Exercise 1A

1 a $P(X=2)=\binom{8}{2}\times\left(\frac{1}{3}\right)^2\times\left(\frac{2}{3}\right)^6$
$=0.273$ (to 3 s.f.)

b $P(X=5)=\binom{8}{5}\times\left(\frac{1}{3}\right)^5\times\left(\frac{2}{3}\right)^3$
$=0.0683$ (to 3 s.f.)

c $P(X\leqslant 1)=P(X=1)+P(X=0)$
$=8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^7+\left(\frac{2}{3}\right)^8$
$=\left(\frac{2}{3}\right)^7\left(\frac{8}{3}+\frac{2}{3}\right)$
$=\left(\frac{2}{3}\right)^7\times\frac{10}{3}$
$=0.195$ (to 3 s.f.)

2 a $P(T=5)=\binom{15}{5}\times\left(\frac{2}{3}\right)^5\times\left(\frac{1}{3}\right)^{10}=0.00670$ (to 3 s.f.)

b $P(T=10)=\binom{15}{10}\times\left(\frac{2}{3}\right)^{10}\times\left(\frac{1}{3}\right)^5=0.214$ (to 3 s.f.)

c $P(3\leqslant T\leqslant 4)=P(T=3)+P(T=4)=\binom{15}{3}\times\left(\frac{2}{3}\right)^3\times\left(\frac{1}{3}\right)^{12}+\binom{15}{4}\times\left(\frac{2}{3}\right)^4\times\left(\frac{1}{3}\right)^{11}$
$=0.00025367...+0.00152206...$
$=0.00178$ (to 3 s.f.)

3 a $X=$ ‘number of defective bolts in a sample of 20’
$X\sim B(20, 0.01)$
$n=20,\ p=0.01$
Assume bolts are defective independently of one another.

b $X=$ ‘number of times wait or stop through 6 sets of lights’
$X\sim B(6, 0.52)$
$n=6,\ p=0.52$
Assume the lights operate independently and the time the lights are on/off is constant.

3 c X = 'number of aces in Stephanie's next 30 serves'

$X \sim B(30, \frac{1}{8})$

$n = 30,\ p = \frac{1}{8}$

Assume serving an ace occurs independently and the probability of an ace is constant.

4 a X = 'number of people in class of 14 who are Rh–'

$X \sim B(14, 0.15)$ is a reasonable model if we assume that being Rh– is independent from pupil to pupil, so no siblings.

b This is not binomial since the number of trials or tosses is not known and fixed. The probability of a head at each toss is constant $(p = 0.5)$ but there is no value for n.

c Assuming, reasonably, that the colours of the cars are independent,

X = 'number of red cars out of 15'

$X \sim B(15, 0.12)$

5 a Let X = 'number of balloons that do not burst'

$$P(X = 20) = (0.95)^{20}$$
$$= 0.358 \text{ (3 s.f.)}$$

b Let Y = 'number of balloons that do burst'

$$P(Y = 2) = \binom{20}{2}(0.95)^{18}(0.05)^2$$
$$= 0.189 \text{ (to 3 s.f.)}$$

6 a There are two possible outcomes of each trial: faulty or not faulty. There are a fixed number trials, 10, and fixed probability of success: 0.08. Assuming each member in the sample is independent, a suitable model is $X \sim B(10, 0.08)$.

b $P(X = 4) = \binom{10}{4}(0.08)^4(0.92)^6 = \frac{10!}{4!6!}(0.08)^4(0.92)^6 = 0.00522$ (to 3 s.f.)

7 a Assumptions are that there is a fixed sample size, that there are only two outcomes for the genetic marker (present or not present), and that there is a fixed probability of people having the marker.

b $X \sim B(50, 0.04)$

$$P(X = 6) = \binom{50}{6}(0.04)^6(0.96)^{44} = \frac{50!}{6!44!}(0.04)^6(0.96)^{44} = 0.0108 \text{ (to 3 s.f.)}$$

8 a We are assuming that each roll of the dice is independent. A suitable model is $X \sim B(15, 0.3)$.

b $X \sim B(15, 0.3)$

$$P(X = 4) = \binom{15}{4}(0.3)^4(0.7)^{11} = \frac{15!}{4!11!}(0.3)^4(0.7)^{11} = 0.219 \text{ (to 3 s.f.)}$$

c $P(X \leqslant 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= (0.7)^{15} + \binom{15}{1}(0.3)^1(0.7)^{14} + \binom{15}{2}(0.3)^2(0.7)^{13}$$

$$= (0.7)^{15} + \frac{15!}{1!14!}(0.3)^1(0.7)^{14} + \frac{15!}{2!13!}(0.3)^2(0.7)^{13}$$

$$= 0.127 \text{ (to 3 s.f.)}$$