

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**4733**

Probability & Statistics 2

MARK SCHEME

**Specimen Paper**

<b>MAXIMUM MARK</b>	<b>72</b>
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**This mark scheme consists of 4 printed pages.**

1	(i) $\frac{12.0}{\sqrt{n}} = 1.50 \Rightarrow \sqrt{n} = \frac{12.0}{1.50} = 8 \Rightarrow n = 64$	B1 M1 A1	3	For any correct equation involving $n$ For correct solution method for $n$ or $\sqrt{n}$ For correct answer 64
	(ii) $n$ is large, the distribution of $\bar{F}$ can be taken to be normal, according to the Central Limit Theorem	M1 A1	2	For relating the size of $n$ to normality For reference to the CLT
		<b>5</b>		
2	(i) Reasons for bias may include: Larger properties more likely to be picked Some regions of the map more/less likely	B1 B1	2	For stating one valid relevant reason For stating a second valid relevant reason
	(ii) Make a list of all the houses in the neighbourhood Number the houses from 1 upwards Select the sample using random numbers	B1 B1 B1	3	For stating or implying a sampling frame For numbering the sampling units For referring to use of random numbers
		<b>5</b>		
3	(i) $\frac{1}{36}$	B1	1	For correct probability
	(ii) Number obtaining two sixes $\sim B(60, \frac{1}{36})$ Approximate distribution is $Po(\frac{5}{3})$ $P(\geq 4) = 1 - e^{-\frac{5}{3}} \left[ 1 + \frac{5}{3} + \frac{(5/3)^2}{2!} + \frac{(5/3)^3}{3!} \right]$  $= 0.0883$	M1 A1✓ M1 M1 A1	5	For stating or implying binomial distribution For the correct Poisson approximation For calculation of correct terms For correct use of Poisson formula For correct answer 0.088(3)
		<b>6</b>		
4	(i) (a) $\frac{15.0 - 20.0}{\sigma} = -0.253$  Hence $\sigma = \frac{5}{0.253} \approx 19.8$	M1 B1 M1 A1	4	For standardising and equating to $\Phi^{-1}(p)$ For correct value 0.253 (or 0.254) seen For solving equation for $\sigma$ For correct value 19.8
	(b) $g = 25.0$ , using symmetry  Hence $P(G > 2g) = 1 - \Phi\left(\frac{50.0 - 20.0}{19.8}\right)$ $= 1 - 0.935 = 0.065$	B1 M1 A1	3	For stating (or finding) the value of $g$ For correct process for upper tail prob For correct answer
	(ii) If normal, $P(G < 0)$ is substantial Hence the assumption seems unjustified	M1 A1	2	For considering relevant normal probability For stating the appropriate conclusion
		<b>9</b>		

<p>5 <math>\bar{x} = \frac{4070}{50} = 81.4</math></p> <p><math>s^2 = \frac{336100}{49} - \frac{4070^2}{49 \times 50} = 98</math></p> <p><math>H_0 : \mu = 84.0; H_1 : \mu &lt; 84.0</math></p> <p><i>EITHER:</i> <math>z = \frac{\bar{x} - 84.0}{\sqrt{(s^2/50)}} = -1.857</math></p> <p>This is significant, since <math>-1.857 &lt; -1.645</math></p> <p><i>OR:</i> <math>\frac{c - 84.0}{\sqrt{(s^2/50)}} = -1.645 \Rightarrow c = 81.697</math></p> <p><math>\bar{x}</math> is in the critical region since <math>81.4 &lt; 81.697</math></p> <p>Hence <math>H_0</math> is rejected</p> <p>There is sufficient evidence to conclude that the mean solubility rating is less than 84.0</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1✓</p> <p>A1✓</p>	<p>For correct value of sample mean</p> <p>For calculation of unbiased or biased estimate</p> <p>For correct value of unbiased estimate</p> <p>For correct statement of null hypothesis</p> <p>For correct statement of alt hypothesis</p> <p>For standardising, including use of <math>\sqrt{50}</math></p> <p>For correct value 1.857</p> <p>For comparing <math>z</math> value to <math>-1.645</math> or equiv</p> <p>For critical value calculation, inc use of <math>\sqrt{50}</math></p> <p>For correct value 81.697</p> <p>For comparing sample mean to critical region</p> <p>For stating or implying rejection of <math>H_0</math></p> <p>For stating the outcome in context</p> <p style="text-align: right;"><b>10</b></p>
<p>6 (i) (a) For one day, the distribution is Po(0.5)</p> <p>Hence <math>P(\text{exactly } 2) = 0.9856 - 0.9098 = 0.0758</math></p> <hr/> <p>(b) No. of days with no cars <math>\sim B(365, 0.6065)</math></p> <p>Normal approximation is <math>N(221.3725, 87.11)</math></p> <p><math>P(&lt; 205) = P\left(Z &lt; \frac{204.5 - 221.3725}{\sqrt{87.11}}\right)</math></p> <p><math>= \Phi(-1.808) = 0.0353</math></p> <hr/> <p>(ii) Events (cars running out of petrol) must occur at a constant average rate. This seems unlikely, given that there will be different volumes of traffic on different days of the week (e.g. weekdays and weekends)</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1✓</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>For use of correct Poisson mean</p> <p>For relevant use of tables (or formula)</p> <p>For correct answer 0.0758</p> <p>For relevant Poisson probability of <math>P(0)</math></p> <p>For identifying correct binomial distribution</p> <p>For correct use of <math>np</math> and <math>npq</math></p> <p>For standardising (with or without c.c. here)</p> <p>For completely correct expression</p> <p>For correct answer 0.0353</p> <p>For correct statement of the condition</p> <p>For a correct explanation</p> <p style="text-align: right;"><b>11</b></p>

7	(i) $1 = k \int_0^3 (9x - x^3) dx = k \left[ \frac{9}{2}x^2 - \frac{1}{4}x^4 \right]_0^3 = \frac{81}{4}k$ Hence $k = \frac{4}{81}$	M1 A1	2	For equating to 1 and integrating For showing given answer correctly
	(ii) $E(X) = \frac{4}{81} \int_0^3 x^2 (9 - x^2) dx = \frac{4}{81} \left[ 3x^3 - \frac{1}{5}x^5 \right]_0^3 = 1.6$	M1 A1 A1	3	For attempt at $\int_0^3 xf(x) dx$ For correct indefinite integral, in any form For correct answer 1.6
	(iii) (a) $\frac{3}{5} = \frac{4}{81} \int_0^y x(9 - x^2) dx = \frac{4}{81} \left[ \frac{9}{2}x^2 - \frac{1}{4}x^4 \right]_0^y$  Hence $\frac{3}{5} = \frac{4}{81} \left\{ \frac{9}{2}y^2 - \frac{1}{4}y^4 \right\}$ i.e. $5y^4 - 90y^2 + 243 = 0$	M1 B1 M1 A1	4	For attempt at $\int_0^y f(x) dx = \frac{3}{5}$ For correct indefinite integral, in any form Use limits to produce relevant equation in y For showing given answer correctly
	(b) $w = \frac{90 \pm \sqrt{(90^2 - 4 \times 5 \times 243)}}{10} = 3.31 \text{ or } 14.7$  Hence $y = \sqrt{3.31} = 1.82$	M1 A1 A1	3	For use of quadratic formula to find w For either value found correctly For correct (unique) answer 1.82
<b>12</b>				
8	(i) $H_0 : p = 0.15; H_1 : p > 0.15$  Under $H_0$ , number left-handed $L \sim B(12, 0.15)$ $P(L \geq 5) = 1 - 0.9761 = 0.0239$  This is significant, since $0.0239 < 0.05$  Hence $H_0$ is rejected Accept the suggestion that the proportion of mathematicians who are left-handed is more than 15%	B1 B1 M1 M1 A1 M1 A1✓	8	For correct statement of null hypothesis For correct statement of alt hypothesis For correct distribution stated or implied For calculation of relevant tail probability, or finding the critical region For correct value 0.0239 or region $l \geq 5$ For comparing tail probability with 0.05 or observed value with critical region For stating or implying rejection of $H_0$
	(ii) $P_I = P(L \text{ in critical region}) = 0.0239$	M1 A1	2	For evaluating $P(\text{reject } H_0)$ For correct answer 0.0239 or equivalent
	(iii) $P_{II} = P(L \leq 4   p = 0.2) = 0.9274$	M1 A1	2	For evaluating $P(\text{accept } H_0)$ with $p = 0.2$ For correct probability
	(iv) $P_{II} = 0.0188$ for $p = \frac{2}{3}$ and $0.0095$ for $p = 0.7$ So the proportion is between 67% and 70%	M1 A1	2	For relevant use of tables For an appropriate conclusion
<b>14</b>				