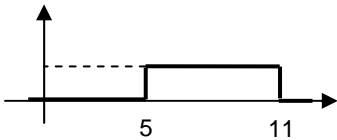


4733 Probability & Statistics 2

Penalise over-specified answers (> 6 SF) first time but only once per paper.

Use **A** or **C** to annotate “over-assertive” or “no context” respectively

1	$\hat{\mu} = \bar{x} = 15.16$ $\hat{\sigma}^2 = \frac{5}{4}s^2$ <p style="text-align: center;">= 1.363</p>	B1 M1 M1 A1	15.16 or 15.2 as answer only Use $\frac{\sum x^2}{5} - \bar{x}^2$ [=1.0904] Multiply by 5/4, or equiv for single formula Final answer 1.36 or 1.363 only, <i>not isw</i>
2 (i)	Not all equally likely – those in range 0 to 199 more likely to be chosen	M1 A1	Not all equally likely stated or implied Justified by reference to numbers, no spurious reasons
(ii)	Ignore random numbers greater than 799, or 399	B1	1 Any valid resolution of this problem, no spurious reasons
3	$B(60, 0.35) \approx N(21, 13.65)$ $\Phi\left(\frac{18.5 - 21}{\sqrt{13.65}}\right) = \Phi(-0.6767)$ $= 1 - 0.7507$ <p style="text-align: center;">= 0.2493</p>	M1 M1 A1 M1 A1 A1	6 B(60, 0.35) stated or implied N(21, ...) Variance or SD = 13.65 Standardise, their np and \sqrt{npq} or npq , wrong or no cc Both \sqrt{npq} and cc correct Answer, a.r.t. 0.249
4	$H_0 : \mu = 60; H_1 : \mu < 60$ $(\alpha) \quad z = \frac{58.9 - 60}{\sqrt{5^2 / 80}} = -1.967$ < -1.645 <p style="text-align: center;"><i>or:</i></p> $(\beta)_c = 60 - 1.645 \times \frac{5}{\sqrt{80}} = 59.08$ $58.9 < 59.08$ <p>Reject H_0</p> <p>Significant evidence that people underestimate time</p>	B2 M1 A1 B1 M1 B1 A1✓ M1 A1✓	7 Both correct, B2 B1 for one error, but not x , t , \bar{x} or \bar{t} Standardise 58.9 & $\sqrt{80}$, allow – or $\sqrt{}$ errors z , art –1.97 or p in range [0.024, 0.025] Explicit comparison with –1.645 or 0.05, or +1.645 or 0.95 if 1.967 or 0.976 used 60 – $z \times 5 / \sqrt{80}$, any $z = \Phi^{-1}$, allow $\sqrt{}$ errors or \pm , not just +; $z = 1.645$ and compare 58.9 59.1 or better, ✓ on wrong z Correct first conclusion, needs essentially correct method including $\sqrt{80}$ or 80 Contextualised, uncertainty acknowledged SR: $\mu = 58.9$: B0M1A0B1 max 2/7 SR: 2-tail: max 5/7
5 (i)	$H_0 : \lambda = 11.0$ $H_1 : \lambda > 11.0$ $(\alpha) \quad P(\geq 19) = 1 - 0.9823$ $= 0.0177$ < 0.05	B2 M1 A1 B1	Allow μ . Both correct, B2 One error: B1, but not C , x etc Find $P(\geq 19)$ [or $P(< 19)$ if later 0.95] art 0.0177 [0.9823, ditto] Compare 0.05 [0.95 if consistent], needs M1
	$(\beta) \quad CR \geq 18,$ $P(\geq 18) = 0.0322$ $19 > 18$	M1 A1 B1	CR or CV 16/17/18/19 stated or clearly implied, but not < 18 and 0.0322 both seen, allow 0.9678 Explicit comparison with 19, needs M1
	Reject H_0 Significant evidence of an increase in number of customers	M1 A1✓	7 Needs essentially correct method & comparison Contextualised, uncertainty acknowledged SR: Normal, or $P(= 19)$ or $P(\leq 19)$ or $P(> 19)$: First B2 only.
(ii)	Can't deduce cause-and-effect, or there may be other factors	B1	1 Conclusion needed. No spurious reasons. If “DNR” in (i), “couldn't deduce even if...”

6	(i)	(a) Probabilities don't total 1	B1	1	Equivalent statement	
		(b) $P(> 70)$ must be $< P(> 50)$	B1	1	Equivalent statement	
		(c) $P(> 50) = 0.3 \Rightarrow \mu < 50$ $P(< 70) = 0.3 \Rightarrow \mu > 70$	B1	1	Any relevant valid statement, e.g. " $P(< 50) = 0.7$ but $P(< 50)$ must be $< P(< 70)$ "	
	(ii)	$\mu = 60$ by symmetry $\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$ $\sigma = 10/0.5243$ = 19.084	B1 M1 B1 A1	 4	$\mu = 60$ obtained at any point, allow from Φ One standardisation, equate to Φ^{-1} , not 0.758 $\Phi^{-1} \in [0.524, 0.5245]$ seen σ in range [19.07, 19.1], e.g. 19.073	
7	(i)		M1 A1	 2	Horizontal line Evidence of truncation <i>[no need for labels]</i>	
		(ii)	$\mu = 8$ $\int_5^{11} \frac{1}{6} t^2 dt = \left[\frac{1}{18} t^3 \right]_5^{11} \quad [= 67]$ $- 8^2$ = 3	B1 M1 B1 M1 A1	 5	8 only, cwd Attempt $\int kt^2 dt$, limits 5 and 11 seen $k = 1/6$ stated or implied Subtract their (non-zero) mean ² Answer 3 only, <i>not</i> from MF1
		(iii)	$N(8, 3/48)$ $1 - \Phi\left(\frac{8.3 - 8}{\sqrt{3/48}}\right) = 1 - \Phi(1.2)$ $= 1 - 0.8848$ = 0.1151 Normal distribution only approx.	M1 A1 A1 M1 A1 B1	 6	Normal stated or implied Mean 8 Variance their (non-zero) (ii)/48 Standardise, \sqrt{n} , ignore sign or $\sqrt{\text{errors}}$. cc: M0 Answer, art 0.115 Any equivalent comment, e.g. CLT used
8	(i)	$P(\leq 4) = 0.0473$ Therefore CR is ≤ 4 $P(\text{Type I error}) =$ 4.73%	M1 B1 A1	 3	$P(\leq r)$ from $B(10, 0.7)$, $r = 3/4/5$, <i>not</i> N " ≤ 4 " stated, not just "4", nothing else Answer, art 0.0473 or 4.73%, must be stated	
		(ii)	$B(10, 0.4)$ and find $P(> 4)$ $1 - P(\leq 4)$ = 0.3669	M1 M1 A1	 3	Must be this, <i>not</i> isw, ✓ on (i) Allow for 0.6177 or 0.1622 Answer, art 0.367
		(iii)	0.5×0.3669 = 0.18345	M1 A1 ✓	 2	$0.5 \times$ (ii) Ans correct to 3 SF, e.g. 0.184 from 0.367

9	(i)	$1 - P(\leq 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	2	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119													
	(ii)	Po(12) $P(\leq 14) - P(\leq 12)$ $[0.7720 - 0.5760]$ $= \mathbf{0.196}$	M1 M1 A1	3	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196													
	(iii)	Po(60) \approx N(60, 60) $\Phi\left(\frac{69.5 - 60}{\sqrt{60}}\right) = \Phi(1.226)$ $= \mathbf{0.8899}$	M1 A1 M1 A1 A1	5	N(60, ...) Variance or SD 60 Standardise, λ & $\sqrt{\lambda}$, allow λ or wrong or no cc $\sqrt{\lambda}$ and cc both correct Answer 0.89 or a.r.t. 0.890													
	(iv)	(a)	$1 - e^{-3m}(1 + 3m)$	M1 A1	2	M1 for one error, e.g. no "1 -", or extra term, or 0 th term missing; answer, aesf												
		(b)	$m = 1.29,$ $p = 0.89842$ $m = 1.3, \quad p = 0.9008$ Straddles 0.9, therefore solution between 1.29 and 1.3	M1 A1 A1 A1	4	Substitute 1.29 or 1.3 into appropriate fn <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Comp</th> <th>0.9</th> <th>0.1</th> <th>0</th> </tr> </thead> <tbody> <tr> <td>1.29</td> <td>0.898</td> <td>0.10158</td> <td>-.00158</td> </tr> <tr> <td>1.3</td> <td>0.901</td> <td>0.09918</td> <td>.0008146</td> </tr> </tbody> </table> Explicit comparison with relevant value, & conclusion, needs both <i>ps</i> correct	Comp	0.9	0.1	0	1.29	0.898	0.10158	-.00158	1.3	0.901	0.09918	.0008146
	Comp	0.9	0.1	0														
1.29	0.898	0.10158	-.00158															
1.3	0.901	0.09918	.0008146															
	or	Method for iteration; 1.296... 1.2965 or better; conclusion stated	M1A1 A1A1		Can be implied by at least 1.296... Need at least 4 dp for M1A2													