

4733 Probability & Statistics 2

<p>1</p>	$\frac{105.0 - \mu}{\sigma} = -0.7; \frac{110.0 - \mu}{\sigma} = -0.5$ <p>Solve: $\sigma = 25$ $\mu = 122.5$</p>	<p>M1 A1 B1 M1 A1 A1</p> <p>6</p>	<p>Standardise once, equate to Φ^{-1}, allow σ^2 Both correct including signs & σ, no cc (continuity correction), allow wrong z Both correct z-values. “1 –” errors: M1A0B1 Get either μ or σ by solving simultaneously σ a.r.t. 25.0 $\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from σ^2 but <i>not</i> from $\sigma = -25$.</p>
<p>2</p>	<p>Po(20) \approx N(20, 20) Normal approx. valid as $\lambda > 15$ $1 - \Phi\left(\frac{24.5 - 20}{\sqrt{20}}\right) = 1 - \Phi(1.006)$ $= 1 - 0.8427 = \mathbf{0.1573}$</p>	<p>M1 A1 B1 M1 A1 A1</p> <p>6</p>	<p>Normal stated or implied (20, 20) or (20, $\sqrt{20}$) or (20, 20^2), can be implied “Valid as $\lambda > 15$”, or “valid as λ large” Standardise 25, allow wrong or no cc, $\sqrt{20}$ errors $1.0 < z \leq 1.01$ Final answer, art 0.157</p>
<p>3</p>	<p>$H_0 : p = 0.6, H_1 : p < 0.6$ where p is proportion in population who believe it’s good value $R \sim B(12, 0.6)$ $\alpha: P(R \leq 4) = 0.0573 > 0.05$ $\beta: CR \text{ is } \leq 3 \text{ and } 4 > 3$ $p = 0.0153$ Do not reject H_0. Insufficient evidence that the proportion who believe it’s good value for money is less than 0.6</p>	<p>B2 M1 A1 B1 B1 A1</p> <p>7</p>	<p>Both, B2. Allow $\pi, \%$ One error, B1, except x or \bar{x} or r or R: 0 B(12, 0.6) stated or implied, e.g. N(7.2, 2.88) Not $P(< 4)$ or $P(\geq 4)$ or $P(= 4)$ Must be using $P(\leq 4)$, or $P(> 4) < 0.95$ and binomial Must be using CR; explicit comparison needed Correct conclusion, needs B(12,0.6) and ≤ 4 Contextualised, some indication of uncertainty [SR: N(7.2, ...) or Po(7.2): poss B2 M1A0] [SR: $P(< 4)$ or $P(= 4)$ or $P(\geq 4)$: B2 M1A0]</p>
<p>4</p>	<p>(i) Eg “not all are residents”; “only those in street asked” (ii) Obtain list of whole population Number it sequentially Select using random numbers [Ignore method of making contact] (iii) Two of: α: Members of population equally likely to be chosen β: Chosen independently/randomly γ: Large sample (e.g. > 30)</p>	<p>B1 B1 B1 B1 B1</p> <p>2 3 2</p>	<p>One valid relevant reason A definitely different valid relevant reason Not “not a random sample”, not “takes too long” “Everyone” or “all houses” must be implied Not “number it with random numbers” unless then “arrange in order of random numbers” SR: “Take a random sample”: B1 SR: Systematic: B1 B0, B1 if start randomly chosen One reason. NB : If “independent”, must be “chosen” independently, not “views are independent” Another reason. Allow “fixed sample size” but not both that and “large sample”. Allow “houses”</p>

5 (i)	Bricks scattered at constant average rate & independently of one another	B1 B1 2	B1 for each of 2 different reasons, in context. (Treat “randomly” ≡ “singly” ≡ “independently”)
(ii)	$Po(12)$ $P(\leq 14) - P(\leq 7) [= .7720 - .0895]$ [or $P(8) + P(9) + \dots + P(14)$] $= \mathbf{0.6825}$	B1 M1 A1 3	$Po(12)$ stated or implied Allow one out at either end or both, eg 0.617, or wrong column, but <i>not</i> from $Po(3)$ nor, eg, .9105 – .7720 Answer in range [0.682, 0.683]
(iii)	$e^{-\lambda} = 0.4$ $\lambda = -\ln(0.4)$ $= 0.9163$ Volume = $0.9163 \div 3 = \mathbf{0.305}$	B1 M1 A1 M1 4	This equation, aef, can be implied by, eg 0.9 Take ln, or 0.91 by T & I λ art 0.916 or 0.92, can be implied Divide their λ value by 3 [SR: Tables, eg 0.9÷3: B1 M0 A0 M1]
6 (i)	$\frac{33.6}{115782.84} - 33.6^2 [= 28.8684]$ $\times \frac{100}{99} = \mathbf{29.16}$	B1 M1 M1 A1 4	33.6 clearly stated [not recoverable later] Correct formula used for biased estimate $\times \frac{100}{99}$, M’s independent. Eg $\frac{\Sigma r^2}{99}[-33.6^2]$ SR B1 variance in range [29.1, 29.2]
(ii)	$\bar{R} \sim N(33.6, 29.16/9)$ $= N(33.6, 1.8^2)$ $1 - \Phi\left(\frac{32 - 33.6}{\sqrt{3.24}}\right) [= \Phi(0.8889)]$ $= \mathbf{0.8130}$	M1 A1 M1 A1 4	Normal, their μ , stated or implied Variance [their (i)]÷9 [not ÷100] Standardise & use Φ , 9 used, answer > 0.5, allow $\sqrt{\quad}$ errors, allow cc 0.05 but <i>not</i> 0.5 Answer, art 0.813
(iii)	No, distribution of R is normal so that of \bar{R} is normal	B2 2	Must be saying this. Eg “9 is not large enough”: B0. Both: B1 max, unless saying that n is irrelevant.
7 (i)	$\frac{2}{9} \int_0^3 x^3(3-x)dx = \frac{2}{9} \left[\frac{3x^4}{4} - \frac{x^5}{5} \right]_0^3 [= 2.7] - (1\frac{1}{2})^2 = \frac{9}{20}$ or $\mathbf{0.45}$	M1 A1 B1 M1 A1 5	Integrate $x^2 f(x)$ from 0 to 3 [not for μ] Correct indefinite integral Mean is $1\frac{1}{2}$, soi [not recoverable later] Subtract their μ^2 Answer art 0.450
(ii)	$\frac{2}{9} \int_0^{0.5} x(3-x)dx = \frac{2}{9} \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^{0.5} = \frac{2}{27}$ AG	M1 A1 2	Integrate $f(x)$ between 0, 0.5, must be seen somewhere Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [] = $\frac{1}{3}$)
(iii)	$B(108, \frac{2}{27})$ $\approx N(8, 7.4074)$ $1 - \Phi\left(\frac{9.5 - 8}{\sqrt{7.4074}}\right)$ $= 1 - \Phi(0.5511)$ $= \mathbf{0.291}$	B1 M1 A1 M1 A1 A1 6	$B(108, \frac{2}{27})$ seen or implied, eg $Po(8)$ Normal, mean 8 variance (or SD) 200/27 or art 7.41 Standardise 10, allow $\sqrt{\quad}$ errors, wrong or no cc, needs to be using $B(108, \dots)$ Correct $\sqrt{\quad}$ and cc Final answer, art 0.291

(iv)	$\bar{X} \sim N(1.5, \frac{1}{240})$	B1 B1√ B1√ 3	Normal Mean their μ Variance or SD (their 0.45)/108 [not (8, 50/729)] NB: <i>not</i> part (iii)
8 (i)	$H_0 : \mu = 78.0$ $H_1 : \mu \neq 78.0$ $z = \frac{76.4 - 78.0}{\sqrt{68.9/120}} = -2.1115$ > -2.576 or $0.0173 > 0.005$	B1 B1 M1 A1 B1	Both correct, B2. One error, B1, but x or \bar{x} : B0. Needs $\pm(76.4 - 78)/\sqrt{(\sigma=120)}$, allow $\sqrt{\quad}$ errors art -2.11 , or $p = 0.0173 \pm 0.0002$ Compare z with $(-)2.576$, or p with 0.005
	$78 \pm z\sqrt{(68.9/120)}$ $= 76.048$ $76.4 > 76.048$	M1 A1√ B1	Needs 78 and 120, can be $-$ only Correct CV to 3 sf, $\sqrt{\quad}$ on z $z = 2.576$ and compare 76.4 , allow from $78 \leftrightarrow 76.4$
	Do not reject H_0 . Insufficient evidence that the mean time has changed	M1 A1√ 7	Correct comparison & conclusion, needs 120, “like with like”, correct tail, \bar{x} and μ right way round Contextualised, some indication of uncertainty
(ii)	$\frac{1}{\sqrt{68.9/n}} > 2.576$ $\sqrt{n} > 21.38,$ $n_{\min} = 458$ Variance is estimated	M1 M1 A1 B1 4	IGNORE INEQUALITIES THROUGHOUT Standardise 1 with n and 2.576 , allow $\sqrt{\quad}$ errors, cc etc but <i>not</i> 2.326 Correct method to solve for \sqrt{n} (<i>not</i> from n) 458 only (<i>not</i> 457), or 373 from 2.326, signs correct Equivalent statement, allow “should use t ”. In principle nothing superfluous, but “variance stays same” B1 bod