Chapter Review 1

1 a $P(X=20)=\binom{30}{20}(0.73)^{20}(0.27)^{10}=\frac{30!}{20!10!}(0.73)^{20}(0.27)^{10}=0.114$ (to 3 s.f.)

b Using the binomial cumulative function on a calculator where $x = 13$, $n = 30$ and $p = 0.73$,
$P(X \leqslant 13) = 0.000580$ (to 3 s.f.)

c Using the binomial cumulative function on a calculator where $x = 11$ and 25, $n = 30$ and p = 0.73,
$P(11 < X \leqslant 25) = P(X \leqslant 25) - P(X \leqslant 11) = 0.937302995 - 0.000033512 = 0.937$ (to 3 s.f.)

2 a Sequence is: H H H H H T

Probability $=\left(\frac{2}{3}\right)^5\times\frac{1}{3}=\frac{32}{729}$ or 0.0439 (to 3 s.f.)

b Let X = 'number of tails in the first 8 tosses', then

$P(X=2)=\binom{8}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^6=0.273$ (to 3 s.f.)

3 X = 'number of patients waiting more than $\frac{1}{2}$ hour'

$X \sim B(12, 0.3)$

a $P(X=0)=(0.7)^{12}=0.01384...=0.0138$ (to 3 s.f.)

b $P(X>2)=1-P(X\leqslant 2)=1-0.2528=0.7472=0.747$ (to 3 d.p.)

4 a i There are n independent trials.
ii n is a fixed number.
iii The outcome of each trial is a success or a failure.
iv The probability of success at each trial is constant.
v The outcome of any trial is independent of any other trial.

b X = 'number of successes'
$X \sim B(10, 0.05)$
$P(X\geqslant 2)=1-P(X\leqslant 1)=1-0.9139=0.0861$ (to 3 s.f.)

c Y_n = 'number of successes in n houses'

$Y_n \sim B(n, 0.05)$

Looking for smallest n such that $P(Y_n\geqslant 1)>0.99$ or, equivalently, $P(Y_n=0)<0.01$.

$P(Y_n=0)=0.95^n<0.01$

So $n=90$ using logarithms.

5 X = 'number of correctly answered questions' and $X \sim B(10, 0.5)$

a $P(X=10)=(0.5)^{10}=0.00097656...=0.000977$ (to 3 s.f.)

b $P(X\geqslant 8)=1-P(X\leqslant 7)=1-0.9453=0.0547$ (to 3 s.f. using tables)

6

x	1	2	3	4	5	6
P(X = x)	p	p	p	p	$2p$	p

$7p = 1 \Rightarrow p = \frac{1}{7}$

a Sequence is: $\bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ 5$

Probability: $\left(\frac{5}{7}\right)^5\left(\frac{2}{7}\right) = 0.0531$ (to 3 s.f.)

b $Y =$ 'number of 5s in 8 throws'

$Y \sim \mathrm{B}(8, \frac{2}{7})$

$\mathrm{P}(Y = 3) = \binom{8}{3}\left(\frac{2}{7}\right)^3\left(\frac{5}{7}\right)^5 = 0.24285 = 0.243$ (to 3 s.f.)

7 $X =$ 'number of green chairs in sample of 10'

a $X \sim \mathrm{B}(10, 0.15)$

b $\mathrm{P}(X \geqslant 5) = 1 - \mathrm{P}(X \leqslant 4) = 1 - 0.9901 = 0.0099$ (tables)

c $\mathrm{P}(X = 2) = \mathrm{P}(X \leqslant 2) - \mathrm{P}(X \leqslant 1) = 0.8202 - 0.5443 = 0.2759$ (tables)

8 $X =$ 'number of yellow beads in a sample of 20' and assume $X \sim \mathrm{B}(20, 0.45)$

a $\mathrm{P}(X < 12) = \mathrm{P}(X \leqslant 11) = 0.8692$ (tables)

b $\mathrm{P}(X = 12) = \mathrm{P}(X \leqslant 12) - \mathrm{P}(X \leqslant 11) = 0.9420 - 0.8692 = 0.0728$ (tables)

9 a $\mathrm{P}(X \geqslant 10) = 1 - \mathrm{P}(X \leqslant 9) = 1 - 0.1275 = 0.8725$ (tables)

b $\mathrm{P}(X \geqslant 10 \text{ in 7 out of 12 sets}) = \binom{12}{7}(0.8725)^7(0.1275)^5$

$= 0.0103$ (to 3 s.f.)

c Let $Y =$ 'number of sets out of 12 that she hits the bullseye with at least 50% of her arrows', then $Y \sim \mathrm{B}(12, 0.8725)$

Using the binomial cumulative function on a calculator where $x = 5$, $n = 12$ and $p = 0.8725$,

$\mathrm{P}(Y < 6) = \mathrm{P}(Y \leqslant 5) = 0.0002407$

10 Let X be the number of damaged seeds that will grow.
$X \sim B(20, 0.075)$

a i $P(X = 2) = {}^{20}C_2 (0.075)^2 (0.925)^{18}$
$= 0.2626\ldots$
$= 0.263$ (3 s.f.)

a ii $P(X > 4) = 1 - P(X \leq 4)$
$= 1 - 0.9858...$
$= 0.0142$ (3 s.f.)

b $X \sim B(80, 0.075)$
$E(X) = 80 \times 0.075$
$= 6$
$Var(X) = 80 \times 0.075 \times 0.925$
$= 5.55$

11 a Let X be the number of misdirected calls in a sample of 10 consecutive calls. Assuming independence between calls, $X \sim B(10, 0.02)$

$P(X > 1) = 1 - P(X \leqslant 1)$

$$= 1 - \binom{10}{1}(0.02)^1(0.98)^9 - \binom{10}{0}(0.98)^{10}$$

$= 0.0162$ (4 d.p.)

b Let Y be the number of misdirected calls in a sample of 500 calls. Again assuming independence between calls, $X \sim B(10, 0.02)$

$E(Y) = 500 \times 0.02 = 10$ and $Var(Y) = 500 \times 0.02 \times 0.98 = 9.8$

12 a Let X be the number of people with the disease in a random sample of 10 people.
So $X \sim B(10, 0.025)$

$$P(X = 2) = \binom{10}{2}(0.025)^2(0.975)^8 = 0.0230 \text{ (4 d.p.)}$$

b Let Y be the number of people with the disease in a random sample of 120 people.
So $Y \sim B(120, 0.025)$

$E(Y) = 120 \times 0.025 = 3$ and $Var(Y) = 120 \times 0.025 \times 0.975 = 2.925$

13 If $X \sim B(6, 0.25)$ then $P(X \geq 3) = 0.169 > 0.1$
If $X \sim B(8, 0.25)$ then $P(X \geq 4) = 0.114 > 0.1$
If $X \sim B(10, 0.25)$ then $P(X \geq 5) = 0.078 > 0.1$
So $n = 5$.

Challenge

$Y \sim B(18, 0.25)$
$P(Y \geqslant 11) = 1 - P(Y \leqslant 10) = 1 - 0.9988 = 0.0012$ (tables)