Chapter Review 2

1 a Let X be the number of accidents in a month. Assume a Poisson distribution.
So $X \sim \text{Po}(0.7)$
$P(X=0)=e^{-0.7}=0.4966$ (4 d.p.)

b Let Y be the number of accidents in a 3-month period. So $Y \sim \text{Po}(2.1)$
$$P(Y=2)=\frac{e^{-2.1}\times 2.1^2}{2!}=0.2700 \text{ (4 d.p.)}$$

c Let A be the number of accident-free months in a sample of six months. So $A \sim B(6, 0.4966)$ from part **a**
$$P(A=2)=\binom{6}{2}\times(0.4966)^2\times(0.5034)^4=0.2376 \text{ (4 d.p.)}$$

2 a X must have a constant mean rate so that the mean number of misprints in a sample is proportional to the number of chapters. Misprints must occur independently of one another and be distinct (i.e. can be counted singly in the text).

b The model is $X \sim \text{Po}(2.25)$. Find $P(X \leqslant 1)$ using a calculator:
$$P(X \leqslant 1)=P(X=0)+P(X=1)$$
$$=e^{-2.25}+\frac{e^{-2.25}\times 2.25^1}{1!}$$
$$=0.1054+0.2371=0.3425 \text{ (4 d.p.)}$$

c Let Y be the number of misprints in two randomly chosen chapters. So $Y \sim \text{Po}(4.5)$
As $\lambda = 4.5$, the required value can be found from the tables in the textbook:
$$P(Y>6)=1-P(Y\leqslant 6)$$
$$=1-0.8311=0.1689$$

3 As $Y \sim \text{Po}(\lambda)$ then $P(Y=5)=\frac{e^{-\lambda}\lambda^5}{5!}$ and $P(Y=3)=\frac{e^{-\lambda}\lambda^3}{3!}$
So as $P(Y=5)=1.25\times P(Y=3)$
$$\frac{e^{-\lambda}\lambda^5}{120}=1.25\times\frac{e^{-\lambda}\lambda^3}{6}$$
$$\lambda^2=25$$
$$\lambda=5 \quad \text{(since } \lambda \text{ must be positive)}$$

4 a The event (receipt of an email) has a constant mean rate through time. Emails are received singly and independently of each other.

b i Let X be the number of emails received in a 10-minute period. Assume a Poisson distribution, so $X \sim \text{Po}(6)$
$$P(X=7)=\frac{e^{-6}\times 6^7}{7!}=0.1377 \text{ (4 d.p.)}$$

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4 b ii Using the tables:

$P(X \geqslant 8) = 1 - P(X \leqslant 7) = 1 - 0.7440 = 0.2560$

5 a A binomial distribution $B(n, p)$ may be approximated by a Poisson distribution $Po(np)$ when n is large and p is small, and typically $np \leqslant 10$.

b $X \sim B(50, 0.08)$

$$P(X \leqslant 3) = \binom{50}{3}(0.08)^3(0.92)^{47} + \binom{50}{2}(0.08)^2(0.92)^{48} + \binom{50}{1}(0.08)(0.92)^{49} + \binom{50}{0}(0.92)^{50}$$

$= 0.4253$ (4 d.p.)

c Use as an approximation $X \sim Po(50 \times 0.08)$, i.e. $X \sim Po(4)$

Using the tables:

$P(X \leqslant 3) = 0.4335$

d $\text{Percentage error} = \frac{0.4335 - 0.4253}{0.4253} \times 100\% = 1.93\%$ (2 d.p)

6 $P(Y > 10) < 0.1$ so $P(Y \leqslant 10) > 0.9$

From the tables, for $\lambda = 7$, $P(Y \leqslant 10) = 0.9015$

For $\lambda = 8$, $P(Y \leqslant 10) = 0.8159$

So the largest integer value for λ satisfying the given condition is $\lambda = 7$

7 Let X be the number of fish caught in a 2-hour period. So $X \sim Po(4)$. Use this to find the probability that the angler catches at least 5 fish on a 2-hour fishing trip.

$P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.6288 = 0.3712$

Then let Y be the number of trips in which the angle catches at least 5 fish from a sample of 5 trips. So using the probability for catching at least 5 fish on a single trip, $Y \sim B(5, 0.3712)$

$$P(Y = 3) = \binom{5}{3}(0.3712)^3(0.6288)^2 = 0.2022 \text{ (4 d.p.)}$$

8 a Let X be the number of cherries in a cake. So $X \sim Po(2.5)$

i $P(X = 4) = \frac{e^{-2.5} \times 2.5^4}{4!} = 0.1336$ (4 d.p.)

ii Using the tables:

$P(X \geqslant 3) = 1 - P(X \leqslant 2)$

$= 1 - 0.5438 = 0.4562$

b Let Y be the number of cherries in 4 cakes. So $Y \sim Po(10)$

Using the tables:

$P(Y > 12) = 1 - P(X \leqslant 12)$

$= 1 - 0.7916 = 0.2084$

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8 c Let A be the number of packets containing more than 12 cherries in a sample of 8 packets. So, using the result from part **b**, $A \sim B(8, 0.2084)$

$$P(A=2)=\binom{8}{2}(0.2084)^2(0.7916)^6=0.2992 \text{ (4 d.p.)}$$

9 a Let X be the number of cars sold in a week. A plausible model for number of cars sold in a week would be $X \quad Po(6)$.

It may be supposed that sales are independent of each other and occur singly (assuming the salesman does not supply businesses); the constant mean rate is consistent with a Poisson model.

b $P(X=5)=\dfrac{e^{-6}\times 6^5}{5!}=0.1606 \text{ (4 d.p.)}$

c Let Y be the number of weeks in which the salesman sells exactly 5 cars, in a sample of 4 consecutive weeks. So, using the result from part **b**, $Y \sim B(4, 0.1606)$

$$P(Y=2)=\binom{4}{2}(0.1606)^2(0.8394)^2=0.1090 \text{ (4 d.p.)}$$

10 Assuming a Poisson distribution for each random variable, with A and B being the number of letters received by Aisha and Biyu in a given day (respectively): then $A \sim Po(1.2)$ and $B \sim Po(0.8)$ so $A+B \sim Po(2)$

a $P(A \geqslant 1 \text{ and } B \geqslant 1) = P(A \geqslant 1) \times P(B \geqslant 1)$ (independence)

$$= (1-P(A=0)) \times (1-P(B=0))$$
$$= (1-e^{-1.2}) \times (1-e^{-0.8})$$
$$= (1-0.30119) \times (1-0.44933)$$
$$= 0.69881 \times 0.55067 = 0.3848 \text{ (4 d.p.)}$$

b $P(A+B=3)=\dfrac{e^{-2}\times 2^3}{3!}=0.1804 \text{ (4 d.p.)}$

c Let Y be the number of days on which they receive a total of 3 letters between them, from a sample of 5 days. So, using the result from part **b**, $Y \sim B(5, 0.1804)$

$$P(Y \geqslant 3) = 1 - P(Y \leqslant 2)$$
$$= 1 - 0.9560 = 0.0440$$

11 Let X be the number of desktops sold in a day and Y the number of laptops sold in a day. Assuming Poisson distributions for both variables and that sales of desktops and laptops are independent, then $X \sim Po(2.4)$ and $Y \sim Po(1.6)$ so $X+Y \sim Po(4)$

a $P(X \geqslant 2 \text{ and } Y \geqslant 2) = P(X \geqslant 2) \times P(Y \geqslant 2)$ (independence)

$$= (1-P(X \leqslant 1)) \times (1-P(Y \leqslant 1))$$
$$= (1-0.30844) \times (1-0.52493)$$
$$= 0.69156 \times 0.47507 = 0.3285 \text{ (4 d.p.)}$$

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11 b $P(X+Y=6)=\frac{e^{-4}\times 4^6}{6!}=0.1042$ (4 d.p.)

c Let A be the combined total of computer sales in a two-day period. So $A\sim \text{Po}(8)$ and the required value can be found from the tables:

$P(A\leqslant 6)=0.3134$

12 a Assuming that accidents can be modelled using a Poisson distribution (so that the mean number of accidents in a given period of time will be proportional to the length of time), let X be the number of accidents in a 6-month period. So $X\sim \text{Po}(7.5)$, and from the tables:

$P(X\leqslant 4)=0.1321$

b Let Y be the number of accidents in a single month. So $Y\sim \text{Po}(1.25)$

$P(Y\geqslant 1)=1-P(Y=0)$

$=1-e^{-1.25}=1-0.2865=0.7135$ (4 d.p.)

c Let A be the number of months in which there is at least one accident, out of a sample of 6 months. From part **b**, $A\sim B(6,0.7135)$

$P(A=4)=\binom{6}{4}(0.7135)^4(0.2865)^2=0.3191$ (4 d.p.)

13 a Assume a Poisson distribution for breakdowns. Let X be the number of breakdowns in a single month, so $X\sim \text{Po}\left(\frac{2}{3}\right)$

$P(X=2)=\frac{e^{-\frac{2}{3}}\times\left(\frac{2}{3}\right)^2}{2!}=0.1141$ (4 d.p.)

b Let Y be the number of months in which there are at least 2 breakdowns.

$P(X\geqslant 2)=1-P(X\leqslant 1)=1-0.8557=0.1443$

So $Y\sim B(4,0.1443)$

$P(Y=3)=\binom{4}{3}(0.1443)^3(0.8557)^1=0.0103$ (4 d.p.)

14 a Visits can be counted singly; assuming visits are independent and at a constant average rate, they may be modelled by a Poisson distribution; the rate of 240 per hour would then scale to a mean rate of 4 in any given minute. Let X be the number of visits in a single minute. So $X\sim \text{Po}(4)$

b $P(X=8)=\frac{e^{-4}\times 4^8}{8!}=0.0298$ (4 d.p.)

c Let Y be the number of visits in a two-minute period. So $Y\sim \text{Po}(8)$

Using the tables:

$P(Y\geqslant 10)=1-P(Y\leqslant 9)$

$=1-0.7166=0.2843$

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15 a Let X be the number of policies sold in the week.

$$\bar{x}=\frac{\sum fx}{\sum f}=\frac{10\times0+23\times1+35\times2+33\times3+24\times4+14\times5+7\times6+3\times7+1\times8}{10+23+35+33+24+14+7+3+1}=2.86$$

$$\sigma^2=\frac{\sum fx^2}{\sum f}-(\bar{x})^2$$

$$=\frac{10\times0^2+23\times1^2+35\times2^2+33\times3^2+24\times4^2+14\times5^2+7\times6^2+3\times7^2+1\times8^2}{10+23+35+33+24+14+7+3+1}-2.86^2$$

$$=2.867\text{ (3 d.p.)}$$

b The values of mean and variance are the same, to 2 significant figures, which would support the validity of a Poisson model.

Challenge

a Assuming the number of planes landing in a given period of time can be modelled by a Poisson distribution, let X be the number of planes landing between 2 pm and 2:30 pm and let Y be the number of planes landing between 2.30 pm and 3 pm. Then $X\sim\text{Po}(7.5)$, $Y\sim\text{Po}(7.5)$ and $X+Y\sim\text{Po}(15)$

The solution uses the formula for conditional probability: $\text{P}(A\mid B)=\dfrac{\text{P}(A\text{ and }B)}{\text{P(B)}}$

$$\text{P}(X=5\mid X+Y=10)=\frac{\text{P}(X=5\text{ and }Y=5)}{\text{P}(X+Y=10)}$$

$$=\frac{\dfrac{e^{-7.5}\times7.5^5}{5!}\times\dfrac{e^{-7.5}\times7.5^5}{5!}}{\dfrac{e^{-15}\times15^{10}}{10!}}$$

$$=\frac{10!}{(5!)^2}\times\frac{7.5^{10}}{15^{10}}=\frac{10\times9\times8\times7\times6}{5\times4\times3\times2}\times\left(\frac{1}{2}\right)^{10}=\frac{3\times2\times7\times6}{2^{10}}=\frac{252}{2^{10}}=\frac{63}{2^8}$$

$$=\frac{63}{256}=0.2461\text{ (4 d.p.)}$$

b $$\text{P}(X>7\mid X+Y=10)=\frac{\text{P}(X>7\text{ and }X+Y=10)}{\text{P}(X+Y=10)}$$

$$=\frac{\text{P}(X=8\text{ and }Y=2)+\text{P}(X=9\text{ and }Y=1)+\text{P}(X=10\text{ and }Y=0)}{\text{P}(X+Y=10)}$$

$$=\frac{\dfrac{e^{-7.5}\times7.5^8}{8!}\times\dfrac{e^{-7.5}\times7.5^2}{2!}}{\dfrac{e^{-15}\times15^{10}}{10!}}+\frac{\dfrac{e^{-7.5}\times7.5^9}{9!}\times\dfrac{e^{-7.5}\times7.5^1}{9!}}{\dfrac{e^{-15}\times15^{10}}{10!}}+\frac{\dfrac{e^{-7.5}\times7.5^{10}}{10!}\times\dfrac{e^{-7.5}\times7.5^0}{0!}}{\dfrac{e^{-15}\times15^{10}}{10!}}$$

$$=\frac{7.5^{10}}{15^{10}}\times10!\times\left(\frac{1}{2\times8!}+\frac{1}{9!}+\frac{1}{10!}\right)=\frac{1}{2^{10}}\times(45+10+1)$$

$$=\frac{7}{128}=0.0547\text{ (4 d.p.)}$$

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Challenge (continued)

An alternative approach to this problem is to treat the 10 landings as each independently having a 0.5 chance of being in the first or second half of the hour and modelling them as a binomial. Let A be the number of landings in the first half hour (2 pm to 2:30 pm) and $A \sim B(10, 0.5)$. To answer part **a**, find $P(A = 5)$; to answer part **b**, find $P(A > 7)$.

$$P(A=5)=\binom{10}{5}(0.5)^5(0.5)^5=\frac{10!}{5!\times 5!}\times\frac{1}{2^{10}}=\frac{63}{256}$$

$$P(A>7)=\binom{10}{8}(0.5)^8(0.5)^2+\binom{10}{9}(0.5)^9(0.5)^1+\binom{10}{10}(0.5)^{10}=\frac{1}{2^{10}}\times 10!\times\left(\frac{1}{2\times 8!}+\frac{1}{9!}+\frac{1}{10!}\right)=\frac{7}{128}$$

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