OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4733**: Probability and Statistics 2

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such " fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer/Indicative content	Marks		Guidance
1		$\hat{\mu} = \bar{x} = \frac{3752}{14}$ **= 268** $\frac{1007448}{14} - \bar{x}^2$ [=136.57...] $\times \frac{14}{13}$; **= 147**(.07…)	B1 M1 M1 A1	**4**	268 only, must be stated separately, *not* isw If single formula used, give M1 for divisor 13 anywhere Multiply by 14/13 Answer, a.r.t. 147, or $\frac{1912}{13} = 147\frac{1}{13}$ MR 3572: 255.14, 7390.6 gets B0M1M1A1
2		$\frac{1.03-1.00}{\sigma} = 1.645$ [$\sigma = 0.0182\ldots \approx {}^{6}/_{329}$] $1 - \Phi\left(\frac{1.05-1.03}{\sigma}\right) = 1 - \Phi(1.0966)$ = 1 – 0.8635 = **0.1365** or 13.6(5)%	M1dep* A1 B1 *M1 M1 A1	**6**	Standardise and equate to Φ^{-1}, allow wrong sign, σ^2, 1–, cc etc All correct apart possibly from value of Φ^{-1} 1.645 seen anywhere, allow –1.645, can be implied Solve to find σ, or eliminate σ, dependent on first M1 Standardise with $\mu = 1.03$, use Φ, answer < 0.5, allow √ errors Final answer in range [0.1355, 0.137] or [13.55%, 13.7%], must be from positive σ, not from σ^2 0.1333 from $\sigma = 0.018$ is 5+A0
3		N(26, 9.1) $\Phi\left(\frac{30.5-26}{\sqrt{9.1}}\right) = \Phi(1.492)$ = **0.9322** "*np* > 5" or "*n* large" stated "14 > 5" or "*p* close to ½" stated	M1 A1 M1 A1 A1 B1 B1	**7**	Normal, mean their attempt at 40×0.65 Mean 26 and variance or SD 9.1 Standardise, their *np*, *npq*, no √*n*, allow cc or √ errors cc and √(their *npq*) correct Final answer, a.r.t. 0.932 One condition asserted Complementary condition, if "*nq*" must see 14 somewhere. *Not npq* [Thus: "*np* > 5, *nq* > 5": B1B0] Extra conditions, e.g. "*n* > 30": max B1B0 SC: Exact (0.935564): maximum B1B1
4		$\frac{\lambda^4}{4!}e^{-\lambda} = \frac{\lambda^5}{5!}e^{-\lambda}$ $\frac{\lambda^4}{4!} = \frac{\lambda^5}{5!}$ $\Rightarrow \lambda$ **= 5** **0.175**(46)	M1 A1 M1 A1 B1	**5**	Poisson formula used [*not* just quoted] correctly once This equation or exact equivalent, needs $e^{-\lambda}$ seen somewhere Correct method for cancelling $e^{-\lambda}$ Solve to get $\lambda = 5$ only, www Probability, in range [0.175, 0.176], allow from $\lambda = 5$ from wrong working

Question		Answer/Indicative content	Marks		Guidance
5	(i)	H_0: $p = 0.55$, H_1: $p > 0.55$ R ~ B(8, 0.55) where R is the number of girls α: $P(R \geq 6) = 1 - 0.7799 = 0.2201$ > 0.1	B2 M1 A1 B1		All correct, B2. One error (e.g. ≠, wrong or no letter) B1, but r, x etc: B0 B(8, 0.55) stated or implied, e.g. N(4.4, 1.98) P(≥ 6) = 0.2201, or P(< 6) = 0.7799 Compare P(≥ 6) with 0.1 or P(< 6) with 0.9
		β: CR is ≥ 7 and 6 < 7 $p = 0.0632$	B1 A1		Correct CR stated and explicit comparison with 6 This probability seen, a.r.t. 0.0632. Award if 0.9368 seen and CR is correct. If CR not clearly stated, cannot get last M1A1
		Do not reject H_0. There is insufficient evidence that the girls are proportionately more likely to become Head Student.	M1 A1	**7**	Correct first conclusion, requires B(8, 0.55), *not* P(> 6) [= 0.0632] or P(≤ 6) [= 0.9368] or P(= 6) [= 0.1569]. Allow 0.7799 if compared with 0.9 Interpreted, in context, acknowledge uncertainty, double negative. SC: Normal: max B2 M1 SC: Two different attempts: max B2 M1 unless both correct
	(ii)	Assume that the last 8 years are a random sample of years when Head Student has been chosen	B1	**1**	Refer to random sample, allow implied by any method described Must be choosing *years*, not *students* *Not* quote conditions for random sample unless explicitly "years" Extras: ignore unless clearly wrong, in which case B0
6	(i)	Cars pass independently of one another and at constant average rate	B1 B1	**2**	"Independently", refer to cars. Not "constant rate", "constant probability". No extra conditions. Ignore all references to "singly" (which is *wrong* in this context!)
	(ii) α	$P(\leq 7) - P(\leq 3) = 0.6728 - 0.1118$ $= \mathbf{0.561}(0)$	M1 A2	**3**	0.680 or 0.681: M1A0 Allow from calculator, no working 0.4491 or 0.5679: M1A1 Allow from calculator, no working
	or β	P(4) + P(5) + P(6) + P(7) = 0.1118 + 0.1454 + 0.1575 + 0.1462 $= \mathbf{0.561}(0)$	M1 A1 A1	**3**	Correct formula for ≥ 3 probabilities from Po(6.5) added, can be implied 3, 4 or 5 correct terms (e.g. P(3) = 0.06880), can be algebraic or implied Answer, a.r.t. 0.561
	(iii)(a)	Po(30) ≈ N(30, 30) $1 - \Phi\left(\frac{35.5 - 30}{\sqrt{30}}\right) = 1 - \Phi(1.004)$ $= 1 - 0.8422 = \mathbf{0.1578}$ Normal suitable as 30 > 15	M1 A1 M1 A1 A1 B1	**6**	Normal, mean 30, stated or implied Variance or SD 30 Standardise, their λ, λ, allow wrong/no cc, var/SD errors cc, $\sqrt{\lambda}$ correct Answer, a.r.t. 0.158 [NB: 0.157 may be from exact. See below] Or "λ large", etc., but no other conditions. If numerical comparison, must involve 15. SC: Exact Poisson, 0.1574, max B1 SC: Po(30), N(15, 15): M0B1 M1A1A0 B1, max 4/6
	(b)	Cars do not pass independently/randomly, as one may be immediately followed by another	B1	**1**	Any plausible relevant explanation in context, needn't be connected to conditions, e.g. "steady stream". *Not* "several cars might pass at once". Allow explanations that might also hold for smaller λ Do not allow comment on size of λ unless explained in valid way, e.g. "λ too large so cars follow one another", but not "λ too large for Poisson"

Question		Answer/Indicative content	Marks	Guidance
7	(i)	[x represents a] possible value(s) taken by X	B1 **1**	Must refer to, or imply, both x and X or "the random variable" Ignore extra unless definitely wrong
	(ii)	$\int_2^\infty ax^{-3}+bx^{-4}\mathrm{d}x=\left[-\frac{a}{2x^2}-\frac{b}{3x^3}\right]_2^\infty=\frac{a}{8}+\frac{b}{24}$ *or* $\int_1^\infty ax^{-3}+bx^{-4}\mathrm{d}x=\left[-\frac{a}{2x^2}-\frac{b}{3x^3}\right]_1^\infty=\frac{a}{2}+\frac{b}{3}$ *or* $\int_1^2 ax^{-3}+bx^{-4}\mathrm{d}x=\left[-\frac{a}{2x^2}-\frac{b}{3x^3}\right]_1^2=\frac{3a}{8}+\frac{7b}{24}$ $\frac{a}{2}+\frac{b}{3}=1$ or $\frac{a}{8}+\frac{b}{24}=\frac{3}{16}$ or $\frac{3a}{8}+\frac{7b}{24}=\frac{13}{16}$ Solve to get $a=1$ $b=\frac{3}{2}$	B1 M1 M1 M1 A1 A1 B1 **7**	Correct indefinite integral [from any set of limits or none] Integrate and substitute limits to obtain one expression Integrate and substitute limits to obtain a second expression The limits must be two of (1, ∞), (1, 2) or (2, ∞), allow (3, ∞) for "≥ 2" Equate two expressions from definite integrals to 1 or $\frac{3}{16}$ or $\frac{13}{16}$ as appropriate, and attempt to solve Both equations correct, any equivalent simplified form, can be implied ["simplified" = one a term, one b term, one number term] Correctly show $a = 1$ **AG**, www Correct value of b obtained from at least one correct equation SC: One equation only: M1B1 M0M0A0 A0B1, max 3/7 Two equations, assume $a = 1$, solve for b, checked in other equation: 7/7
	(iii)	$\int_1^\infty ax^{-2}+bx^{-3}\mathrm{d}x=\left[-\frac{a}{x}-\frac{b}{2x^2}\right]_1^\infty$ $\left\{=a+\frac{b}{2}\right\}$ $=1\frac{3}{4}$	M1 B1ft A1 **3**	Integrate $xf(x)$, limits 1 and ∞ seen somewhere Correct indefinite integral, their b, can be implied by correct answer *Expect to see* $\int_1^\infty x^{-2}+\frac{3}{2}x^{-3}\mathrm{d}x=\left[-\frac{1}{x}-\frac{3}{4x^2}\right]_1^\infty$ Correctly obtain 1¾ or a.r.t. 1.75 www, allow from calculator

Question		Answer/Indicative content	Marks	Guidance
8	(i)	H_0: $\mu = 13.3$, H_1: $\mu < 13.3$	B2	Both correct: B2. One error [e.g. p, ≠, no symbol] B1, but x, $\bar{x}$ etc B0
	α:	$z = \frac{12.48 - 13.3}{\sqrt{12.25/50}} = -1.6566$ [$p = 0.0488$]	M1 A1	Standardise with √50, allow √ errors, allow cc, allow 13.3 – 12.48 z in range [–1.66, –1.65], or p in range [0.04875, 0.0489], allow 0.9512 only if consistent
		[12.25/50 = 0.245] < –1.645 [$p < 0.05$]	B1	Compare with –1.645, allow +1.6566 with +1.645, or p with 0.05/0.95 as consistent
	β:	CV $13.3 - 1.645\sqrt{\frac{12.25}{50}} = 12.4857\ldots$	M1 B1	13.3 – $z\sigma/\sqrt{50}$, any recognisable z, allow √ errors etc, ignore 13.3 + … $z = 1.645$
		12.48 < CV	A1	Compare 12.49 (or better) with 12.48, ignore 13.3 + … SC: 2-tailed, 12.33 gets B1B0 M1B0A1ft M1A1
		Reject H_0. Significant evidence that animals in zoos have shorter expected lifetime	M1 A1ft **7**	Consistent, needs √50, like-with-like comparison, hypotheses *not* 12.48 Contextualised, acknowledge uncertainty, their z SC1: 2-tailed: can get B1 M1A1B0 M1A1 max 5/7 SC2: No √50: can get B2 M0A0 B1 M0 max 3/7 SC3: $\bar{x}$ and μ confused consistently: can get B0 M1A1 B1 M0 SC4: 50/49 used in (i): can get B2 M1A0B1 M1A1 (6) in (i), M1 in (ii)
	(ii)	$\hat{\sigma}^2 = \frac{50}{49} \times 12.25$ [= 12.5]	M1	Multiply 12.25 by 50/49, allow √ etc, allow if done in part (i) but then 0
		$z = \frac{12.48 - 13.3}{\sqrt{12.5/50}} = -1.64$ [$p = 0.0505$]	M1 A1	Standardise with √50 Obtain a.r.t. –1.64, allow +1.64 if consistent with (i).
		> –1.645 [$p > 0.05$]	B1	Compare with same CV as in (i)
		Opposite conclusion	A1ft **5**	State opposite conclusion (ft), any form, allow $\bar{x}/\mu$ here , needs M1M1 *Identical mark scheme for method β, CV 12.4775* SC1: 50 omitted consistently in both: M1M0A0B1A1 max 3/5 SC2: no √50 in (i), √50 but not 50/49 in (ii): M0M1A0B1A1 max 3/5
	(iii)	Yes as population not known to be normal	B1 **1**	Not "n large" unless "Yes, not known normal, but n large so can use" No wrong extras, e.g. "depends on whether it's sample or population"
9		P(≤ 19 | $\lambda = 11$) = 0.9907	M1	Attempt to find critical region from $\lambda = 11$, allow even if tail wrong e.g. P(≤ 3) = 0.0049, allow from $\lambda = 10$ or 12
		so critical region ≥ 20	A1	Critical region is ≥ 20. [CV 19 or 20 can imply first M1] [If only one probability shown, assume this is CR]
		P(≥ 20) = 0.0093	A1	Probability 0.9907 or 0.0093 seen (even if CR is wrong) [from $\lambda = 11$]
		P(≤ 19 | $\lambda = 14$)	M1	Find P(not in CR | $\lambda = 14$), must now be LH tail, e.g. 0.8826
		= **0.9235**	A1	Answer in range [0.923, 0.924]
		Type II error	B1 **6**	"Type II error" stated, allow "Type 2" SC1: P(≤ 19) = 0.9907 so CR is ≥ 19: M1A0A1M1A0B1 max 4/6 SC2: $\lambda = 14$ used throughout, e.g. P(≥ 23) = 0.0093: max B1

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