Exercise 3A

1 a i $\mathrm{P}(X=4)=\binom{100}{4}\times(0.05)^4\times(0.95)^{96}=0.1781$ (4 d.p.)

ii Using a calculator:
$\mathrm{P}(X\leqslant 2)=0.1183$ (4 d.p.)

b Use the approximation $X\sim \mathrm{Po}(100\times 0.05)$, i.e. $X\sim \mathrm{Po}(5)$

i $\mathrm{P}(X=4)=\dfrac{\mathrm{e}^{-5}5^4}{4!}=0.1755$ (4 d.p.)

ii Using the tables in the textbook:
$\mathrm{P}(X\leqslant 2)=0.1247$

2 a i $\mathrm{P}(X=5)=\binom{150}{5}\times(0.04)^5\times(0.96)^{145}=0.1628$ (4 d.p.)

ii Using a calculator:
$\mathrm{P}(X\leqslant 3)=0.1458$ (4 d.p.)

b Use the approximation $X\sim \mathrm{Po}(150\times 0.04)$, i.e. $X\sim \mathrm{Po}(6)$

i $\mathrm{P}(X=5)=\dfrac{\mathrm{e}^{-6}6^5}{5!}=0.1606$ (4 d.p.)

ii Using the tables in the textbook:
$\mathrm{P}(X\leqslant 3)=0.1512$

3 a Let $X=200-Y$ so that $X\sim \mathrm{B}(200,0.02)$

i $\mathrm{P}(Y=197)=\mathrm{P}(X=3)=\binom{200}{3}\times(0.02)^3\times(0.98)^{197}=0.1963$ (4 d.p.)

ii Using a calculator:
$\mathrm{P}(Y\geqslant 198)=\mathrm{P}(X\leqslant 2)=0.2351$ (4 d.p.)

b Use the approximation $X\sim \mathrm{Po}(200\times 0.02)$, i.e. $X\sim \mathrm{Po}(4)$

i $\mathrm{P}(X=3)=\dfrac{\mathrm{e}^{-4}4^3}{3!}=0.1954$ (4 d.p.)

ii Using the tables in the textbook:
$\mathrm{P}(X\leqslant 2)=0.2381$

4 Let X be the number of pupils with a birthday on 1 April, assuming a uniform distribution of birthdays across 365 days of the year.

Then $X \sim \mathrm{B}\left(800, \frac{1}{365}\right)$

a $\mathrm{P}(X=4) = \binom{800}{4} \times \left(\frac{1}{365}\right)^4 \times \left(\frac{364}{365}\right)^{796} = 0.1075$ (4 d.p.)

b Use the approximation $X \sim \mathrm{Po}\left(800 \times \frac{1}{365}\right)$, i.e. $X \sim \mathrm{Po}\left(\frac{160}{73}\right)$

$\mathrm{P}(X=4) = \frac{e^{-\frac{160}{73}}\left(\frac{160}{73}\right)^4}{4!} = 0.1074$ (4 d.p.)

c The two answers are very similar, which shows that the Poisson approximation is a good approximation in this case. This is to be expected as the Poisson approximation is extremely accurate for this very low value p and high n.

5 Let X be the number of faulty items in a batch of 100. Then $X \sim \mathrm{B}(100, 0.03)$ and this can be approximated by $X \sim \mathrm{Po}(100 \times 0.03)$, i.e. $X \sim \mathrm{Po}(3)$

a Using the tables in the textbook:
$\mathrm{P}(X \leqslant 3) = 0.6472$

b $\mathrm{P}(X=2) = \frac{e^{-3}3^2}{2!} = 0.2240$ (4 d.p.)

6 Let X be the number of patients with the condition in a sample of 180. Then $X \sim \mathrm{B}(180, 0.02)$ and this can be approximated by $X \sim \mathrm{Po}(180 \times 0.02)$, i.e. $X \sim \mathrm{Po}(3.6)$

a $\mathrm{P}(X=1) = \frac{e^{-3.6}3.6^1}{1!} = 0.0984$ (4 d.p.)

b As $\lambda = 3.6$ use a calculator to find the required value:
$\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X \leqslant 1)$
$= 1 - 0.1257 = 0.8743$ (4 d.p.)

7 **a** Let X be the number of people who catch the virus in a sample of 20. Then $X \sim \mathrm{B}\left(20, \frac{1}{120}\right)$

$\mathrm{P}(X=1) = \binom{20}{1}\left(\frac{1}{120}\right)^1\left(\frac{119}{120}\right)^{19} = 0.1422$ (4 d.p.)

7 b Let Y be the number of people who catch the virus in a sample of 900.

Then $Y \sim B\left(900, \frac{1}{120}\right)$ and this can be approximated by $Y \sim Po(7.5)$

Using the tables in the textbook:

$P(Y \leqslant 6) = 0.3782$

8 a Let X be the number of faulty articles in a sample of 10. Then $X \sim B(10, 0.025)$

$$P(X = 1) = \binom{10}{1} = (0.025)^1(0.975)^9 = 0.1991 \text{ (4 d.p.)}$$

b Let Y be the number of faulty articles in a sample of 120. Then $Y \sim B(120, 0.025)$ and this can be approximated by $Y \sim Po(120 \times 0.025)$, i.e. $Y \sim Po(3)$

Using the tables in the textbook:

$P(Y \leqslant 3) = 0.6472$

9 a Let X be the number of broken pots in a sample of 10. Then $X \sim B(10, 0.05)$

b $P(X = 3) = \binom{10}{3}(0.05)^3(0.95)^7 = 0.0105 \text{ (4 d.p.)}$

c Let Y be the number of broken pots in a sample of 140. Then $Y \sim B(140, 0.05)$ and this can be approximated by $Y \sim Po(140 \times 0.05)$, i.e. $Y \sim Po(7)$

Using the tables in the textbook:

$$P(6 \leqslant Y \leqslant 9) = P(Y \leqslant 9) - P(Y \leqslant 5)$$
$$= 0.8305 - 0.3007 = 0.5298$$

10 Let X be the number of tomato plants growing over 2 metres in a sample of 50. Then $X \sim B(50, 0.08)$ and this can be approximated by $X \sim Po(50 \times 0.08)$, i.e. $X \sim Po(4)$

Using the tables in the textbook:

$$P(5 \leqslant X \leqslant 8) = P(X \leqslant 8) - P(X \leqslant 4)$$
$$= 0.9786 - 0.6288 = 0.3498$$

11 a Let X be the number of damaged genes in an insect cell. Assuming genes are damaged independently, $X \sim B(1200, 0.005)$

b $E(X) = np = 1200 \times 0.005 = 6$

$\text{Var}(X) = np(1-p) = 1200 \times 0.005 \times 0.995 = 5.97$

c Use the approximation $X \sim Po(1200 \times 0.005)$, i.e. $X \sim Po(6)$

Using the tables in the textbook:

$P(X \leqslant 4) = 0.2851$

12 a Let X be the number of faulty nails in a sample of 200. Then $X \sim B(200, 0.025)$ and this can be approximated by $X \sim Po(200 \times 0.025)$, i.e. $X \sim Po(5)$
Using the tables in the textbook:
$P(X > 6) = 1 - P(X \leqslant 6)$
$= 1 - 0.7622 = 0.2378$

b Let Y be the number of packets with more than 6 defective nails in a sample of 6 packets. Then $Y \sim B(6, 0.2378)$.
Using a calculator:
$P(Y > 3) = 1 - P(Y \leqslant 3)$
$= 1 - 0.9685 = 0.0315$ (4 d.p.)

13 a Let X be the number of faulty components in a sample of 400. Then $X \sim B(400, 0.0125)$ and this can be approximated by $X \sim Po(400 \times 0.0125)$, i.e. $X \sim Po(5)$
Using the tables in the textbook:
$P(X > 3) = 1 - P(X \leqslant 3)$
$= 1 - 0.2650 = 0.7350$

b Let Y be the number of boxes containing more than 3 faulty components in a sample of 5. Then using the answer from part **a**, $Y \sim B(5, 0.735)$

$$P(Y = 3) = \binom{5}{3} \times (0.735)^3 \times (0.265)^2 = 0.2788 \text{ (4 d.p.)}$$

Challenge

Consider $P(X = 0 \mid X \sim B(n, p)) = (1 - p)^n$
Consider $P(X = 0 \mid X \sim Po(np)) = e^{-np}$
It will overestimate when $e^{-np} > (1 - p)^n$
Taking logs gives:
$-np > n \ln (1 - p)$
$-p > \ln (1 - p)$
It can be shown graphically