OCR
RECOGNISING ACHIEVEMENT

# Friday 18 January 2013 – Afternoon

## A2 GCE MATHEMATICS

**4733/01** Probability & Statistics 2

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4733/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**1** A random variable has the distribution $B(n, p)$. It is required to test $H_0: p = \frac{2}{3}$ against $H_1: p < \frac{2}{3}$ at a significance level as close to 1% as possible, using a sample of size $n = 8$, 9 or 10. Use tables to find which value of $n$ gives such a test, stating the critical region for the test and the corresponding significance level. **[4]**

**2** A random variable $C$ has the distribution $N(\mu, \sigma^2)$. A random sample of 10 observations of $C$ is obtained, and the results are summarised as

$$n = 10, \Sigma c = 380, \Sigma c^2 = 14602.$$

**(i)** Calculate unbiased estimates of $\mu$ and $\sigma^2$. **[4]**

**(ii)** Hence calculate an estimate of the probability that $C > 40$. **[2]**

**3** A factory produces 9000 music DVDs each day. A random sample of 100 such DVDs is obtained.

**(i)** Explain how to obtain this sample using random numbers. **[3]**

**(ii)** Given that 24% of the DVDs produced by the factory are classical, use a suitable approximation to find the probability that, in the sample of 100 DVDs, fewer than 20 are classical. **[5]**

**4** A continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ and $a$ are constants.

**(i)** State what the letter $x$ represents. **[1]**

**(ii)** Find $k$ in terms of $a$. **[2]**

**(iii)** Find $\text{Var}(X)$ in terms of $a$. **[6]**

**5** In a mine, a deposit of the substance *pitchblende* emits radioactive particles. The number of particles emitted has a Poisson distribution with mean 70 particles per second. The warning level is reached if the total number of particles emitted in one minute is more than 4350.

**(i)** A one-minute period is chosen at random. Use a suitable approximation to show that the probability that the warning level is reached during this period is 0.01, correct to 2 decimal places. You should calculate the answer correct to 4 decimal places. **[5]**

**(ii)** Use a suitable approximation to find the probability that in 30 one-minute periods the warning level is reached on at least 4 occasions. (You should use the given rounded value of 0.01 from part **(i)** in your calculation.) **[3]**

**6** Gordon is a cricketer. Over a long period he knows that his population mean score, in number of runs per innings, is 28, and the population standard deviation is 12. In a new season he adopts a different batting style and he finds that in 30 innings using this style his mean score is 28.98.

**(i)** Stating a necessary assumption, test at the 5% significance level whether his population mean score has increased. **[8]**

**(ii)** Explain whether it was necessary to use the Central Limit Theorem in part **(i)**. **[2]**

**7** The continuous random variable $X$ has the distribution $\mathrm{N}(\mu, \sigma^2)$. The mean of a random sample of $n$ observations of $X$ is denoted by $\overline{X}$. It is given that $\mathrm{P}(\overline{X} < 35.0) = 0.9772$ and $\mathrm{P}(\overline{X} < 20.0) = 0.1587$.

**(i)** Obtain a formula for $\sigma$ in terms of $n$. **[5]**

Two students are discussing this question. Aidan says "If you were told another probability, for instance $\mathrm{P}(\overline{X} > 32) = 0.1$, you could work out the value of $\sigma$." Binya says, "No, the value of $\mathrm{P}(\overline{X} > 32)$ is fixed by the information you know already."

**(ii)** State which of Aidan and Binya is right. If you think that Aidan is right, calculate the value of $\sigma$ given that $\mathrm{P}(\overline{X} > 32) = 0.1$. If you think that Binya is right, calculate the value of $\mathrm{P}(\overline{X} > 32)$. **[4]**

**8** In a large city the number of traffic lights that fail in one day of 24 hours is denoted by $Y$. It may be assumed that failures occur randomly.

**(i)** Explain what the statement "failures occur randomly" means. **[1]**

**(ii)** State, in context, two different conditions that must be satisfied if $Y$ is to be modelled by a Poisson distribution, and for each condition explain whether you think it is likely to be met in this context. **[4]**

**(iii)** For this part you may assume that $Y$ is well modelled by the distribution $\mathrm{Po}(\lambda)$. It is given that $\mathrm{P}(Y = 7) = \mathrm{P}(Y = 8)$. Use an algebraic method to calculate the value of $\lambda$ and hence calculate the corresponding value of $\mathrm{P}(Y = 7)$. **[5]**

**9** The random variable $A$ has the distribution $\mathrm{B}(30, p)$. A test is carried out of the hypotheses $\mathrm{H}_0$: $p = 0.6$ against $\mathrm{H}_1$: $p < 0.6$. The critical region is $A \leqslant 13$.

**(i)** State the probability that $\mathrm{H}_0$ is rejected when $p = 0.6$. **[1]**

**(ii)** Find the probability that a Type II error occurs when $p = 0.5$. **[2]**

**(iii)** It is known that on average $p = 0.5$ on one day in five, and on other days the value of $p$ is 0.6. On each day two tests are carried out. If the result of the first test is that $\mathrm{H}_0$ is rejected, the value of $p$ is adjusted if necessary, to ensure that $p = 0.6$ for the rest of the day. Otherwise the value of $p$ remains the same as for the first test. Calculate the probability that the result of the second test is to reject $\mathrm{H}_0$. **[5]**

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.**