Exercise 3C

1 $X \sim \text{Po}(30)$
Since λ is quite large, you can use a normal approximation with
$\mu = \lambda = 30$ and $\sigma^2 = \lambda = 30$
So $Y \sim \text{N}(30, 30)$

a Applying the continuity correction gives:
$\text{P}(X \leq 20) \approx \text{P}(Y < 20.5)$
$= \text{P}\left(Z < \frac{20.5 - 30}{\sqrt{30}}\right)$
$= \text{P}(Z < -1.734\ldots)$
$= 1 - \text{P}(Z < 1.734\ldots)$
$= 1 - 0.9585\ldots$
$= 0.0415$ (3 s.f.)

b Applying the continuity correction gives:
$\text{P}(X > 43) \approx \text{P}(Y > 43.5)$
$= 1 - \text{P}\left(Z < \frac{43.5 - 30}{\sqrt{30}}\right)$
$= 1 - \text{P}(Z < 2.464\ldots)$
$= 1 - 0.99314$
$= 0.00686$ (3 s.f.)

c Applying the continuity correction gives:
$\text{P}(25 \leq X \leq 35) \approx \text{P}(24.5 \leq Y \leq 35.5)$
$= \text{P}\left(\frac{24.5 - 30}{\sqrt{30}} \leq Z \leq \frac{35.5 - 30}{\sqrt{30}}\right)$
$= \text{P}(-1.004... \leq Z \leq 1.004...)$
$= 0.8423\ldots - (1 - 0.8423\ldots)$
$= 0.685$ (3 s.f.)

2 $X \sim \text{Po}(45)$
Since λ is quite large, you can use a normal approximation with
$\mu = \lambda = 45$ and $\sigma^2 = \lambda = 45$
So $Y \sim \text{N}(45, 45)$

a Applying the continuity correction gives:
$\text{P}(X < 40) \approx \text{P}(Y < 39.5)$
$= \text{P}\left(Z < \frac{39.5 - 45}{\sqrt{45}}\right)$
$= \text{P}(Z < -0.8198\ldots)$
$= 1 - \text{P}(Z < 0.8198\ldots)$
$= 1 - 0.7938\ldots$
$= 0.206$ (3 s.f.)

2 b Applying the continuity correction gives:

$P(X \geq 50) \approx P(Y > 49.5)$

$$= 1 - P\left(Z < \frac{49.5 - 45}{\sqrt{45}}\right)$$

$$= 1 - (Z < 0.6708...)$$

$= 1 - 0.7488\ldots$

$= 0.251$ (3 s.f.)

c Applying the continuity correction gives:

$P(43 < X \leq 52) \approx P(43.5 < Y \leq 52.5)$

$$= P\left(\frac{43.5 - 45}{\sqrt{45}} \leq Z \leq \frac{52.5 - 45}{\sqrt{45}}\right)$$

$$= P(-0.2236... \leq Z \leq 1.118...)$$

$= 0.8682\ldots - (1 - 0.5884\ldots)$

$= 0.457$ (3 s.f.)

3 $X \sim \text{Po}(60)$

Since λ is quite large you can use a normal approximation with $\mu = \lambda = 60$ and $\sigma^2 = \lambda = 60$

So $Y \sim N(60, 60)$

a Applying the continuity correction gives:

$P(X \leq 62) \approx P(Y < 62.5)$

$$= P\left(Z < \frac{62.5 - 60}{\sqrt{60}}\right)$$

$= P(Z < 0.3227\ldots)$

$= 0.6265\ldots$

$= 0.627$ (3 s.f.)

b Applying the continuity correction gives:

$P(X = 63) \approx P(62.5 \leq Y \leq 63.5)$

$$= P\left(\frac{62.5 - 60}{\sqrt{60}} \leq Z \leq \frac{63.5 - 60}{\sqrt{60}}\right)$$

$$= P(0.3227... \leq Z \leq 0.4518...)$$

$= 0.6743\ldots - 0.6265\ldots$

$= 0.0478$ (3 s.f.)

c Applying the continuity correction gives:

$P(55 \leq X < 65) \approx P(54.5 < Y \leq 64.5)$

$$= P\left(\frac{54.5 - 60}{\sqrt{60}} \leq Z \leq \frac{64.5 - 60}{\sqrt{60}}\right)$$

$$= P(-0.5809... \leq Z \leq 0.7100...)$$

$= 0.7193\ldots - 0.2388\ldots$

$= 0.480$ (3 s.f.)

4 Let R be the rate that the radioactive object breaks down.
$R \sim \text{Po}(14)$
Since λ is quite large you can use a normal approximation with
$\mu = \lambda = 14$ and $\sigma^2 = \lambda = 14$
So $Y \sim \text{N}(14, 14)$

a $\text{P}(20 \le R \le 22) \approx \text{P}(19.5 < Y < 22.5)$
$$= \text{P}\left(\frac{19.5-14}{\sqrt{14}} < Z < \frac{22.5-14}{\sqrt{14}}\right)$$
$$= \text{P}(1.469... < Z < 2.271...)$$
$$= 0.9884... - 0.9292$$
$$= 0.0592 \text{ (3 s.f.)}$$

b $\text{P}(R > 10) \approx \text{P}(Y > 10.5)$
$$= \text{P}\left(Z > \frac{10.5-14}{\sqrt{14}}\right)$$
$$= \text{P}(Z > -0.9354...)$$
$$= \text{P}(Z < 0.9354...)$$
$$= 0.8252...$$
$$= 0.825 \text{ (3 s.f.)}$$

c $\text{P}(12 < R < 16) \approx \text{P}(12.5 < Y < 15.5)$
$$= \text{P}\left(\frac{12.5-14}{\sqrt{14}} < Z < \frac{15.5-14}{\sqrt{14}}\right)$$
$$= \text{P}(-0.4008... < Z < 0.4008...)$$
$$= 2(0.6557 - 0.5)$$
$$= 0.311 \text{ (3 s.f.)}$$

5 Let H be the number of boats hired.
$H \sim \text{Po}(15)$
Since λ is quite large you can use a normal approximation with
$\mu = \lambda = 15$ and $\sigma^2 = \lambda = 15$
So $Y \sim \text{N}(15, 15)$

a $\text{P}(H \le 5) \approx \text{P}(Y < 5.5)$
$$= \text{P}\left(Z < \frac{5.5-15}{\sqrt{15}}\right)$$
$$= \text{P}(Z < -2.452...)$$
$$= 1 - \text{P}(Z < 2.452...)$$
$$= 1 - 0.9929...$$
$$= 0.0071...$$
In 100 days, you would expect five or fewer boats to be hired.
$100 \times 0.0071 = 0.71$ days.
So 1 day to the nearest day.

5 b $P(H = 10) \approx P(9.5 < Y < 10.5)$

$$= P\left(\frac{9.5-15}{\sqrt{15}} < Z < \frac{10.5-15}{\sqrt{15}}\right)$$
$$= P(-1.420... < Z < -1.161...)$$
$$= (1 - P(Z < 1.161...)) - (1 - P(Z < 1.420...))$$
$$= (1 - 0.8773\ldots) - (1 - 0.9222\ldots)$$
$$= 0.1226\ldots - 0.0777\ldots$$
$$= 0.0448\ldots$$

In 100 days you would expect exactly 10 boats to be hired.
$100 \times 0.0448 = 4.48$ days.
So 4 days to the nearest day.

c $P(H > 20) \approx P(Y > 20.5)$

$$= 1 - P\left(Z < \frac{20.5-15}{\sqrt{15}}\right)$$
$$= 1 - P(Z < 1.420...)$$
$$= 1 - 0.9222\ldots$$
$$= 0.0777\ldots$$

In 100 days you would expect exactly 10 boats to be hired $100 \times 0.0777\ldots = 7.77$ days.
So 8 days to the nearest day.