Exercise 1C

1 **a** $\mathrm{E}(X) = 12 \times 0.7 = 8.4$

b $\mathrm{Var}(X) = 12 \times 0.7 \times (1 - 0.7) = 2.52$

2 **a** $\mathrm{E}(X) = 0.4n = 3.2$
So $n = 8$

b $\mathrm{P}(X = 5) = \binom{8}{5}(0.4)^5(0.6)^3$
$= 56 \times 0.1024 \times 0.216 = 0.1239$ (4 d.p.)

c The answer can be found from the binomial cumulative distribution function tables in the textbook or by using a calculator.
$\mathrm{P}(X \leqslant 2) = 0.3154$

3 $\mathrm{Var}(X) = 10p(1 - p) = 2.4$
So $p(1 - p) = 0.24$
$p^2 - p + 0.24 = 0$
$(p - 0.4)(p - 0.6) = 0$ factoring
$p = 0.4$ or 0.6

4 $\mathrm{Var}(X) = 15p(1 - p) = 2.4$
So $p(1 - p) = 0.16$
$p^2 - p + 0.16 = 0$
$(p - 0.2)(p - 0.8) = 0$ factoring
$p = 0.2$ or 0.8

5 $\mathrm{E}(X) = np = 4.8$ **(1)**
$\mathrm{Var}(X) = np(1 - p) = 2.88$ **(2)**
(2) ÷ **(1)** gives: $(1 - p) = \frac{2.88}{4.8} = 0.6 \Rightarrow p = 0.4$
From **(1)**: $np = 4.8 \Rightarrow n = \frac{4.8}{0.4} = 12$

6 **a** Let X be the number of heads obtained in 20 spins, so $X \sim \mathrm{B}(20, p)$
$\mathrm{Var}(X) = 20p(1 - p) = 4.2$
So $p(1 - p) = 0.21$
$p^2 - p + 0.21 = 0$
$(p - 0.3)(p - 0.7) = 0$ factoring
$p = 0.3$ (as $p < 0.5$)

6 b $P(X=7)=\binom{20}{7}(0.3)^7(0.7)^{13}$

$=77520\times0.0002187\times0.0096889=0.1643\text{ (4 d.p.)}$

7 a Let Y be the number of times in 10 attempts that the salesman gets a reply.
So $Y\sim B(10,0.65)$

i $P(Y=5)=\binom{10}{5}(0.65)^5(0.35)^5$

$=252\times0.11603\times0.005252=0.1536\text{ (4 d.p.)}$

ii $P(Y\geqslant5)=1-P(Y\leqslant4)$ find value by using a calculator

$=1-0.0949=0.9051$

b Let X be the number of times in n attempts that the salesman gets a reply.
So $X\sim B(n,0.65)$

i Require that $E(X)=0.65n=78$
So $n=120$

ii $\text{Var}(X)=np(p-1)=120\times0.65\times0.35=27.3$

8 a Let X be the number of biscuits in a box of 48 that are broken.
So $X\sim B(48,0.04)$
$P(X\geqslant2)=1-P(X\leqslant1)$ find value by using a calculator

$=1-0.4228=0.5772$

b Let Y be the number of boxes out of 120 that contain at least 2 broken biscuits.
So $Y\sim B(120,0.5772)$
$E(X)=120\times0.5772=69.264$
$\text{Var}(X)=120\times0.5772\times(1-0.5772)$

$=120\times0.5772\times0.4228$

$=29.28\text{ (2 d.p.)}$

9 a $X\sim B(5,p)$
$P(X\geqslant1)=1-P(X=0)$

$=1-\binom{5}{0}p^0(1-p)^5=1-(1-p)^5=0.83193$

So $(1-p)^5=0.16807$
$1-p=0.7\Rightarrow p=0.3$

b $E(X)=5\times0.3=1.5$
$\text{Var}(X)=5\times0.3\times0.7=1.05$

10 a Let X be the number of sixes in 5 throws of the dice.

From the data:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{163\times0+208\times1+98\times2+28\times3+3\times4}{163208+98+28+3} = 1$$

$$\sigma^2 = \frac{\sum fx^2}{\sum f} - 1^2 = \frac{163\times0^2+208\times1^2+98\times2^2+28\times3^2+3\times4^2}{163208+98+28+3} = 1.8-1 = 0.8$$

b $X \sim \mathrm{B}(5, p)$

$\mathrm{E}(X) = 5p = 1$

So $p = 0.2$

c Using $X \sim \mathrm{B}(5, 0.2)$, the expected frequencies can be found using the formula

$$\text{expected frequency for } x \text{ sixes} = 500\times \mathrm{P}(X=x) = 500\times\binom{5}{x}(0.2)^x(0.8)^{5-x}$$

Number of sixes	0	1	2	3	4	5
Expected frequency	164	205	102	26	3	0

The data are very similar to the expected results from the proposed binomial model.

d Using the model, $\mathrm{Var}(X) = 5\times0.2\times0.8 = 0.8$, which exactly matches the observed variance of the data. This further supports the suitability of the binomial model.

Challenge

a $\mathrm{E}(X) = \sum x\mathrm{P}(X-x)$

$$= \sum_{x=0}^{3} x\binom{3}{x}p^x(1-p)^{3-x}$$

$$= 0+1\times3p(1-p)^2+2\times3p^2(1-p)+3\times p^3$$

$$= 3p(1-2p+p^2)+6p^2(1-p)+3p^3$$

$$= 3p(1-2p+p^2+2p-2p^2+p^2) = 3p$$

b $\mathrm{Var}(X) = \sum x^2\mathrm{P}(X-x) - (\mathrm{E}(X))^2$

$$= \sum_{x=0}^{3} x^2\binom{3}{x}p^x(1-p)^{3-x} - (3p)^2$$

$$= 0+1^2\times3p(1-p)^2+2^2\times3p^2(1-p)+3^2\times p^3-9p^2$$

$$= 3p(1-2p+p^2)+12p^2(1-p)+9p^3-9p^2$$

$$= 3p(1-2p+p^2+4p-4p^2+3p^2-3p) = 3p(1-p)$$