Exercise 1B

1 $X \sim B(9, 0.2)$

a $P(X \leqslant 4) = 0.9804$ (tables)

b $P(X < 3) = P(X \leqslant 2) = 0.7382$ (tables)

c $P(X \geqslant 2) = 1 - P(X \leqslant 1) = 1 - 0.4362 = 0.5638$ (tables)

d $P(X = 1) = P(X \leqslant 1) - P(X = 0) = 0.4362 - 0.1342 = 0.3020$ (tables)

2 $X \sim B(20, 0.35)$

a $P(X \leqslant 10) = 0.9468$ (tables)

b $P(X > 6) = 1 - P(X \leqslant 6) = 1 - 0.4166 = 0.5834$ (tables)

c $P(X = 5) = P(X \leqslant 5) - P(X \leqslant 4) = 0.2454 - 0.1182 = 0.1272$ (tables)

d $P(2 \leqslant X \leqslant 7) = P(X \leqslant 7) - P(X \leqslant 1) = 0.6010 - 0.0021 = 0.5989$ (tables)

3 a Using the binomial cumulative function on a calculator where $x = 19$, $n = 40$ and $p = 0.47$,
$P(X < 20) = P(X \leqslant 19) = 0.5888$

b Using the binomial cumulative function on a calculator where $x = 16$, $n = 40$ and $p = 0.47$,
$P(X > 16) = 1 - P(X \leqslant 16) = 0.7662$

c Using the binomial cumulative function on a calculator where $x = 10$ and 15, $n = 40$ and $p = 0.47$,
$P(11 \leqslant X \leqslant 15) = P(X \leqslant 15) - P(X \leqslant 10) = 0.1478 - 0.0036 = 0.1442$

d Using the binomial cumulative function on a calculator where $x = 10$ and 16, $n = 40$ and $p = 0.47$,
$P(10 < X < 17) = P(X \leqslant 16) - P(X \leqslant 10) = 0.2338 - 0.0036 = 0.2302$

4 a Using the binomial cumulative function on a calculator where $x = 20$, $n = 37$ and $p = 0.65$,
$P(X > 20) = 1 - P(X \leqslant 20) = 0.8882$

b Using the binomial cumulative function on a calculator where $x = 26$, $n = 37$ and $p = 0.65$,
$P(X \leqslant 26) = 0.7992$

c Using the binomial cumulative function on a calculator where $x = 19$ and 14, $n = 37$ and $p = 0.65$,
$P(15 \leqslant X < 20) = P(X \leqslant 19) - P(X \leqslant 14) = 0.06061 - 0.00068 = 0.05993$

5 $X =$ 'number of heads'
$X \sim B(8, 0.5)$ (coins are fair so $p = 0.5$)

a $P(X = 0) = (0.5)^8 = 0.0039$ (tables)

b $P(X \geqslant 2) = 1 - P(X \leqslant 1) = 1 - 0.0352 = 0.9648$ (tables)

5 c $P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.6367 = 0.3633$ (tables)

6 $X =$ 'number of plants with blue flowers on tray of 15'

a $P(X = 4) = P(X \leqslant 4) - P(X \leqslant 3) = 0.6865 - 0.4613 = 0.2252$ (tables)

b $P(X \leqslant 3) = 0.4613$ (tables)

c $P(3 \leqslant X \leqslant 6) = P(X \leqslant 6) - P(X \leqslant 2) = 0.9434 - 0.2361 = 0.7073$ (tables)

7 $X \sim B(50, 0.40)$

a $P(X \leqslant 13) = 0.0280$
$P(X \leqslant 14) = 0.0540$ (tables)
$\therefore k = 13$

b $P(X \leqslant 27) = 0.9840$
$\Rightarrow P(X > 27) = 0.0160 > 0.01$
$P(X \leqslant 28) = 0.9924$
$\Rightarrow P(X > 28) = 0.0076 < 0.01$
$\therefore r = 28$

8 $X \sim B(40, 0.10)$

a $P(X = 0) = 0.0148 < 0.02$
$P(X \leqslant 1) = 0.0805 > 0.02$ (tables)
$P(X < 1) = 0.0148 < 0.02$
$\therefore k = 1$

b $P(X \leqslant 8) = 0.9845$ (tables)
$\Rightarrow P(X > 8) = 0.0155 > 0.01$
$P(X \leqslant 9) = 0.9949$
$\Rightarrow P(X > 9) = 0.0051 < 0.01$
$\therefore r = 9$

c $P(k \leqslant X \leqslant r) = P(X \leqslant r) - P(X \leqslant k-1)$
$= P(X \leqslant 9) - P(X = 0)$
$= 0.9949 - 0.0148$
$= 0.9801$

9 a A suitable distribution is $X \sim B\ (10, 0.30)$. Assumptions: There are two possible outcomes of each trial, listen or don't listen. There is a fixed number of trials, 10, and fixed probability of success, 0.3. Each resident in the sample is assumed to listen independently.

b $P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.8497 = 0.1503$ (tables)

c $P(X \leqslant 6) = 0.9894$ so $P(X \geq 7) = 1 - 0.9894 = 0.0106 > 0.01$
$P(X \leqslant 7) = 0.9984$ so $P(X \geq 8) = 1 - 0.9984 = 0.0016 < 0.01$ (tables)
Therefore $s = 8$ is the smallest such value.

10 $X =$ 'number of defects in 50 components'
$X \sim B(50, 0.05)$

a $P(X < 2) = P(X \leqslant 1) = 0.2794$ (tables)

b $P(X > 5) = 1 - P(X \leqslant 5) = 1 - 0.9622 = 0.0378$ (tables)

c Seek smallest d such that $P(X > d) < 0.05$
$P(X \leqslant 4) = 0.8964$ so $P(X > 4) = 0.1036 > 0.05$
$P(X \leqslant 5) = 0.9622$ so $P(X > 5) = 0.0378 < 0.05$
$\therefore d = 5$