OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

# A2 GCE MATHEMATICS

**4733** Probability & Statistics 2

**QUESTION PAPER**



Candidates answer on the printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4733
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

## INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**1** A random sample of 50 observations of the random variable $X$ is summarised by

$$n = 50, \Sigma x = 182.5, \Sigma x^2 = 739.625.$$

Calculate unbiased estimates of the expectation and variance of $X$. **[4]**

**2** The random variable $Y$ has the distribution B(140, 0.03). Use a suitable approximation to find P($Y$ = 5). Justify your approximation. **[5]**

**3** The random variable $G$ has a normal distribution. It is known that

$$P(G < 56.2) = P(G > 63.8) = 0.1.$$

Find P($G$ > 65). **[6]**

**4** The discrete random variable $H$ takes values 1, 2, 3 and 4. It is given that E($H$) = 2.5 and Var($H$) = 1.25. The mean of a random sample of 50 observations of $H$ is denoted by $\bar{H}$.
Use a suitable approximation to find P($\bar{H}$ < 2.6). **[5]**

**5** **(i)** Six prizes are allocated, using random numbers, to a group of 12 girls and 8 boys. Calculate the probability that exactly 4 of the prizes are allocated to girls if

**(a)** the same child may win more than one prize, **[2]**

**(b)** no child may win more than one prize. **[2]**

**(ii)** Sixty prizes are allocated, using random numbers, to a group of 1200 girls and 800 boys. Use a suitable approximation to calculate the probability that at least 30 of the prizes are allocated to girls. Does it affect your calculation whether or not the same child may win more than one prize? Justify your answer. **[6]**

**6** The number of fruit pips in 1 cubic centimetre of raspberry jam has the distribution Po($\lambda$). Under a traditional jam-making process it is known that $\lambda$ = 6.3. A new process is introduced and a random sample of 1 cubic centimetre of jam produced by the new process is found to contain 2 pips. Test, at the 5% significance level, whether this is evidence that under the new process the average number of pips has been reduced. **[8]**

**7** **(i)** The continuous random variable $X$ has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{2\sqrt{x}} & 1 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find **(a)** $E(X)$, **[3]**

**(b)** the median of $X$. **[3]**

**(ii)** The continuous random variable $Y$ has the probability density function

$$g(y) = \begin{cases} \dfrac{1.5}{y^{2.5}} & y \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given that $E(Y) = 3$, show that $\text{Var}(Y)$ is not finite. **[3]**

**8** In a certain fluid, bacteria are distributed randomly and occur at a constant average rate of 2.5 in every 10 ml of the fluid.

**(i)** State a further condition needed for the number of bacteria in a fixed volume of the fluid to be well modelled by a Poisson distribution, explaining what your answer means. **[2]**

Assume now that a Poisson model is appropriate.

**(ii)** Find the probability that in 10 ml there are at least 5 bacteria. **[2]**

**(iii)** Find the probability that in 3.7 ml there are exactly 2 bacteria. **[3]**

**(iv)** Use a suitable approximation to find the probability that in 1000 ml there are fewer than 240 bacteria, justifying your approximation. **[7]**

**9** It is desired to test whether the average amount of sleep obtained by school pupils in Year 11 is 8 hours, based on a random sample of size 64. The population standard deviation is 0.87 hours and the sample mean is denoted by $\bar{H}$. The critical values for the test are $\bar{H} = 7.72$ and $\bar{H} = 8.28$.

**(i)** State appropriate hypotheses for the test, explaining the meaning of any symbol you use. **[3]**

**(ii)** Calculate the significance level of the test. **[4]**

**(iii)** Explain what is meant by a Type I error in this context. **[1]**

**(iv)** Given that in fact the average amount of sleep obtained by all pupils in Year 11 is 7.9 hours, find the probability that the test results in a Type II error. **[3]**

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.**