 20 \times 15 - 225 - \frac{25}{3} \\ &= 300 - 225 - \frac{25}{3} = 75 - \frac{25}{3} = \frac{225 - 25}{3} = \frac{200}{3} \text{ cm}^2 \end{aligned}$$