OCR Maths S2

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1	(i) (a) Po(2): $1 - P(\leq 3)$ $= 0.1429$	M1 A1	**2**	Po(2) tables, "1 – " used Answer, a.r.t. 0.143
	(b) Po(2/3): $e^{-2/3}\frac{(\frac{2}{3})^2}{2!}$ $= 0.114$	M1 M1 A1	**3**	Parameter 2/3 Poisson formula correct, $r = 2$, any μ Answer, a.r.t. 0.114
	(ii) Foxes may congregate so not independent	B1 B1	**2**	Independent/not constant rate/singly used Any valid relevant application in context
2	N(80/7, 400/49) $\frac{13.5 - \frac{80}{7}}{\frac{20}{7}}$ $= 0.725$ $1 - \Phi(0.725)$ $= 0.2343$	B1 B1 M1 A1 A1 M1 A1	**7**	80/7, a.e.f (11.43) 400/49 or 20/7 seen, a.e.f. (8.163 or 2.857) Standardise with np & npq or $\sqrt{npq}$ or nq, no $\sqrt{n}$ $\sqrt{npq}$ correct 13.5 correct Normal tables used, answer < 0.5 Answer, a.r.t. 0.234 [SR: Binomial, complete expression M1, 0.231 A1 Po(80/7) B1, complete expression M1, 0.260 A1 Normal approx to Poisson, B1B0 M1A0A1 M1A0]
3	H_0: $p = 0.3$ H_1: $p \neq 0.3$ B(8, 0.3) $P(\leq 4) = 0.9420$; $P(> 4) = 0.0580$ $P(\leq 5) = 0.9887$; $P(> 5) = 0.0113$ Compare 0.025 or critical value 6 Do not reject H_0 Insufficient evidence that manufacturer's claim is wrong	B1 B1 M1 A1 M1 M1 A1√	**7**	NH stated, must be this form (or π) AH stated, must be this form (or π) [μ: B1 both] B(8, 0.3) stated or implied Any one of these four probabilities seen *Either* compare $P(\geq 5)$ & 0.025 / $P(\leq 4)$ & 0.975 *Or* critical region ≥ 6 with 5 H_0 not rejected, can be implied, needs essentially correct method Correct conclusion in context [SR: Normal, Poisson: can get B2M1A0M0M1A1 $P(\leq 5)$: first 4 marks. $P(= 5)$: first 3 marks only.]
4	(i) B(80, 0.02) approx Po(1.6) $1 - P(\leq 1) = 1 - 0.5249$ $= 0.4751$	M1 M1 M1 A1	**4**	B(80, 0.02) seen or implied, e.g. N(1.6, 1.568) Po(np) used $1 - P(\leq 1)$ used Answer, a.r.t. 0.475 [SR: Exact: M1 M0 M0, 0.477 A1]
	(ii) $P(\leq 4) = 0.9763$, $P(\geq 5) = 0.0237$ $P(\leq 5) = 0.9940$, $P(\geq 6) = 0.0060$ Therefore least value is 6	M1 A1 A1	**3**	Evidence for correct method, e.g. answer 6 At least one of these probabilities seen Answer 6 only [SR N(1.6,1.568): $2.326 = (r - 1.6)/\sqrt{1.568}$ M1 $r = 5$ or (with cc) 6 A1 Exact: M1 A0 A1]

5	(i) $\frac{0-\mu}{\mu/2} = -2$, independent of μ $1 - \Phi(2) = 1 - 0.9772 =$ 0.0228	M1 A1 A1 A1 **4**	Standardise, allow –, allow $\mu^2/4$ $z = 2$ or -2 z-value independent of μ and any relevant statement Answer, a.r.t. 0.023
	(ii) $\Phi[(9-6)/3]$ $\Phi(1.0) = 0.8413$ $[\Phi(1.0)]^3$ $= 0.59546$	M1 A1 M1 A1 **4**	Standardise and use Φ *[no √n]* 0.8413 [not 0.1587] Cube previous answer Answer, in range [0.595, 0.596]
	(iii) Annual increases not independent	B1 **1**	Independence mentioned, in context. Allow "one year affects the next" but not "years not random"
6	H_0: $\mu = 32$; H_1: $\mu > 32$, where μ is population mean waist measurement $\overline{W} = 32.3$ $s^2 = 52214.50/50 - \overline{W}^2$ [= 1] $\hat{\sigma}^2 = 50/49 \times s^2$ [= 50/49 or 1.0204]	B1 B1 B1 M1 M1	One hypothesis correctly stated, *not x* or $\bar{x}$ or $\overline{w}$ Both completely correct, μ used Sample mean 32.3 seen Correct formula for s^2 used Multiply by 50/49 or √
	α: $z = (32.3 - 32) \times \sqrt{49}$ $= 2.1$ Compare 2.1 with 3.09 or 0.0179 with 0.001	M1 A1 B1	Correct formula for z, can use s, aef, need $\mu = 32$ $z = 2.1$ or $1 - \Phi(z) = 0.0179$, *not* –2.1 Explicitly compare their 2.1 with 3.09(0) or their 0.0179 with 0.001
	β: CV $= 32 + 3.09 \div \sqrt{49}$ $= 32.44$ Compare CV with 32.3	M1 B1 A1√	$32 + z \times \sigma/\sqrt{n}$ [allow ±, s, any z] $z = 3.09$ and (later) compare $\bar{x}$ CV in range [32.4, 32.5], √ on k
	Do not reject H_0 Insufficient evidence that waists are actually larger	M1√ A1√ **10**	Correct conclusion, can be implied, needs essentially correct method including $\sqrt{n}$, any reasonable σ, but not from $\mu = 32.3$ Interpreted in context
7	(i) $\frac{80-c}{8/\sqrt{12}} = 2.326$ $c = 74.63$	M1 A1 1 **4** B1 A1	Equate standardised variable to Φ^{-1}, allow $-\sqrt{12}$, 8 correct 2.326 or a.r.t 2.33 seen, signs must be correct Answer, a.r.t. 74.6, cwo, allow ≤ or ≥
	(ii) (a) Type I error (b) Correct	B **1** 1√ **1** B 1√	"Type I error" stated, needs evidence "Correct" stated or clearly implied Wrong c: $74 < c < 75$, B1√ B1√ $c < 74$, both "correct", B1. $75 < c < 80$, both "Type I", B1 Also allow if only one is answered
	(iii) $\frac{74.63-\mu}{8/\sqrt{12}} = -1.555$ Solve for μ $\mu = 78.22$	M1*dep A1√ dep* M1 A1 **4**	$\frac{c-\mu}{8/\sqrt{12}} = (\pm)\Phi^{-1}$, allow no √12 but not 80, not 0.8264 Correct including sign, √ on their c Solve to find μ, dep, answer consistent with signs Answer, a.r.t. 78.2

8	(i)	$\int_0^1 x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$	M1	Integrate x^n, limits 0 and 1
		$k/(n+1) = 1$ so $k = n+1$	M1 A1 **3**	Equate to 1 and solve for k Answer $n+1$, *not* 1^{n+1}, c.w.o.
	(ii)	$\int_0^1 x^{n+1} dx = \left[\frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+2}$	M1 A1	Integrate x^{n+1}, limits 0 and 1, not just $x.x^n$ Answer $\frac{1}{n+2}$
		$\mu = \frac{k}{n+2} = \frac{n+1}{n+2}$ **AG**	A1 **3**	Correctly obtain given answer
	(iii)	$\int_0^1 x^5 dx = \left[\frac{x^6}{6}\right]_0^1$ $[= \frac{1}{6}]$	M1 M1	Integrate x^5, limits 0 and 1, allow with n Subtract $\left(\frac{4}{5}\right)^2$
		$\sigma^2 = \frac{4}{6} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$	A1 **3**	Answer $\frac{2}{75}$ or a.r.t. 0.027
	(iv)	$N\left(\frac{4}{5}, \frac{2}{7500}\right)$	B1 B1 B1√ **3**	Normal stated Mean $\frac{4}{5}$ or $\frac{n+1}{n+2}$ Variance their (iii)/100, a.e.f., allow √
	(v)	Same distribution, translated	M1	Can be negative translation; *or* integration, must include correct method for integral
		Mean 0	A1√	(Their mean) – $\frac{4}{5}$, c.w.d.
		Variance $\frac{2}{75}$	B1√ **3**	Variance same as their (iii), or $\frac{2}{75}$ by integration

1		$\mu = \frac{3}{37}\int_3^4 x^3 dx = \frac{3}{37}\left[\frac{x^4}{4}\right]_3^4$ [= $3\frac{81}{148}$]	M1	Integrate $xf(x)$, limits 3 & 4 *[can be implied]*
				[$\frac{525}{148}$ or 3.547]
		$\frac{3}{37}\int_3^4 x^4 dx = \frac{3}{37}\left[\frac{x^5}{5}\right]_3^4$	M1	Attempt to integrate $x^2f(x)$, limits 3 & 4
			A1	Correct indefinite integral, any form
		= $12\frac{123}{185}$ or 12.665	A1	$\frac{2343}{185}$ or in range [12.6, 12.7] *[can be implied]*
		$\sigma^2 = 12\frac{123}{185} - 3\frac{81}{148}^2$ = **0.0815**	M1	Subtract their μ^2
			A1 **6**	Answer, in range [0.0575, 0.084]
2	(i)	Find P($R \geq 6$) or P($R < 6$)	M1	Find P(= 6) from tables/calc, OR RH critical region
		= 0.0083 or 0.9917	A1	P(≥ 6) in range [0.008, 0.0083] or P(< 6) = 0.9917
		Compare with 0.025 [can be from N]	B1	OR CR is 6 with probability 0.0083/0.9917
		[0.05 if "empty LH tail stated]	**4**	Explicitly compare with 0.025 [or 0.975 if consistent]
		Reject H_0	A1√	OR state that result is in critical region
				Correct comparison and conclusion, √ on their *p*
	(ii)	*n* = 9, P(≤ 1) = 0.0385 [> 0.025]	M1	At least one, or *n* = 8, P(≤ 1) = 0.0632
		n = 10, P(≤ 1) = 0.0233 [< 0.025]	A1	Both of these probabilities seen, don't need 0.025
		Therefore *n* = **9**	B1 **3**	Answer *n* = 9 only, indep't of M1A1, *not* from P(= 1)
3	(i)	($140 - \mu$)/σ = –2.326	M1	One standardisation equated to Φ^{-1}, allow "1–", σ^2
		($300 - \mu$)/σ = 0.842	B1	Both 2.33 and 0.84 at least, ignore signs
			A1√	Both equations completely correct, √ on their *z*
		Solve to obtain:	M1	Solve two simultaneous equations to find one variable
		μ = **257.49**	A1	μ value, in range [257, 258]
		σ = **50.51**	A1 **6**	σ in range [50.4, 50.55]
	(ii)	Higher	B1	"Higher" or equivalent stated
		as there is positive skew	B1 **2**	Plausible reason, allow from normal calculations
4	(i)	Each element equally likely to be selected (and all selections independent) OR each possible sample equally likely	B1 **1**	One of these two. "Selections independent" alone is insufficient, but don't need this. An example is insufficient.
	(ii)	B(6, 5/8)	M1	B(6, 5/8) stated or implied, allow e.g. 499/799
		${}^6C_4\, p^4\,(1-p)^2$	M1	Correct formula, any *p*
		= **0.32187**	A1√ **3**	Answer, a.r.t. 0.322, can allow from wrong *p*
	(iii)	N(37.5, 225/16)	B1	Normal, mean 37.5, or 37.47 from 499/799, 499/800
			B1	14.0625 or 3.75 seen, allow 14.07/14.1 or 3.75
		$\frac{39.5-37.5}{3.75}$ = 0.5333	M1 dep	Standardise, wrong or no cc, *np*, *npq*, no √*n*
			A1	Correct cc, √*npq*, signs can be reversed
		1 – Φ(0.5333)	dep M1	Tables used, answer < 0.5, *p* = 5/8
		= **0.297**	A1	Answer, a.r.t. 0.297
			6	SR: *np* < 5: Po(*np*) stated or implied, B1

5	(i)	B(303, 0.01) ≈ Po(3.03)	B1 B1 **2**	B(303, 0.01) stated, allow p = 0.99 or 0.1 Allow Bin implied clearly by parameters Po(3.03) stated or implied, can be recovered from (ii)
	(ii)	$e^{-3.03}(1+3.03+\frac{3.03^2}{2})$ = 0.4165 **AG**	M1 A1 **2**	Correct formula, ± 1 term or "1 – " or both Convincingly obtain 0.4165(02542) [Exact: 0.41535]
	(iii)	302 seats ⇒ μ = 3.02 $e^{-3.02}(1+3.02)$ = 0.1962 0.196 < 0.2 So **302** seats.	M1 M1 A1 A1 A1 **5**	Try smaller value of μ Formula, at least one correct term Correct number of terms for their μ 0.1962 [or 0.1947 from exact] Answer 302 only
	SR:	B(303, 0.99): B1B0; M0; M1 then N(298.98,2.9898) or equiv, standardise: M1A1 total 4/9		
	SR:	p = 0.1: B(303, 0.1), N(30.3, 27.27) B1B0; Standardise 2 with np & $\sqrt{npq}$, M1A0; N(0.1n, 0.09n); standardise with np & $\sqrt{npq}$; solve quadratic for $\sqrt{n}$; n = 339: M1M1M1A1, total 6/9		
	SR:	B(303, 0.01) ≈ N(3.03, 2.9997): B1B0; M0A0; M1A0		
6	(i)	Customers arrive independently	B1 **1**	Valid reason in context, allow "random"
	(ii)	1 – 0.9921 = **0.0079**	M1 A1 **2**	Poisson tables, "1 –", or correct formula ± 1 term Answer, a.r.t. 0.008 [1 – 0.9384 = 0.0606: M1A0]
	(iii)	N(48, 48) $z = \frac{55.5-48}{\sqrt{48}}$ = 1.0825 1 – Φ(1.0825) = **0.1394**	B1 B1√ M1 dep A1 dep M1 A1 **6**	Normal, mean 48 Variance or SD same as mean√ Standardise, wrong or no cc, μ = λ Correct cc, √λ Use tables, answer < 0.5 Answer in range [0.139, 0.14]
	(iv)	$e^{-\lambda}$ < 0.02 λ > –ln 0.02 = 3.912 0.4t = 3.912: t = 9.78 minutes t = 9 minutes 47 seconds	M1 M1 A1 M1 A1 **5**	Correct formula for P(0), OR P(0 | λ = 4) at least ln used OR λ = 3.9 at least by T & I 3.91(2) seen OR λ = 3.91 at least by T & I Divide λ by 0.4 or multiply by 150, any distribution 587 seconds ± 1 sec *[inequalities not needed]*

7	(i)	$\frac{c - 4000}{60 / \sqrt{50}} = 1.645$ Solve $c = 4014$ [4013.958] Critical region is **> 4014**	M1 B1 A1√ M1 A1 A1√ **6**	Standardise unknown with √50 or 50 *[ignore RHS]* $z = 1.645$ or -1.645 seen Wholly correct eqn, √ on their z [1 – 1.645: M1B1A0] Solve to find c Value of c, a.r.t. 4014 Answer "> 4014", allow ≥, √ on their c, needs M1M1
	(ii)	Use "Type II is: accept when H_0 false" $\frac{4020 - 4014}{60 / \sqrt{50}}$ = 0.7071 [0.712 from 4013.958] $1 - \Phi(0.7071)$ = **0.240** [0.238 from 4013.958]	M1dep depM1 A1√ A1 M1 A1 **6**	Standardise 4020 and 4014√ , allow 60^2, cc With √50 or 50 Completely correct LHS, √ on their c z-value in range [0.707, 0.712] Normal tables, answer < 0.5 Answer in range [0.2375, 0.2405]
	(iii)	Smaller Smaller cv, better test etc	B1 B1 **2**	"Smaller" stated, no invalidating reason Plausible reason
	(iv)	Smaller Smaller cv, larger prob of Type I etc	B1 B1 **2**	"Smaller" stated, no invalidating reason Plausible reason
	(v)	No, parent distribution known to be normal	B2 **2**	"No" stated, convincing reason SR: If B0, "No", reason that is not invalidating: B1

For over-specified answers (> 6SF where inappropriate) deduct 1 mark, no more than once in paper.

1	$\frac{22-\mu}{5} = -\Phi^{-1}(0.242)$ $= -0.7$ $\mu =$ **25.5**	M1 A1 B1 A1	**4**	Standardise with Φ^{-1}, allow +, "1 –" errors, cc, $\sqrt{5}$ or 5^2 Correct equation including signs, no cc, can be wrong Φ^{-1} 0.7 correct to 3 SF, can be + Answer 25.5 correct to 3 SF
2	(i) 900 ÷ 12 = **75**	B1	**1**	75 only
	(ii) (a) True, first choice is random	B1	**1**	True stated with reason based on first choice
	(b) False, chosen by pattern	B1	**1**	False stated, with any non-invalidating reason
	(iii) Not equally likely e.g. P(1) = 0, or triangular	M1 A1	**2**	"Not equally likely", or "Biased" stated Non-invalidating reason
3	Let R be the number of 1s $R \sim$ B(90, 1/6) $\approx$ N(15, 12.5) $\frac{13.5-15}{\sqrt{12.5}}$ [= –0.424] **0.6643**	B1 B1 B1 M1 A1 A1	**6**	B(90, 1/6) stated or implied, e.g. Po(15) Normal, $\mu = 15$ stated or implied 12.5 or $\sqrt{12.5}$ or 12.5^2 seen Standardise, np and npq, allow errors in √ or cc or both √ and cc both right Final answer, a.r.t. 0.664. [Po(15): 1/6]
4	(i) $\overline{w} = 100.8 \div 14 = 7.2$ $\frac{938.70}{14} - \overline{w}^2$ [= 15.21] × 14/13 = **16.38**	B1 M1 M1 A1	**4**	7.2 seen or implied Use Σw^2 – their $\overline{w}^2$ Multiply by $n/(n-1)$ Answer, a.r.t. 16.4
	(ii) N(7.2, 16.38 ÷ 70) [= N(7.2, 0.234)]	B1 B1√ B1√	**3**	Normal stated Mean their $\overline{w}$ √ Variance [their (i)√ ÷ 70], allow arithmetic slip
5	(i) $\lambda = 1.2$ Tables or formula used **0.6626**	B1 M1 A1	**3**	Mean 1.2 stated or implied Tables or formula [allow ± 1 term, or "1 –"] correctly used Answer in range [0.662, 0.663] [.3012, .6990, .6268 or .8795: B1M1A0]
	(ii) B(20, 0.6626√) $^{20}C_{13}\ 0.6626^{13} \times 0.3374^{7}$ **0.183**	M1 M1 A1	**3**	B(20, p), p from (i), stated or implied Correct formula for their p Answer, a.r.t. 0.183
	(iii) Let S be the number of stars $S \sim$ Po(24) $\approx$ N(24, 24) $\frac{29.5-24}{\sqrt{24}}$ [= 1.1227] **0.8692**	B1 B1 B1√ M1 A1 A1	**6**	Po(24) stated or implied Normal, mean 24 Variance 24 or 24^2 or √24, √ if 24 wrong Standardise with λ, λ, allow errors in cc or √ or both √λ and cc both correct Answer, in range [0.868, 0.8694]

6	(i)	$\left[ax+\frac{bx^2}{2}\right]_0^2=1$	M1	Use total area = 1
			B1	Correct indefinite integral, or convincing area method
		$2a + 2b = 1$ **AG**	A1 **3**	Given answer correctly obtained, "1" appearing before last line [if + *c*, must see it eliminated]
	(ii)	$\left[\frac{ax^2}{2}+\frac{bx^3}{3}\right]_0^2=\frac{11}{9}$	M1	Use $\int xf(x)dx = 11/9$, limits 0, 2
			B1	Correct indefinite integral
		$2a+\frac{8b}{3}=\frac{11}{9}$	A1	Correct equation obtained, a.e.f.
		Solve simultaneously	M1	Obtain one unknown by correct simultaneous method
			A1	*a* correct, 1/6 or a.r.t 0.167
		$a = \frac{1}{6}$, $b = \frac{1}{3}$	A1 **6**	*b* correct, 1/3 or a.r.t. 0.333
	(iii)	e.g. P(< 11/9) = 0.453, *or*	M1	Use P(x < 11/9), *or* integrate to find median *m*
		$\left[ax+\frac{bx^2}{2}\right]_0^m=0.5,\ m=1.303 \text{ or } \frac{\sqrt{13}-1}{2}$	M1	Substitute into $\int f(x)dx$, √ on *a*, *b*, limits 0 and 11/9 or *m* [if finding *m*, need to solve 3-term quadratic]
		Hence median > mean	A1	Correct numerical answer for probability or *m*
			A1√ **4**	Correct conclusion, cwo ["Negative skew", M2; median > mean, A2]
7	(i)	$H_0: p = 0.35$ [*or* $p \geq 0.35$]	B1	Each hypothesis correct, B1+B1, allow $p \geq .35$ if .35 used
		$H_1: p < 0.35$	B1	[Wrong or no symbol, B1, but *r* or *x* or $\bar{x}$: B0]
		B(14, 0.35)	M1	Correct distribution stated or implied, can be implied by N(4.9, …), but *not* Po(4.9)
	α:	P(≤ 2) = 0.0839 > 0.025	A1	0.0839 seen, *or* P(≤ 1) = 0.0205 if clearly using CR
	β:	CR ≤ 1, probability 0.0205	B1	Compare binomial tail with 0.025, *or* *R* = 2 binomial CR
		Do not reject H_0. Insufficient evidence that proportion that can receive Channel C is less than 35%	M1	Do not reject H_0, √ on their probability, *not* from N or Po or P(< 2); Contextualised conclusion √
			A1√ **7**	
	(ii)	B(8, 0.35): P(0) = 0.0319	M1	Attempt to find P(0) from B(*n*, 0.35)
		B(9, 0.35): P(0) = 0.0207	A1	One correct probability [P(≤ 2) = .0236, *n* = 18: M1A1]
			A1	Both probabilities correct
		Hence largest value of *n* is **8**	A1 **4**	Answer 8 or ≤ 8 only, needs minimum M1A1
	or	$0.65^n > 0.025$; $n \ln 0.65 > \ln 0.025$	M1M1	$p^n > 0.025$, any relevant *p*; take ln, *or* T&I to get 1 SF
		8.56; largest value of *n* = 8	A1A1	In range [8.5, 8.6]; answer 8 or ≤ 8 only
8	(i) α:	$\frac{100.7-102}{5.6/\sqrt{80}}=-2.076$	M1	Standardise 100.7 with √80 or 80
			A1	a.r.t. –2.08 obtained, must be –, *not* from μ = 100.7
		Compare with –2.576	B1 **3**	–2.576 or –2.58 seen and compare *z*, allow both +
	or β:	Φ(–2.076) = 0.0189 [or Φ(2.076) = 0.981]	M1	Standardise 100.7 with √80 or 80
			A1	a.r.t. 0.019, allow 0.981 only if compared with 0.995
		and compare with 0.005 [or 0.995]	B1 (3)	Compare correct tail with 0.005 or 0.995
	or γ:	$102-\frac{k\times5.6}{\sqrt{80}}$	M1	This formula, allow +, 80, wrong SD, any *k* from Φ^{-1}
		k = 2.576, compare 100.7	B1	*k* = 2.576/2.58, – sign, and compare 100.7 with CV
		100.39	A1 (3)	CV a.r.t. 100.4
		Do not reject H_0	M1	Reject/Do not reject, √, needs normal, 80 or √80, Φ^{-1} or equivalent, correct comparison, *not* if clearly μ = 100.7
		Insufficient evidence that quantity of SiO_2 is less than 102	A1 **2**	Correct contextualised conclusion
	(ii) (a)	$\frac{c-102}{5.6/\sqrt{n}}=-2.326$	M1	One equation for *c* and *n*, equated to Φ^{-1}, allow cc, wrong sign, σ^2; 2.326 or 2.33
		$102-c=\frac{13.0256}{\sqrt{n}}$ **AG**	B1	
			A1 **3**	Correctly obtain given equation, needs in principle to have started from *c* – 102, –2.326
	(b)	$\frac{c-100}{5.6/\sqrt{n}}=1.645$ or $c-100=\frac{9.212}{\sqrt{n}}$	M1	Second equation, as before
			A1 **2**	Completely correct, aef
	(c)	Solve simultaneous equations	M1	Correct method for simultaneous equations, find *c* or √*n*
		$\sqrt{n}$ =11.12	A1	√*n* correct to 3 SF
		n_{min} = **124**	A1	n_{min} = 124 only
		c = **100.83**	A1 **4**	Critical value correct, 100.8 or better

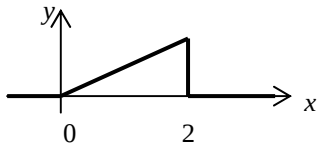
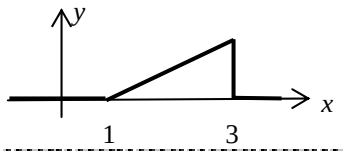
1	(i)	$\hat{\mu}$ = 4830.0/100 = 48.3 249509.16/100 – (their $\bar{x}^2$) × 100/99 = 163.84	B1 M1 M1 A1	**4**	48.3 seen Biased estimate: 162.2016: can get B1M1M0 Multiply by $n/(n-1)$ Answer, 164 or 163.8 or 163.84
	(ii)	No, Central Limit theorem applies, so can assume distribution is normal	B2	**2**	"No" with statement showing CLT is understood (though CLT does not need to be mentioned) [SR: No with reason that is not wrong: B1]
2		B(130, 1/40) ≈ Po(3.25) $e^{-\lambda}\frac{\lambda^3}{4!}$ = 0.180	B1 M1 A1√ M1 A1	**5**	B(130, 1/40) stated or implied Poisson, *or* correct N on their B(n, p) Parameter their np, *or* correct parameter(s)√ Correct formula, or interpolation Answer, 0.18 or a.r.t. 0.180 [SR: N(3.25, 3.17) or N(3.25, 3.25): B1M1A1]
3	(i)	Binomial	B1	**1**	Binomial stated or implied
	(ii)	Each element equally likely Choices independent	B1 B1	**2**	All elements, or selections, equally likely stated Choices independent [not just "independent"] [can get B2 even if (i) is wrong]
4	(i)	*Two of:* Distribution symmetric No substantial truncation Unimodal/Increasingly unlikely further from μ, etc	B1 B1	**2**	One property Another definitely different property Don't give both marks for just these two "Bell-shaped": B1 only unless "no truncation"
	(ii)	Variance $8^2/20$ $z = \frac{47.0 - 50.0}{\sqrt{8^2/20}} = -1.677$ Φ(1.677) = 0.9532	M1 A1 A1 A1	**4**	Standardise, allow cc, don't need n Denominator (8 or 8^2 or √8) ÷ (20 or √20 or 20^2) z-value, a.r.t. –1.68 or +1.68 Answer, a.r.t. 0.953
5	(i)	H_1: $\lambda > 2.5$ or 15	B1	**1**	$\lambda > 2.5$ or 15, allow μ, don't need "H_1"
	(ii)	Use parameter 15 P(> 23) 1 – 0.9805 = 0.0195 or 1.95%	M1 M1 A1	**3**	λ = 15 used [N(15, 15) gets this mark only] Find P(> 23 or ≥ 23), final answer < 0.5 eg 0.0327 or 0.0122 Answer, 1.95% or 2% or 0.0195 or 0.02 [SR: 2-tailed, 3.9% gets 3/3 here]
	(iii)	P(≤ 23 | λ = 17) = 0.9367 P(≤ 23 | λ = 18) = 0.8989 Parameter = 17 λ = 17/6 or 2.83	M1 A1 M1	**3**	One of these, or their complement: .9367, .8989, 0.9047, 0.8551, .9317, .8933, .9907, .9805 Parameter 17 *[17.1076]*, needs P(≤ 23), cwo [SR: if insufficient evidence can give B1 for 17] Their parameter ÷ 6 *[2.85]* [SR: Solve $(23.5 - \lambda)/\sqrt{\lambda} = 1.282$ M1; 18.05 A0]
6	(i)	H_0: $p = 0.19$, H_1: $p < 0.19$ where p is population proportion $0.81^{20} + 20 \times 0.81^{19} \times 0.19$ = 0.0841 Compare 0.1	B2 M1 A1 A1 B1		Correct, B2. One error, B1, but x or $\bar{x}$ or r: B0 Binomial probabilities, allow 1 term only Correct expression [0.0148 + 0.0693] Probability, a.r.t. 0.084 Explicit comparison of "like with like"
	or	Add binomial probs until ans > 0.1 Critical region ≤ 1	A1 B1		[P(≤ 2) = 0.239]
		Reject H_0 Significant evidence that proportion of *e*'s in language is less than 0.19	M1 A1√	**8**	Correct deduction and method [needs P(≤ 1)] Correct conclusion in context [SR: N(3.8, 3.078): B2M1A0B1M0]
	(ii)	Letters not independent	B1	**1**	Correct modelling assumption, stated in context Allow "random", "depends on message", etc

7	(i)		B1 B1 B1 **3**	Horizontal straight line Positive parabola, symmetric about 0 Completely correct, including correct relationship between two Don't need vertical lines or horizontal lines outside range, but don't give last B1 if horizontal line continues past "±1"
	(ii)	*S* is equally likely to take any value in range, *T* is more likely at extremities	B2 **2**	Correct statement about distributions (*not* graphs) *[Partial statement, or correct description for one only: B1]*
	(iii)	$\int_t^1 \frac{3}{2}x^2 dx = \left[\frac{x^3}{2}\right]_t^1$ ½(1 – t^3) = 0.2 or ½(t^3 + 1) = 0.8 t^3 = 0.6 t = 0.8434	M1 B1 M1 M1 A1 **5**	Integrate f(x) with limits (–1, t) or (t, 1) [recoverable if t used later] Correct indefinite integral Equate to 0.2, or 0.8 if [–1, t] used Solve cubic equation to find t Answer, in range [0.843, 0.844]
8	(i)	$\frac{64.2-63}{\sqrt{12.25/23}} = 1.644$ P(z > 1.644) = 0.05	M1dep A1 dep M1 A1 **4**	Standardise 64.2 with $\sqrt{n}$ z = 1.644 or 1.645, must be + Find Φ(z), answer < 0.5 Answer, a.r.t. 0.05 or 5.0%
	(ii)	(a) $63 + 1.645 \times \frac{3.5}{\sqrt{50}}$ ≥ 63.81	M1 B1 A1 **3**	63 + 3.5 × k / √50, k from Φ^{-1}, *not* – k = 1.645 (allow 1.64, 1.65) Answer, a.r.t. 63.8, allow >, ≥, =, c.w.o.
		(b) P(< 63.8 | μ = 65) $\frac{63.8-65}{3.5/\sqrt{50}} = -2.3956$ 0.0083	M1 M1 A1 A1 **4**	Use of correct meaning of Type II Standardise their c with √50 z = (±) 2.40 [or –2.424 or – 2.404 etc] Answer, a.r.t. 0.008 [eg, 0.00767]
	(iii)	B better: Type II error smaller (and same Type I error)	B2√ **2**	This answer: B2. "B because sample bigger": B1. *[SR: Partial answer: B1]*
9	(a)	np > 5 and nq > 5 0.75n > 5 is relevant n > 20	M2 A1 **3**	Use either nq > 5 or npq > 5 [SR: If M0, use np > 5, or "n = 20" seen: M1] Final answer n > 20 or n ≥ 20 only
	(b)	(i) 70.5 – μ = 1.75σ μ – 46.5 = 2.25σ Solve simultaneously μ = 60 σ = 6	M1 A1 B1 M1 A1√ A1√ **6**	Standardise once, and equate to Φ^{-1}, ± cc Standardise twice, signs correct, cc correct Both 1.75 and 2.25 Correct solution method to get one variable μ, a.r.t. 60.0 or ± 154.5 σ, a.r.t. 6.00 [Wrong cc (below): A1 both] [SR: σ^2: M1A0B1M1A1A0]
		(ii) np = 60, npq = 36 q = 36/60 = 0.6 p = 0.4 n = 150	M1dep depM1 A1√ A1√ **4**	np = 60 and npq = 6^2 or 6 Solve to get q or p or n p = 0.4 √ on wrong cc or z n = 150 √ on wrong cc or z

		σ	μ	q	p (±0.01)	n
70.5	46.5	6	60	0.6	0.4	150
			60.062			
71	46	6.25	5	0.6504	0.3496	171.8
			60.562			
71.5	46.5	6.25	5	0.6450	0.3550	170.6
			59.562			
70.5	45.5	6.25	5	0.6558	0.3442	173.0
71.5	45.5	6.5	60.125	0.7027	0.2973	202.2
70	46	6	59.5	0.6050	0.3950	150.6

4733 Probability & Statistics 2

1		$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{\mu-50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$ Solve simultaneously $\mu = 58.7$, $\sigma = 12.9$	M1 B1 A1 M1 A1 A1	**6**	Standardise once with Φ^{-1}, allow σ^2, cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong *z*, *not* 1–1.645 Solve two standardised equations μ, a.r.t 58.7 σ, a.r.t. 12.9 [*not* σ^2] [σ^2: M1B1A0M1A1A0]
2	(i)	Let *R* denote the number of choices which are 500 or less. $R \sim B(12, \frac{5}{6})$ $P(R = 12) = (\frac{5}{6})^{12}$ [=0.11216] = **0.112**	M1 M1 A1	**3**	$B(12, \frac{5}{6})$ stated or implied, allow 501/600 etc p^{12} or q^{12} or equivalent Answer, a.r.t. 0.112 [SR: $\frac{500}{600} \times \frac{499}{599} \times \frac{498}{598} \times \ldots$; 0.110: M1A1] [M1 for 0.910 or 0.1321 or vague number of terms]
	(ii)	Method unbiased; unrepresentative by chance	B1 B1	**2**	State that method is unbiased Appropriate comment (e.g. "not unlikely") [SR: partial answer, e.g. not necessarily biased: B1]
3	(i)	P(≤ 1) = 0.0611 P(≥ 9) = 1 – P(≤ 8) = 1 – 0.9597 = 0.0403 0.0611 + 0.0403 [= 0.1014] = **10.1**%	B1 M1 A1 M1 A1	**5**	0.0611 seen Find P(≥ 9), allow 8 or 10 [0.0866, 0.0171] 0.0403 correct Add probabilities of tails, *or* 1 tail × 2 Answer [10.1, 10.2]% or probability
	(ii)	$P(2 \le G \le 8)$ = 0.8944 – 0.0266 [= 0.8678] = **0.868**	M1 M1 A1	**3**	Attempt at $P(2 \le G \le 8)$, *not* isw, allow $1 \le G \le 9$ etc Po(5.5) tables, P(≤ top end) – P(≤ bottom end) Answer, a.r.t. 0.868, allow %
4	(i)	$\hat{\mu} = \bar{y} = \frac{3296.0}{40} = 82.4$ $\frac{286800.4}{40} - 82.4^2$ [= 380.25] $S^2 \times \frac{40}{39}$; = 390	B1 M1 M1 A1	**4**	Mean 82.4, c.a.o. Use correct formula for biased estimate Multiply by $n/(n-1)$ [SR: all in one, M2 or M0] Variance 390, c.a.o.
	(ii)	$\Phi\left(\frac{60-82.4}{\sqrt{390}}\right) = \Phi(-1.134)$ = 1 – 0.8716 = **0.128**	M1 A1	**2**	Standardise, allow 390, cc or biased estimate, +/–, do not allow $\sqrt{n}$ Answer in range [0.128, 0.129]
	(iii)	No, distribution irrelevant	B1	**1**	"No" stated or implied, any valid comment
5	(i)	$H_0 : \mu = 500$ where μ denotes $H_1 : \mu < 500$ the population mean α: $z = \frac{435-500}{100/\sqrt{4}} = -1.3$ Compare –1.282	B2 M1 A1 B1		Both hypotheses stated correctly [SR: 1 error, B1, but $\bar{x}$ etc: B0] Standardise, use √4, can be + $z = -1.3$ (allow –1.29 from cc) *or* $\Phi(z) = 0.0968$ (.0985) Compare *z* & –1.282 *or p* (< 0.5) & 0.1 or equivalent
		β: $500 - 1.282 \times 100/\sqrt{4}$ = 435.9; compare 435	M1 A1√;B1		$500 - z \times 100/\sqrt{4}$, allow √ errors, any Φ^{-1}, must be – CV correct, √ on their *z*; 1.282 correct and compare
		Reject H_0 Significant evidence that number of visitors has decreased	M1√ A1√	**7**	Correct deduction, needs √4, μ = 500, like-with-like Correct conclusion interpreted in context
	(ii)	CLT doesn't apply as *n* is small So need to know distribution	M1 B1	**2**	Correct reason ["*n* is small" is sufficient] Refer to distribution, e.g. "if not normal, can't do it"

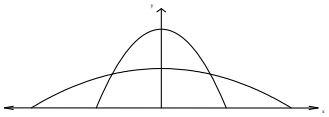
6	(i)	(a) $1 - 0.8153$ $= 0.1847$ (b) $0.8153 - 0.6472$ $= \mathbf{0.168}$	M1 A1 **2** M1 A1 **2**		Po(3) tables, "1 –" used, e.g. 0.3528 or 0.0839 Answer 0.1847 or 0.185 Subtract 2 tabular values, or formula $[e^{-3}\ 3^4/4!]$ Answer, a.r.t. 0.168
	(ii)	N(150, 150) $1 - \Phi\left(\frac{165.5 - 150}{\sqrt{150}}\right)$ $= 1 - \Phi(1.266) = \mathbf{0.103}$	B1 B1 M1 A1 A1 **5**		Normal, mean 3×50 stated or implied Variance or SD = 3 × 50, or same as μ Standardise 165 with λ, √λ or λ, any or no cc √λ and 165.5 Answer in range [0.102, 0.103]
	(iii)	(a) The sale of one house does not affect the sale of any others (b) The average number of houses sold in a given time interval is constant	B1 B1 **2**		Relevant answer that shows evidence of correct understanding [but *not* just examples] Different reason, in context [Allow "constant rate" or "uniform" but not "number constant", "random", "singly", "events".]
7	(i)	$\int_0^2 kx\,dx = \left[\frac{kx^2}{2}\right]_0^2 = 2k$ $= 1$ so $k = ½$	M1 A1 **2**		Use $\int_0^2 kx\,dx = 1$, or area of triangle Correctly obtain $k = ½$ **AG**
	(ii)		B1 B1 **2**		Straight line, positive gradient, through origin Correct, some evidence of truncation, no need for vertical
	(iii)	$\int_0^2 \frac{1}{2}x^2\,dx = \left[\frac{1}{6}x^3\right]_0^2 = \frac{4}{3}$ $\int_0^2 \frac{1}{2}x^3\,dx = \left[\frac{1}{8}x^4\right]_0^2$ [= 2] $2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$	M1 A1 M1 M1 A1 **5**		Use $\int_0^2 kx^2\,dx$; $\frac{4}{3}$ seen or implied Use $\int_0^2 kx^3\,dx$; subtract their mean^2 Answer $\frac{2}{9}$ or a.r.t. 0.222, c.a.o.
	(iv)		M1 A1√ **2**		Translate horizontally, allow stated, or "1, 2" on axis One unit to right, 1 and 3 indicated, nothing wrong seen, no need for vertical or emphasised zero bits [If in doubt as to → or ↓, M0 in this part]
	(v)	$\frac{7}{3}$ $\frac{2}{9}$	B1√ B1√ **2**		Previous mean + 1 Previous variance [If in doubt as to → or ↓, B1B1 in this part]

8	(i)	$H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ B(12, 0.65)	B2 M1	Both hypotheses correctly stated, in this form [One error (but not r, x or $\bar{x}$): B1] B(12, 0.65) stated or implied
		α: $P(\leq 6) = 0.2127$ Compare 0.10	A1 B1	Correct probability from tables, *not* P(= 6) Explicit comparison with 0.10
		β: Critical region ≤ 5; $6 > 5$ Probability 0.0846	B1 A1	Critical region ≤ 5 or ≤ 6 or $\{\leq 4\} \cap \{\geq 11\}$ & compare 6 Correct probability
		Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1√ A1√ **7**	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: N(7.8, 2.73): can get B2M1A0B1M0: 4 ex 7]
	(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1√ B1 **2**	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)"
	(iii)	$R \sim B(2n, 0.65)$, $P(R \leq n) > 0.15$ B(18, 0.65), $p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1 **4**	B($2n$, 0.65), $P(R \leq n) > 0.15$ stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR $< n$: M1A0, $n = 4$, 0.1061 A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR B(12, 0.35): M1A0, $n = 4$, 0.1061 A1A0]

3	0.3529	7	0.1836	12	0.0942
4	0.2936	8	0.1594	13	0.0832
5	0.2485	9	0.1391	14	0.0736
6	0.2127	10	0.1218	15	0.0652

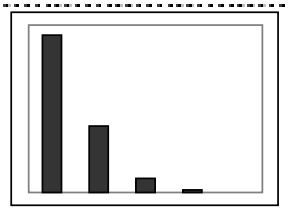
4733 Probability & Statistics 2

General: Conclusions to hypothesis tests must acknowledge uncertainty. Thus "time is unchanged" is A0. Similarly, "Significant evidence that time is unchanged" is also A0.

1	**(i)**	Biased in favour of those with strong political interest	B2	**2**	"Biased", "unrepresentative", "not indept" or equiv [but *not* "not random"] stated, with sensible reason. [SR: partial answer, B1]
	(ii)	Obtain list of all pupils Allocate numbers sequentially Choose using random numbers	B1 B1 B1	**3**	List, can be implied; number serially or randomly, not just "number pupils" Select consistently with method of numbering, not just "select randomly" [SR: systematic: List B1, every n^{th} B1, random start B1] [SR: names in a hat: B2]
2	**(i)**	$\Phi\left(\frac{24-30}{12}\right) - \Phi\left(\frac{20-30}{12}\right)$ $= \Phi(-0.5) - \Phi(-0.833)$ $= (1 - 0.6915) - (1 - 0.7976) =$ **0.1061**	M1 A1 M1 A1	**4**	Standardise one, allow $\sqrt{12}$, 12^2, $\sqrt{n}$ Both standardisations correct, allow cc here Correct handling of tails [0.3085 – 0.2024] Answer, a.r.t. 0.106, c.a.o.
	(ii)	Not symmetrical (skewed) Therefore inappropriate	M1 A1	**2**	Any comment implying not symmetric Conclude "not good model" [Partial answer: B1]
3		$H_0 : \mu = 28$ $H_1 : \mu \neq 28$ $\sigma^2 = 37.05 \times 40/39$ [= 38]	B2 M1		Both hypotheses correctly stated; one error, allow wrong or no letter, but not x or t or $\bar{x}$, B1 Multiply 37.05 or $\sqrt{37.05}$ by $n/(n-1)$ or $\sqrt{[n/(n-1)]}$
	α	$z = \frac{26.44 - 28}{\sqrt{38/40}} = -1.601$ Compare –1.645, or 0.0547 with 0.05	M1 A1 B1		Standardise with $\sqrt{n}$, allow $\sqrt{}$ errors, cc, + Correct z, a.r.t –1.60, or $p \in$ [0.0547, 0.0548] Explicit comparison of z with –1.645 or p with 0.05
	β	Critical value $28 - z\sigma/\sqrt{n}$ [= 26.397] $z = 1.645$ Compare 26.44 with 26.40	M1 B1 A1√		Allow "±", √ errors, cc, ignore other tail $z = 1.645$ in CV expression, and compare 26.44 CV, √ on their z, rounding to 3 SF correct
		Do not reject H_0 [can be implied] Insufficient evidence that time taken has changed.	M1 A1√	**8**	Needs $\sqrt{n}$, correct method & comparison, *not* $\mu = 26.44$ Conclusion interpreted in context, √ on z,
4	**(i)**	$\frac{53-50}{\sigma/\sqrt{10}} < 2.326$ $\sigma >$ **4.08** **AG** [Allow ≥]	M1 A1 B1 A1	**4**	Standardise with 10 or $\sqrt{10}$ and Φ^{-1} Both sides same sign, $\sqrt{10}$, don't worry about < 2.326 or 2.33 seen Convincingly obtain $\sigma > 4.08$ to 3 SF, one other step [SR: Substitution: standardise & substitute 4.08 M1; 0.0101 A1; 4.07 or 4.075 tried, M1; full justification A1]
	(ii)	P(Type I) = 0.01 used, e.g. Geo(0.01) $0.99^4 \times 0.01$ = **0.0096**	M1 M1 A1	**3**	Not enough merely to state $p = 0.01$ $p^4 \times q$ Answer, a.r.t. 0.0096
5	**(i)**	$\int_{-1}^{1} \frac{3}{4}(x^2 - x^4)dx = \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1}$ [= 1/5] $1/5 - 0^2$ = **1/5**	M1 A1 B1 A1	**4**	Attempt $\int_{-1}^{1} x^2 f(x)dx$ Correct indefinite integral Mean 0 clearly indicated Answer 1/5 or a.r.t. 0.200, don't need $\mu = 0$
	(ii)	(a)  (b) Areas equal, more spread out, so g_{max} lower (c) W greater as more spread out	B1 M1 A1 B1dep depB1	**5**	Correct graph, don't need f(x) as well. Don't allow if graph goes further below axis than "pips". Don't worry too much about exact shape Mention areas or total probability Convincing argument, not just "flatter" W greater… …with convincing reason

6	(a)	Po(2.375) $e^{-2.375}\left(\frac{2.375^3}{3!}+\frac{2.375^4}{4!}\right)$ [= 0.2079 + 0.1233] = **0.3310**	M1 M1 A1 A1 **4**	Po(19/8) stated or implied One correct Poisson formula, *not* tables Complete correct expression, including addition Answer, a.r.t. 0.331 [SR: Po(2) or Po(2.4) and tables, M1]
	(b)	(i) *n* large OR $n > 50$ *p* small OR $np < 5$	B1 B1 **2**	Or equivalent [Allow ≤ and ≥ throughout] Or equivalent, e.g. $np \approx npq$, or $p < 0.1$ [Treat "$np < 5$, $npq < 5$" as single wrong statement]
		(ii) B(108, $\frac{1}{36}$) ≈ Po(3) 1 – P(≤ 3) = 1 – 0.6472 = **0.3528**	M1 M1 A1 M1 A1 **5**	Correct binomial distribution stated or implied Po(*np*), √ on their *n*, *p* Po(3) Use Po tables, "1 –", *or* correct formula, ± 1 term, e.g. 0.1847; a.r.t. 0.353, allow from exact Binomial
7	(i)	Dropped catches must occur independently of one another and at constant average rate	B1 B1 **2**	"independently", in context, allow "random" "Constant average rate", in context ["Singly" doesn't gain B1]
Marks can be awarded in either part	(ii)	Use: "Reject H_0 when correct" Po(10) P(≥ 16) = 1 – P(≤ 15) = 1 – 0.9513 Probability 0.0487	M1 M1 M1 A1	Find P(≥ *r*) where $r > \lambda$, e.g. P(≥ 6) from Po(2) Po(10) stated or implied [can be recovered in (iii)] Seek biggest prob < 0.05, e.g. 0.0835 or 0.0166, allow 0.0293 but no other LH tail Answer in range [0.0487, 0.0488], cwd, cwo
	(iii)	$H_0 : \lambda = 10$ or 2 [or μ] $H_1 : \lambda > 10$ or 2 [or μ] α: P(≥ 14) = 1 – 0.8645 = 0.1355 > 0.05	B2 A1 B1	Hypotheses fully correct, allow λ or μ [SR: one error, B1, but *r* or *R* or *x* or $\bar{x}$: B0] $p \in$ [0.135, 0.136] from Po(10) Compare explicitly with 0.05 or 0.0487
		β: Critical region $r \geq 16$, $p = 0.0487$ Compare $r = 14$	A1√ B1√	√ on answer from (ii)
		Do not reject H_0 [can be implied] Insufficient evidence of an increase in the number of dropped catches	M1 A1√ **10**	Method correct, √ on *p*, must be upper tail and "≥" Conclusion interpreted in context [SR: P(≤ 14) = 0.9165 < 0.95: (B2 M1) A0 B1 M0A0; same for P(> 14) or P(= 14)] [SR: N(10,10): (ii) 0.05 M0. (iii) (B2) M1 A0 B1 M0A0]
8	(i)	$H_0 : p = 0.4$ or $\mu = 4.8$ $H_1 : p > 0.4$ or $\mu > 4.8$ B(12, 0.4) P(≥ 9) = 1 – 0.9847 = 0.0153 < 0.05 Reject H_0 [can be implied] Significant evidence of increase in proportion of audience members who know sponsor's name	B2 M1 A1 B1√ M1 A1√ **7**	Both fully correct, B2. [SR: one error, B1, but *x* or *R* or *r* or $\bar{x}$: B0] B(12, 0.4) stated or implied, e.g. 0.9972 or 0.9847 Or: CR is ≥ 9 and $p \in$ [0.015, 0.0153] Explicitly compare with 0.05, or 9 with ≥ 9, √ on < Reject H_0, √ on probability, must be "≥" Conclusion interpreted in context [SR: P(≤ 9) or P(= 9) or P(> 9): (B2 M1) A0 B1 M0A0] [SR: N(4.8, 2.88): (B2) M1 A0 B0 M0A0]
	(ii)	N(160, 96) $\frac{(x-0.5)-160}{\sqrt{96}} = 1.645$ Solve to find *x* [= 176.6] Minimum value is **177**	B1 B1 M1 A1 B1 M1 A1 **7**	Normal, mean 160 Variance (or SD) 96 [96/400: B2M0] Standardise unknown with *np* and √*npq* or *npq*, & equate to Φ^{-1}; √96 and signs correct, ignore cc RHS = 1.645 Solve [implied by 177 or 176.6 or 176.1] 177 only, from 176.6, CWO [cc error: 6 ex 7]

4733 Probability & Statistics 2

1		$U \sim$ B(800, 0.005) ≈ Po(4) P($U \le 6$) = **0.8893** $n > 50$/large, $np < 5$/p small	B1 M1 A1 B1	**4**	Po(np) stated or implied Tables or formula ± 1 term, e.g. 0.7851, 0 .9489, 0.1107, *not* 1– Answer 0.889 or a.r.t. 0.8893 Both conditions
2		$\frac{23.625-23}{5/\sqrt{n}} = 2$ $\sqrt{n} = 16$ n = **256**	M1 A1 M1 A1	**4**	Standardise with $\sqrt{n}$, allow $\sqrt{}^2$ errors Equate to 2 or a.r.t. 2.00, signs correct Solve for $\sqrt{n}$, needs Φ^{-1}, *not* from /n 256 only, allow from wrong signs
3	**(i)**	(a) $e^{-0.42}$ = **0.657** (b) $0.42\ e^{-0.42}$ = **0.276**	M1 A1 A1	**3**	Correct formula for R = 0 or 1 P(0), a.r.t. 0.657 P(1), a.r.t. 0.276
	(ii)	Po(2.1): 1 – P(≤ 3) = 1 – 0.8386 = **0.1614**	M1 M1 A1	**3**	Po(2.1) stated or implied Tables or formula, e.g. 0.8386 or 0.6496 or 0.9379 or complement; Answer, in range [0.161, 0.162]
	(iii)		B2	**2**	At least 3 separate bars, all decreasing *Allow histogram. Allow convex* P(0) < P(1) but otherwise OK: B1 Curve: B1 *[no hint of normal allowed]*
4	**(i)**	$H_0 : p = 0.14$ $H_1 : p < 0.14$ B(22, 0.14) P(≤ 2) = $.86^{22}$ + (22×$.86^{21}$×.14) + (231×$.86^{20}$×$.14^2$) = **0.3877** > 0.1 Do not reject H_0. Insufficient evidence that company overestimates viewing proportion	B2 M1 A1 A1 B1 M1 A1	**8**	Both correct. 1 error, B1, but x or r or $\bar{x}$ etc: 0 B(22, 0.14) stated or implied, e.g. N(3.08, 2.6488) or Po(3.08) Correct formula for 2 or 3 terms, *or* P(≤ 0) = 0.036 and CR Correct answer, a.r.t. 0.388, *or* CR is = 0 Explicitly compare 0.1 or CR with 2, OK from Po but *not* from N Correct comparison type and conclusion, needs binomial, at least 2 terms, *not* from P(< 2) Contextualised, some acknowledgement of uncertainty [SR: Normal: B2 M1 A0 B0 M0] [SR: 2-tailed, or $p > 0.14$, P(≥ 2): B1M1A2B0M1A1]
	(ii)	Selected independently Each adult equally likely to be chosen	B1 B1	**2**	Independent *selection* Choice of sample elements equally likely (no credit if not focussed on selection) [Only "All samples of size n equally likely": B1 only unless related to Binomial conditions]
5	**(i)**		B1 B1 B1	**3**	Horizontal straight line Symmetrical U-shaped curve Both correct, including relationship between the two and not extending beyond [–2, 2], curve through (0,0)
	(ii)	S is equally likely to take any value T is more likely at extremities	B2	**2**	Correct statement about both distributions, √ on their graph [Correct for one only, or partial description: B1] *Not* "probability of S is constant", etc.
	(iii)	$\frac{5}{64}\int_{-2}^{2} x^6 dx = \frac{5}{64}\left[\frac{x^7}{7}\right]_{-2}^{2}$ $\left[= \frac{20}{7}\right]$ -0^2 = $\frac{20}{7}$	M1 A1 B1 A1	**4**	Integrate x^2g(x), limits –2, 2 Correct indefinite integral [= $5x^7/448$] 0 or 0^2 subtracted or E(X) = 0 seen, *not* $\int x^2 f(x)dx - \int xf(x)dx$ Answer $\frac{20}{7}$ or $2\frac{6}{7}$ or a.r.t. 2.86, don't need 0

6	**(i)**	$50.0 \pm 1.96\sqrt{\frac{20.25}{81}} = 50.0 \pm 0.98$ = 49.02, 50.98 $\overline{W} < 49.02$ and $\overline{W} > 50.98$	M1 B1 A1A1 A1√ **5**	$50.0 \pm z\sqrt{(1.96/81)}$, allow one sign only, allow √ errors $z = 1.96$ in equation (*not* just stated) Both critical values, min 4 SF at some stage (if both 3SF, A1) CR, allow ≤ / ≥, don't need $\overline{W}$, √ on their CVs, can't recover [Ans 50 ± 0.98: A1 only] [SR: 1 tail, M1B0A0; 50.8225 or 49. 1775: A1]
	(ii)	$\frac{50.98-50.2}{0.5} = 1.56$ $\frac{49.02-50.2}{0.5} = -2.36$ $\Phi(1.56) - \Phi(-2.36) =$ **0.9315**	M1 A1 A1 M1 A1 **5**	Standardise one limit with same SD as in (i) A.r.t. 1.56, allow – ⎫ Can allow √ here A.r.t. –2.36, allow + ⎭ if very unfair Correct handling of tails for Type II error Answer in range [0.931, 0.932] [SR 1-tail M1; –1.245 or 2.045 A1; 0.893 or 0.9795 A1]
	(iii)	It would get smaller	B1 **1**	No reason needed, but withhold if definitely wrong reason seen. Allow from 1-tail
7	**(i)**	$\hat{\mu} = \bar{t} = 13.7$ $\frac{12657.28}{64} - 13.7^2$ [=10.08]; $\times \frac{64}{63}$ = **10.24** $H_0 : \mu = 13.1$, $H_1 : \mu > 13.1$ $\frac{13.7-13.1}{\sqrt{10.24/64}} = 1.5$ or $p = 0.0668$ 1.5 < 1.645 or 0.0668 > 0.05 Do not reject H_0. Insufficient evidence that time taken on average is greater than 13.1 min	B1 M1 M1 A1 B2 M1 A1 B1 M1 A1 **11**	13.7 stated Correct formula for biased estimate $\times \frac{64}{63}$ used, or equivalent, can come in later Variance or SD 10.24 or 10.2 Both correct. [SR: One error, B1, but x or t or $\bar{x}$ or $\bar{t}$, 0] Standardise, or find CV, with √64 or 64 z = a.r.t. 1.50, or $p = 0.0668$, or CV 13.758 [√ on z] Compare z & 1.645, or p & 0.05 (must be correct tail), or $z = 1.645$ & 13 with CV Correct comparison & conclusion, needs 64, *not* $\mu = 13.7$ Contextualised, some acknowledgement of uncertainty [13.1 – 13.7: (6), M1 A0 B1 M0]
	(ii)	Yes, not told that dist is normal	B1 **1**	Equivalent statement, *not* "n is large", don't need "yes"
8	**(i)**	N(14.7, 4.41) Valid because $np = 14.7 > 5$; $nq = 6.3 > 5$ $1 - \Phi\left(\frac{15.5-14.7}{\sqrt{4.41}}\right) = 1 - \Phi(0.381)$ = 1 – 0.6484 = **0.3516**	M1 A1 B1 B1 M1 A1 A1 **7**	Normal, attempt at np Both parameters correct Check $np > 5$; ⎫ If both asserted but not both nq or $npq > 5$ ⎭ 14.7 and 6.3 seen: B1 only [Allow "n large, p close to ½"] Standardise, answer < 0.5, no √n z, a.r.t. 0.381 Answer in range [0.351, 0.352] [Exact: M0]
	(ii)	$\bar{K} \sim N(14.7, 4.41/36)$ [= N(14.7, 0.35^2)] Valid by Central Limit Theorem as 36 is large $\Phi\left(\frac{14.0+\frac{1}{72}-14.7}{\sqrt{4.41/36}}\right) = \Phi(-1.96)$ = **0.025**	M1 A1√ B1 M1 A1 A1 A1 **7**	Normal, their np from (i) Their variance/36 Refer to CLT or large n (= 36, *not* 21), or "$K \sim N$ so $\bar{K} \sim N$", *not* same as (i), *not* $np > 5$, $nq > 5$ for $\bar{K}$ Standardise 14.0 with 36 or √36 cc included, allow 0.5 here, e.g. 14.5 – 14.7 $z = -1.96$ or –2.00 or –2.04, allow + if answer < 0.5 0.025 or 0.0228 [0.284 loses last 2] [Po(25.2) etc: probably 0]
	OR:	B(756, 0.7) ≈ N(529.2, 158.76) $\Phi\left(\frac{504.5-529.2}{\sqrt{158.76}}\right) = \Phi(-1.96)$ = **0.025**	M1M1A1 B1 M1 A1 A1	×36; N(529.6,…); 158.76 CLT as above, or $np > 5$, $nq > 5$, can be asserted here Standardise 14×36 cc correct and √npq 0.025 or 0.0228

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1	$\frac{105.0-\mu}{\sigma} = -0.7; \frac{110.0-\mu}{\sigma} = -0.5$ Solve: $\sigma = 25$ $\mu = 122.5$	M1 A1 B1 M1 A1 A1 **6**	Standardise once, equate to Φ^{-1}, allow σ^2 Both correct including signs & σ, no cc (continuity correction), allow wrong z Both correct z-values. "1 –" errors: M1A0B1 Get either μ or σ by solving simultaneously σ a.r.t. 25.0 $\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from σ^2 but *not* from $\sigma = -25$.
2	Po(20) ≈ N(20, 20) Normal approx. valid as $\lambda > 15$ $1-\Phi\left(\frac{24.5-20}{\sqrt{20}}\right) = 1 - \Phi(1.006)$ $= 1 - 0.8427$ **= 0.1573**	M1 A1 B1 M1 A1 A1 **6**	Normal stated or implied (20, 20) or (20, √20) or (20, 20^2), can be implied "Valid as $\lambda > 15$", *or* "valid as λ large" Standardise 25, allow wrong or no cc, √20 errors $1.0 < z \leq 1.01$ Final answer, art 0.157
3	$H_0 : p = 0.6$, $H_1 : p < 0.6$ where p is proportion in population who believe it's good value $R \sim B(12, 0.6)$ α: $P(R \leq 4) = 0.0573$ > 0.05	B2 M1 A1 B1	Both, B2. Allow π, % One error, B1, except x or $\bar{x}$ or r or R: 0 B(12, 0.6) stated or implied, e.g. N(7.2, 2.88) *Not* P(< 4) or P(≥ 4) or P(= 4) Must be using P(≤ 4), or P(> 4) < 0.95 and binomial
	β: CR is ≤ 3 and 4 > 3 $p = 0.0153$	B1 A1	Must be using CR; explicit comparison needed
	Do not reject H_0. Insufficient evidence that the proportion who believe it's good value for money is less than 0.6	M1 A1 **7**	Correct conclusion, needs B(12,0.6) and ≤ 4 Contextualised, some indication of uncertainty [**SR**: N(7.2, …) or Po(7.2): poss B2 M1A0] [**SR**: P(< 4) or P(= 4) or P(≥ 4): B2 M1A0]
4 (i)	Eg "not all are residents"; "only those in street asked"	B1 B1 **2**	One valid relevant reason A definitely different valid relevant reason *Not* "not a random sample", *not* "takes too long"
(ii)	Obtain list of whole population Number it sequentially Select using random numbers [Ignore method of making contact]	B1 B1 B1 **3**	"Everyone" or "all houses" must be implied *Not* "number it with random numbers" unless then "arrange in order of random numbers" **SR**: "Take a random sample": B1 **SR**: Systematic: B1 B0, B1 if start randomly chosen
(iii)	Two of: α: Members of population equally likely to be chosen β: Chosen independently/randomly γ: Large sample (e.g. > 30)	B1 B1 **2**	One reason. NB : If "independent", must be "chosen" independently, not "views are independent" Another reason. Allow "fixed sample size" but not both that and "large sample". Allow "houses"

5	(i)	Bricks scattered at constant average rate & independently of one another	B1 B1 **2**	B1 for each of 2 different reasons, in context. (Treat "randomly" ≡ "singly" ≡ "independently")
	(ii)	Po(12) P(≤ 14) – P(≤ 7) [= .7720 – .0895] [or P(8) + P(9) + … + P(14)] **= 0.6825**	B1 M1 A1 **3**	Po(12) stated or implied Allow one out at either end or both, eg 0.617, or wrong column, but *not* from Po(3) nor, eg, .9105 – .7720 Answer in range [0.682, 0.683]
	(iii)	$e^{-\lambda} = 0.4$ $\lambda = -\ln(0.4)$ = 0.9163 Volume = 0.9163 ÷ 3 = **0.305**	B1 M1 A1 M1 **4**	This equation, aef, can be implied by, eg 0.9 Take ln, or 0.91 by T & I λ art 0.916 or 0.92, can be implied Divide their λ value by 3 [**SR**: Tables, eg 0.9÷3: B1 M0 A0 M1]
6	(i)	33.6 $\frac{115782.84}{100} - 33.6^2$ [= 28.8684] $\times\frac{100}{99}$ **= 29.16**	B1 M1 M1 A1 **4**	33.6 clearly stated [not recoverable later] Correct formula used for biased estimate $\times\frac{100}{99}$, M's independent. Eg $\frac{\Sigma r^2}{99}[-33.6^2]$ **SR** B1 variance in range [29.1, 29.2]
	(ii)	$\bar{R} \sim N(33.6, 29.16/9)$ $= N(33.6, 1.8^2)$ $1-\Phi\left(\frac{32-33.6}{\sqrt{3.24}}\right)$ [= Φ(0.8889)] **= 0.8130**	M1 A1 M1 A1 **4**	Normal, their μ, stated or implied Variance [their (i)]÷9 [*not* ÷100] Standardise & use Φ, 9 used, answer > 0.5, allow √ errors, allow cc 0.05 but *not* 0.5 Answer, art 0.813
	(iii)	No, distribution of R is normal so that of $\bar{R}$ is normal	B2 **2**	Must be saying this. Eg "9 is not large enough": B0. Both: B1 max, unless saying that n is irrelevant.
7	(i)	$\frac{2}{9}\int_0^3 x^3(3-x)dx = \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$ [= 2.7] – $(1\frac{1}{2})^2$ $= \frac{9}{20}$ or **0.45**	M1 A1 B1 M1 A1 **5**	Integrate $x^2f(x)$ from 0 to 3 [*not* for μ] Correct indefinite integral Mean is 1½, soi [not recoverable later] Subtract their μ^2 Answer art 0.450
	(ii)	$\frac{2}{9}\int_0^{0.5} x(3-x)dx = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^{0.5}$ $= \frac{2}{27}$ AG	M1 A1 **2**	Integrate f(x) between 0, 0.5, must be seen somewhere Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [ ] = ⅓)
	(iii)	B(108, $\frac{2}{27}$) ≈ N(8, 7.4074) $1-\Phi\left(\frac{9.5-8}{\sqrt{7.4074}}\right)$ = 1 – Φ(0.5511) **= 0.291**	B1 M1 A1 M1 A1 A1 **6**	B(108, $\frac{2}{27}$) seen or implied, eg Po(8) Normal, mean 8 … … variance (or SD) 200/27 or art 7.41 Standardise 10, allow √ errors, wrong or no cc, needs to be using B(108,…) Correct √ and cc Final answer, art 0.291

	(iv)	$\bar{X} \sim N(1.5, \frac{1}{240})$	B1 B1√ B1√ **3**	Normal NB: *not* part (iii) Mean their μ Variance or SD (their 0.45)/108 [not (8, 50/729)]
8	(i)	$H_0 : \mu = 78.0$ $H_1 : \mu \neq 78.0$ $z = \frac{76.4 - 78.0}{\sqrt{68.9/120}} = -2.1115$ > -2.576 or $0.0173 > 0.005$	B1 B1 M1 A1 B1	Both correct, B2. One error, B1, but x or $\bar{x}$: B0. Needs $\pm(76.4 - 78)/\sqrt{(\sigma \div 120)}$, allow √ errors art –2.11, or $p = 0.0173 \pm 0.0002$ Compare z with (–)2.576, or p with 0.005
		$78 \pm z\sqrt{(68.9/120)}$ $= 76.048$ $76.4 > 76.048$	M1 A1√ B1	Needs 78 and 120, can be – only Correct CV to 3 sf, √ on z $z = 2.576$ and compare 76.4, allow from $78 \leftrightarrow 76.4$
		Do not reject H_0. Insufficient evidence that the mean time has changed	M1 A1√ **7**	Correct comparison & conclusion, needs 120, "like with like", correct tail, $\bar{x}$ and μ right way round Contextualised, some indication of uncertainty
	(ii)	$\frac{1}{\sqrt{68.9/n}} > 2.576$ $\sqrt{n} > 21.38$, $n_{\min} =$ **458** Variance is estimated	M1 M1 A1 B1 **4**	IGNORE INEQUALITIES THROUGHOUT Standardise 1 with n and 2.576, allow √ errors, cc etc but *not* 2.326 Correct method to solve for $\sqrt{n}$ (*not* from n) 458 only (*not* 457), *or* 373 from 2.326, signs correct Equivalent statement, allow "should use t". In principle nothing superfluous, but "variance stays same" B1 bod

Specimen Answers

Question 4: Part (i)

α	Takes too long/too slow	B0
β	Interviewing people in the street isn't a random sample	B0
γ	Many tourists so not representative	B1
δ	Those who don't shop won't have their views considered	B1
ε	Interviewers biased as to who they ask	B1
ζ	Views influenced by views of others	B1

Part (ii)

α	Choose a random sample of the town and ask their opinion	B1
β	Choose names at random from the town's phone book	B1
γ	A random number machine determines which house numbers should be used, and every street should have the same proportion of residents interviewed	B0B0B1
δ	Visit everyone door to door and give them a questionnaire	B1B0B0
ε	Assign everyone a number and select randomly	B1B0B0
ζ	Assign everyone a number and select using random numbers	B1B0B1
η	Ditto + "ignoring numbers that don't correspond to a resident"	B1B1B1
θ	Assign each eligible person a number and pick numbers from a hat	B1B1B0
ι	Put names of all residents into a hat and pick them out [NB: postal survey is biased]	B1B1B0

Part (iii)

α	One person's view should not affect another's	B0
β	It is without bias	B0
γ	Results occur randomly	B0
δ	Should be asked if they are for or against (binomial testing)	B0
ε	It will survey a diverse group from different areas so should be representative	B0
ζ	Everyone's should be chose independently of everyone else	B1
η	The sample size must be large	B1
θ	Participants are chosen at random and independently from one another [though η & θ together would get B2]	B1 only

Question 5 (i)

α	Number of bricks must always be the same		B0
β	Results occur randomly		B0
γ	The chance of a brick being in one place is always the same		B0
δ	Events must occur independently and at constant average rate		B0
ε	They must occur independently and at constant average rate		B1 only
ζ	Bricks' locations must be random and independent	*[effectively the same]*	B1 only
η	Only one brick in any one place; bricks independent	*[effectively the same]*	B1 only

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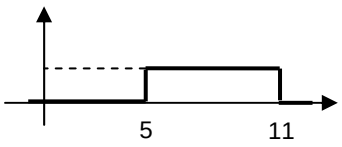
Penalise over-specified answers (> 6 SF) first time but only once per paper.
Use Ⓐ or Ⓒ to annotate "over-assertive" or "no context" respectively

1		$\hat{\mu} = \bar{x} = 15.16$ $\hat{\sigma}^2 = \frac{5}{4}s^2$ $= \mathbf{1.363}$	B1 M1 M1 A1	**4**	15.16 or 15.2 as answer only Use $\frac{\Sigma x^2}{5} - \bar{x}^2$ [=1.0904] Multiply by 5/4, or equiv for single formula Final answer 1.36 or 1.363 only, *not* isw
2	(i)	Not all equally likely – those in range 0 to 199 more likely to be chosen	M1 A1	**2**	Not all equally likely stated or implied Justified by reference to numbers, no spurious reasons
	(ii)	Ignore random numbers greater than 799, or 399	B1	**1**	Any valid resolution of this problem, no spurious reasons
3		B(60, 0.35) ≈ N(21, 13.65) $\Phi\left(\frac{18.5-21}{\sqrt{13.65}}\right) = \Phi(-0.6767)$ = 1 – 0.7507 $= \mathbf{0.2493}$	M1 M1 A1 M1 A1 A1	**6**	B(60, 0.35) stated or implied N(21, …) Variance or SD = 13.65 Standardise, their *np* and √*npq* or *npq*, wrong or no cc Both √*npq* and cc correct Answer, a.r.t. 0.249
4		$H_0 : \mu = 60$; $H_1 : \mu < 60$ (α) $z = \frac{58.9-60}{\sqrt{5^2/80}} = -1.967$ < -1.645	B2 M1 A1 B1		Both correct, B2 B1 for one error, but not *x*, *t*, $\bar{x}$ or $\bar{t}$ Standardise 58.9 & √80, allow – or √ errors *z*, art –1.97 or *p* in range [0.024, 0.025] Explicit comparison with –1.645 or 0.05, or +1.645 or 0.95 if 1.967 or 0.976 used
	or:	(β) $c = 60 - 1.645 \times \frac{5}{\sqrt{80}} = 59.08$ 58.9 < 59.08	M1 B1 A1✓		60 – *z*×5/√80, any $z = \Phi^{-1}$, allow √ errors or ±, not just +; *z* = 1.645 and compare 58.9 59.1 or better, ✓on wrong *z*
		Reject H_0 Significant evidence that people underestimate time	M1 A1✓	**7**	Correct first conclusion, needs essentially correct method including √80 or 80 Contextualised, uncertainty acknowledged SR: μ = 58.9: B0M1A0B1 max 2/7 SR: 2-tail: max 5/7
5	(i)	$H_0 : \lambda = 11.0$ $H_1 : \lambda > 11.0$ (α) P(≥ 19) = 1 – 0.9823 = 0.0177 < 0.05	B2 M1 A1 B1		Allow μ. Both correct, B2 One error: B1, but not *C*, *x* etc Find P(≥ 19) [or P(< 19) if later 0.95] art 0.0177 [0.9823, ditto] Compare 0.05 [0.95 if consistent], needs M1
		(β) CR ≥ 18, P(≥ 18) = 0.0322 19 > 18	M1 A1 B1		CR or CV 16/17/18/19 stated or clearly implied, but not < 18 and 0.0322 both seen, allow 0.9678 Explicit comparison with 19, needs M1
		Reject H_0 Significant evidence of an increase in number of customers	M1 A1✓	**7**	Needs essentially correct method & comparison Contextualised, uncertainty acknowledged SR: Normal, or P(= 19) or P(≤ 19) or P(> 19): First B2 only.
	(ii)	Can't deduce cause-and-effect, or there may be other factors	B1	**1**	Conclusion needed. No spurious reasons. If "DNR" in **(i)**, "couldn't deduce even if…"

6	(i)	(a) Probabilities don't total 1	B1	**1**	Equivalent statement
		(b) P(> 70) must be < P(> 50)	B1	**1**	Equivalent statement
		(c) P(> 50) = 0.3 $\Rightarrow \mu < 50$ P(< 70) = 0.3 $\Rightarrow \mu > 70$	B1	**1**	Any relevant valid statement, e.g. "P(< 50) = 0.7 but P(< 50) must be < P(< 70)"
	(ii)	$\mu = 60$ by symmetry	B1		$\mu = 60$ obtained at any point, allow from Φ
		$\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$	M1		One standardisation, equate to Φ^{-1}, not 0.758
		$\sigma = 10/0.5243$	B1		$\Phi^{-1} \in [0.524, 0.5245]$ seen
		= **19.084**	A1	**4**	σ in range [19.07, 19.1], e.g. 19.073
7	(i)		M1 A1	**2**	Horizontal line Evidence of truncation *[no need for labels]*
	(ii)	$\mu = 8$	B1		8 only, cwd
		$\int_5^{11} \frac{1}{6}t^2 dt = \left[\frac{1}{18}t^3\right]_5^{11}$ [= 67]	M1		Attempt $\int kt^2$ dt, limits 5 and 11 seen
			B1		$k = 1/6$ stated or implied
		$- 8^2$	M1		Subtract their (non-zero) mean^2
		= **3**	A1	**5**	Answer 3 only, *not* from MF1
	(iii)	N(8, 3/48)	M1		Normal stated or implied
			A1		Mean 8
		$1 - \Phi\left(\frac{8.3-8}{\sqrt{3/48}}\right) = 1 - \Phi(1.2)$	A1		Variance their (non-zero) **(ii)**/48
		$= 1 - 0.8848$	M1		Standardise, $\sqrt{n}$, ignore sign or $\sqrt{}$ errors. cc: M0
		= **0.1151**	A1		Answer, art 0.115
		Normal distribution only approx.	B1	**6**	Any equivalent comment, e.g. CLT used
8	(i)	P(≤ 4) = 0.0473	M1		P(≤ r) from B(10, 0.7), r = 3/4/5, *not* N
		Therefore CR is ≤ 4	B1		"≤ 4" stated, not just "4", nothing else
		P(Type I error) = **4.73%**	A1	**3**	Answer, art 0.0473 or 4.73%, must be stated
	(ii)	B(10, 0.4) and find P(> 4)	M1		Must be this, *not* isw, ✓on **(i)**
		$1 - P(\leq 4)$	M1		Allow for 0.6177 or 0.1622
		= **0.3669**	A1	**3**	Answer, art 0.367
	(iii)	0.5×0.3669	M1		$0.5 \times$ **(ii)**
		= **0.18345**	A1✓	**2**	Ans correct to 3 SF, e.g. 0.184 from 0.367

9	(i)	$1 - P(\leq 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	**2**	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119
	(ii)	Po(12) $P(\leq 14) - P(\leq 12)$ [0.7720 – 0.5760] $= \mathbf{0.196}$	M1 M1 A1	**3**	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196
	(iii)	Po(60) ≈ N(60, 60) $\Phi\left(\frac{69.5-60}{\sqrt{60}}\right) = \Phi(1.226)$ $= \mathbf{0.8899}$	M1 A1 M1 A1 A1	**5**	N(60, …) Variance or SD 60 Standardise, λ & $\sqrt{\lambda}$, allow λ or wrong or no cc $\sqrt{\lambda}$ and cc both correct Answer 0.89 or a.r.t. 0.890
	(iv)	(a) $1 - e^{-3m}(1 + 3m)$	M1 A1	**2**	M1 for one error, e.g. no "1 –", or extra term, or 0^{th} term missing; answer, aesf
		(b) $m = 1.29$, $p = 0.89842$ $m = 1.3$, $p = 0.9008$ Straddles 0.9, therefore solution between 1.29 and 1.3	M1 A1 A1 A1	**4**	Substitute 1.29 or 1.3 into appropriate fn Comp | 0.9 | 0.1 | 0 1.29 | 0.898 | 0.10158 | –.00158 1.3 | 0.901 | 0.09918 | .0008146 Explicit comparison with relevant value, & conclusion, needs both *p*s correct
	or	Method for iteration; 1.296… 1.2965or better; conclusion stated	M1A1 A1A1		Can be implied by at least 1.296… Need at least 4 dp for M1A2

1	(i)(a)	$1 - P(\leq 6) = 1 - 0.8675$ $= \mathbf{0.1325}$	M1 A1	**2**	$1 - .9361$ or $1 - .8786$ or $1 - .8558$: M1. .9721: M0 Or 0.132 or 0.133
	(b)	Po(0.42) $e^{-0.42}\ \frac{0.42^2}{2!}$ $= \mathbf{0.05795}$	M1 M1 A1	**3**	Po(0.42) stated or implied Correct formula, any numerical λ Answer, art 0.058. Interpolation in tables: M1B2
	(ii)	E.g. "Contagious so incidences do not occur independently", or "more cases in winter so not at constant average rate"	B2	**2**	Contextualised reason, referred to conditions: B2. No marks for mere learnt phrases or spurious reasons, e.g. not just "independently, singly and constant average rate". See notes.
2	(i)	B(10, 0.35) P(< 3) $= \mathbf{0.2616}$	M1 M1 A1	**3**	B(10, 0.35) stated or implied Tables used, e.g. 0.5138 or 0.3373, or formula ± 1 term Answer 0.2616 or better or 0.262 only
	(ii)	Binomial requires being chosen independently, which this is not, but unimportant as population is large	B2	**2**	Focus on "Without replacement" negating independence condition. It doesn't negate "constant probability" condition but can allow B1 if "selected". See notes
3	(i)	$\left(\frac{32-40}{\sigma}\right) = \Phi^{-1}(0.2) = -0.842$ $\sigma = 9.5[06]$	M1 B1 A1	**3**	Standardise and equate to Φ^{-1}, allow "1 –" errors, σ^2, cc 0.842 seen Answer, 9.5 or in range [9.50, 9.51], c.w.o.
	(ii)	B(90, 0.2) $\approx$ N(18, 14.4) $1 - \Phi\left(\frac{19.5-18}{\sqrt{14.4}}\right) = 1 - \Phi(0.3953)$ $= 1 - 0.6537$ $= \mathbf{0.3463}$	B1 M1 A1 M1 A1 A1	**6**	B(90, 0.2) stated or implied N, their np … … variance their npq, allow √ errors Standardise with np and npq, allow √, cc errors, e.g. .396, .448, .458, .486, .472; $\sqrt{npq}$ and cc correct Answer, a.r.t. 0.346 [NB: 0.3491 from Po: 1/6]
4	(α)	$H_0 : p = 0.4$, $H_1 : p > 0.4$ $R \sim$ B(16, 0.4): $P(R \geq 11) = 0.0191$ > 0.01	B1 B1 M1 A1 A1		Fully correct, B2. Allow π. p omitted or μ used in both, or > wrong: B1 only. x or $\bar{x}$ or 6.4 etc: B0 B(16, 0.4) stated or implied, allow N(6.4, 3.84) Allow for $P(\leq 10) = 0.9808$, and < 0.99, or $z = 2.092$ or $p = 0.018$, but *not* $P(\leq 11) = 0.9951$ or $P(= 11) = 0.0143$ Explicit comp with .01, or z<2.326, *not* from ≤11 or =11
	(β)	CR $R \geq 12$ and $11 < 12$ Probability 0.0049	A1 A1		Must be clear that it's ≥ 12 and not ≤ 11 Needs to be seen, allow 0.9951 here, or p = .0047 from N
		Do not reject H_0. Insufficient evidence that proportion of commuters who travel by train has increased	M1 A1 FT	**7**	Needs like-with-like, $P(R \geq 11)$ or CR $R \geq 12$ Conclusion correct on their p or CR, contextualised, not too assertive, e.g. "evidence that" needed. Normal, $z = 2.34$, "reject" [no cc] can get 6/7
5	(i)	(a) $30 + 1.645 \times \frac{5}{\sqrt{10}}$ $= 32.6$ Therefore critical region is $\bar{t} > 32.6$	M1 B1 A1 A1 FT	**4**	$30 + 5z/\sqrt{10}$, allow ± but not just –, allow √ errors $z = 1.645$ seen, allow – Critical value, art 32.6 " > c" or "≥ c", FT on c provided > 30, can't be recovered. Withhold if not clear which is CR
		(b) $P(\bar{t} < 32.6 \mid \mu = 35)$ $\frac{32.6-35}{5/\sqrt{10}}$ [= –1.5178] **0.0645**	M1* dep*M1 A1	**3**	Need their c, final answer < 0.5 and $\mu = 35$ at least, but allow answer > 0.5 if consistent with their (i) Standardise their CV with 35 and √10 or 10 Answer in range [0.064, 0.065], or 0.115 from 1.96 in (a)
	(ii)	$(32.6 - \mu) = 0$ $\mu = 32.6$ $20 + 0.6m = 32.6$ $m =$ **21**	M1 A1 FT M1 A1	**4**	Standardise c with μ, equate to Φ^{-1}, can be implied by: μ = their c Equate and solve for m, allow from 30 or 35 Answer, a.r.t. 21, c.a.o. MR: 0.05: M1 A0 M1, 16.7 A1 FT Ignore variance throughout (ii)

6	(a)	N(24, 24) $1-\Phi\left(\frac{30.5-24}{\sqrt{24}}\right)=1-\Phi(1.327)$ $=$ **0.0923**	B1 B1 M1 A1 A1 **5**	Normal, mean 24 stated or implied Variance or SD equal to mean Standardise 30 with λ and √λ, allow cc or √ errors, e.g. .131 or .1103 ; 30.5 and √λ correct Answer in range [0.092, 0.0925]
	(b)(i)	*p* or *np* [= 196] is too large	B1 **1**	Correct reason, no wrong reason, don't worry about 5 or 15
	(ii)	Consider (200 – *E*) (200 – *E*) ~ Po(4) P(≥ 6) [= 1 – 0.7851] = **0.2149**	M1 M1 M1 A1 **4**	Consider complement Po(200×0.02) Poisson tables used, correct tail, e.g. 0.3712 or 0.1107 Answer a.r.t. 0.215 only
7		$H_0 : \mu = 56.8$ $H_1 : \mu \neq 56.8$ $\bar{x}$ = 17085/300 = 56.95 $\frac{300}{299}\left(\frac{973847}{300}-56.95^2\right)$ = 2.8637… $z=\frac{56.95-56.8}{\sqrt{2.8637/300}}=1.535$ 1.535 < 1.645 or 0.0624 > 0.05	B2 B1 M1 M1 A1 M1 A1 A1	Both correct One error: B1, but *not* $\bar{x}$, etc 56.95 or 57.0 seen or implied Biased [2.8541] : M1M0A0 Unbiased estimate method, allow if ÷ 299 seen anywhere Estimate, a.r.t. 2.86 [not 2.85] Standardise with √300, allow √ errors, cc $z \in$ [1.53, 1.54] or $p \in$ [0.062, 0.063], *not* – 1.535 Compare explicitly *z* with 1.645 or *p* with 0.05, or 2*p* > 0.1, not from μ = 56.95
	(α)			
	(β)	CV $56.8 \pm 1.645 \times \sqrt{\frac{2.8637}{300}}$ 56.96 > 56.95	M1 A1 A1 FT	56.8 + *z*σ/√300, needn't have ±, allow √ errors *z* = 1.645 *c* = 56.96, FT on *z*, and compare 56.95 [c_L = 56.64]
		Do not reject H_0; insufficient evidence that mean thickness is wrong	M1 A1 FT **11**	Consistent first conclusion, needs 300, correct method and comparison Conclusion stated in context, not too assertive, e.g. "evidence that" needed
8	(i)	$\int_1^\infty kx^{-a}\,dx=\left[k\frac{x^{-a+1}}{-a+1}\right]_1^\infty$ Correctly obtain $k = a - 1$ **AG**	M1 B1 A1 **3**	Integrate f(*x*), limits 1 and ∞ (at some stage) Correct indefinite integral Correctly obtain given answer, don't need to see treatment of ∞ but mustn't be wrong. Not k^{-a+1}
	(ii)	$\int_1^\infty 3x^{-3}dx=\left[3\frac{x^{-2}}{-2}\right]_1^\infty=1\frac{1}{2}$ $\int_1^\infty 3x^{-2}dx=\left[3\frac{x^{-1}}{-1}\right]_1^\infty-(1\frac{1}{2})^2$ Answer ¾	M1 M1 A1 M1 A1 **5**	Integrate *x*f(*x*), limits 1 and ∞ (at some stage) [x^4 is *not* MR] Integrate x^2f(*x*), correct limits Either μ = 1½ or E(X^2) = 3 stated or implied, allow *k*, *k*/2 Subtract their numerical μ^2, allow letter if subs later Final answer ¾ or 0.75 only, cwo, e.g. not from μ =–1½. [SR: Limits 0, 1: can get (i) B1, (ii) M1M1M1]
	(iii)	$\int_1^2 (a-1)x^{-a}dx=\left[-x^{-a+1}\right]_1^2=0.9$ $1-\frac{1}{2^{a-1}}=0.9,\ 2^{a-1}=10$ *a* = 4.322	M1* dep*M1 M1 indept A1 **4**	Equate ∫f(*x*)d*x*, one limit 2, to 0.9 or 0.1. [Normal: 0 ex 4] Solve equation of this form to get 2^{a-1} = number Use logs or equivalent to solve 2^{a-1} = number Answer, a.r.t. 4.32. T&I: (M1M1) B2 or B0

Specimen Verbal Answers

1

α	"Cases of infection must occur randomly, independently, singly and at constant average rate"	B0
β	Above + "but it is contagious"	B1
γ	Above + "but not independent as it is contagious"	B2
δ	"Not independent as it is contagious"	B2
ε	"Not constant average rate", or "not independent"	B0
λ	"Not constant average rate because contagious" *[needs more]*	B1
ζ	"Not constant average rate because more likely at certain times of year"	B2
μ	Probabilities changes because of different susceptibilities	B0
ν	Not constant average rate because of different susceptibilities	B2
η	Correct but with unjustified or wrong extra assertion *[scattergun]*	B1
θ	More than one correct assertion, all justified	B2
π	Valid reason (e.g. "contagious") but not referred to conditions	B1

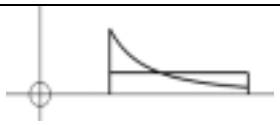
[Focus is on explaining why the required assumptions might not apply. No credit for regurgitating learnt phrases, such as "events must occur randomly, independently, singly and at constant average rate, even if contextualised.]

2 Don't need either "yes" or "no".

α	"No it doesn't invalidate the calculation" *[no reason]*	B0
β	"Binomial requires not chosen twice" *[false]*	B0
γ	"Probability has to be constant but here the probabilities change"	B0
δ	Same but "probability <u>of being chosen</u>" *[false, but allow B1]*	B1
ε	"Needs to be independently chosen but probabilities change" *[confusion]*	B0
ζ	"Needs to be independent but one choice affects another" *[correct]*	B2
η	"The sample is large so it makes little difference" *[false]*	B0
θ	"The population is large so it makes little difference" *[true]*	B2
λ	Both correct and wrong reasons (scattergun approach)	B1

[Focus is on modelling conditions for binomial: On every choice of a member of the sample, each member of the population is equally likely to be chosen; and each choice is independent of all other choices.
Recall that in fact even without replacement the probability that any one person is chosen is the same for each choice. Also, the binomial "independence" condition <u>does</u> require the possibility of the same person being chosen twice.]

1		$\hat{\mu} = \bar{x} = \frac{468}{9} = 52$ $\frac{24820}{9} - 52^2$ [= 53.78] $\hat{\sigma}^2 = \frac{9}{8} \times 53.78$ = **60.5**	B1 M1 M1 A1 **4**	52 stated Correct method for biased estimator Multiply by 9/8 [if single formula, allow M0 M1 if wrong but divisor 8 seen anywhere] Answer 60.5 or exact equivalent
2		$\frac{53.28 - \mu}{5/\sqrt{n}} = 1.96$ $\frac{\mu - 51.65}{5/\sqrt{n}} = 1.3$ $\sqrt{n} = 10$, n = **100** μ = **52.3**	M1dep A1 B1 depM1 A1 B1 **6**	Standardise with $\sqrt{n}$ once & equate to z, allow sign, square/$\sqrt{}$ errors twice, signs correct, zs may be wrong Both correct z values seen Solve to get $\sqrt{n}$ or μ, needs first M1 n = 100, not from wrong signs a.r.t. 52.3, right arithmetic needed but $\sqrt{n}$ can be omitted
3		B(200, 0.0228) Po(4.56) $e^{-4.56}(1 + 4.56 + \frac{4.56^2}{2})$ **= 0.167** n large *or* $n > 50$; p small *or* $np < 5$	M1 A1 M1 A1 A1 B1 **6**	B(200, 0.0228) stated or implied Po(4.56) stated or implied, allow 4.6 here Correct formula for P(≤ 2) ± 1 term, any λ (tables: M0) Correct formula, 4.56 needed Answer, a.r.t. 0.167 [0.16694] Both, can be merely asserted. If numbers, must be these SR interpolation: clear method M1, answer A2 MR: typically B(200, 0.228) ≈ N(45.6, 3.52): M1A1; standardise correctly, M1; state np, $nq > 5$, B1
4	(i)	*Either* $z = \frac{213.4 - 230}{45/\sqrt{50}}$ = –2.608 –2.608 < –2.576 *or* 0.0047 < 0.005	M1 A1 B1	Standardise z with $\sqrt{50}$, ignore sign or $\sqrt{}$ or squaring errors z-value, a.r.t. –2.61, or p in range [0.0044, 0.005) Correctly compare (–)2.576, signs consistent, *or* p explicitly with 0.005
	Or	CV is $230 - 2.576 \times \frac{45}{\sqrt{50}}$ = 213.6 213.4 < 213.6	M1 B1 A1	$230 - z\sigma/\sqrt{50}$, allow $\sqrt{}$ or squaring errors, allow ± but not just +; z = 2.576 Explicitly compare 213.4 with 213.6
		Reject H_0. Significant evidence that population mean is not 230	M1 A1 FT **5**	"Reject", FT, needs correct method and form of comparison; interpreted, acknowledge uncertainty
	(ii)	Yes, population distribution is not known to be normal	B2 **2**	*Not*, "yes, sample size is large" but ignore "*can* use it as …" SR: Both right and wrong answers: B1 α "Yes as it must be assumed normal": B1
5		H_0: $\lambda = 12$; H_1: $\lambda > 12$ *Either* : P(≥ 19) = 1 – P(≤ 18) = 1 – 0.9626 = 0.0374 < 0.1	B2 M1 A1 B1	Both correct: B2. Allow μ. One error, B1, but *not* x, r etc. Po(12) stated or implied, e.g. 0.9787 0.0374, *or* 0.9626 if compared with 0.9 Explicitly compare P(≥ 19) with 0.1, *or* P(≤ 18) with 0.9
		Or: CR is ≥ 18, p = 0.063 19 ≥ 18	A1 B1	≥ 18 and 0.063 stated Explicit comparison of CV (right-hand CR) with 19
		Reject H_0. Significant evidence of increase in mean number of applicants	M1 A1 FT **7**	"Reject" FT, needs correct method and comparison, e.g. *not* from ≤ 19 or = 19, withhold if inconsistent Interpreted in context, acknowledge uncertainty

6	(i)	If one customer arrives, it does not change the probability that another one does so; customers probably arrive in groups of at least 2	B1 B1 **2**	Answer that shows correct understanding of "independent", in context; *not* just equivalent to "singly" Plausible reason, in context, nothing wrong, nothing that suggests "constant average rate"
	(ii)	**0.1730**	M1 A1 **2**	Correct use of tables or formula, e.g. .3007, or .4405 from Po(5) if Po(7) stated; answer 0.173, 0.1730 or better
	(iii)	Po(35) N(35, 35) $1-\Phi\left(\frac{40.5-35}{\sqrt{35}}\right) = 1 - \Phi(0.9297)$ = **0.1763**	B1 M1 A1 M1 A1 A1 **6**	Po(5×7) stated or implied Normal, μ = their λ Both parameters correct, allow 35^2, $\sqrt{35}$ Standardise 40 with λ, $\sqrt{\lambda}$, allow $\sqrt{}$, cc errors Both $\sqrt{\lambda}$ and cc correct Answer, a.r.t. 0.176 [penalise 0.1765]
7	(i)		B1 B1 B1 **3**	Horizontal line above axis Concave decreasing curve above axis Both correct including approx relationship, not extending beyond [1, 3], verticals and scale not needed
	(ii)	$\int_1^3 \frac{a}{x^2}\mathrm{d}x = 1,\ \left[\frac{-a}{x}\right]_1^3 = 1;\ a = \frac{3}{2}$	M1 B1 A1 **3**	Attempt $\int f_X(x)\mathrm{d}x$, limits 1, 3 at some stage, and equate to 1 Correct indefinite integral Correctly obtain 3/2 or 1.5 or exact equivalent
	(iii)	$\int_1^3 \frac{a}{x}\mathrm{d}x = \left[a\ln x\right]_1^3$ $= \frac{3}{2}\ln 3$	M1 B1 FT A1 FT **3**	Attempt $\int x f_X(x)\mathrm{d}x$, limits 1, 3 at some stage Correct indefinite integral, FT on a Answer, any exact equivalent or a.r.t 1.65, FT on a, or $a\ln 3$
	(iv)	T is equally likely to take any value between 1 and 3	B1 **1**	Must be "values taken by T" (or "of T") or clear equivalent Any hint that they think T is an *event* gets B0. α "Same chance of occurring anywhere between 1 and 3": 0 β "For values of T between 1 and 3, T is equally likely": 0 γ "Each value of T is equally likely to occur": 1
8	(i)	B(40, 0.225) ≈ N(9, 6.975) $\frac{5.5-9}{\sqrt{6.975}} = -1.325$ **0.9074** $np = 9 > 5$ *or* n large; and $nq = 31 > 5$ *or* p close to 0.5	M1 M1 A1 M1 A1 A1 B2 **8**	B(40, 0.225) stated or implied Normal, mean 9 Variance 6.975 or SD 2.641 or 6.975 Standardise with np and $\sqrt{npq}$, allow npq, no or wrong cc CC and $\sqrt{npq}$ correct, allow from N(3600, 0.225) Answer, in range [0.907, 0.908] Full conditions B2; partial, B1 (assertions OK). Allow npq, allow from e.g. n = 3600
	(ii)	Number list sequentially and select using random numbers If # > 3600, ignore (etc)	B1 B1 B1 **3**	Number list, don't need "sequentially" Mention random numbers (*not* "select numbers randomly") Deal with issue of # > 3600, *or* "ignore repeats" α "Randomly pick numbers from 0 to 3599": (B1) B0 B1
9	(i)	B(14, 0.7) CR is **≥ 13** with probability 0.0475	M1 A1 A1 **3**	B(14, 0.7) stated or implied, e.g. N(9.8, 2.94), can be recovered CV 13, or > 12 or {13, 14}, allow = but no other inequalities Exactly correct CR, and supporting prob .0475 or .9525 seen
	(ii)	H_0: $p = 0.7$, H_1: $p > 0.7$ 12 < 13 Do not reject H_0. Insufficient evidence that proportion who show improvement is greater than 0.7	B2 B1 M1 A1 FT **5**	Both, B2. Allow π. One error, B1, but r, x etc: B0 Compare CV *from correct tail and inequality* with 12, *or* P(≥ 12) = 0.1608 and > 0.05 *or* P(< 12) = 0.8392 and < 0.95 Correct method & conclusion, requires like-with-like; CV method needs ≥ 13 or < 12; p method needs ≥ 12 or < 12 Withhold if inconsistent Contextualised, acknowledge uncertainty [SR: Normal or Po: (i) M1, (ii) B2 maximum] [0.9932 or 0.0068 probably B2 maximum]
	(iii)	B(14, 0.8) P(≤ 12) from B(14, 0.8) **0.8021**	M1 M1 A1 **3**	B(14, 0.8) stated or implied, allow from B(14, 0.75) Attempt prob of acceptance region, e.g. 0.8990, √ on (i) Answer 0.802 or a.r.t. 0.8021

1		Number all the houses sequentially, or use house numbers Select using random numbers Ignore numbers > 263	B1 B1 B1 **3**	Any mention of using house numbers, or houses, or other numbering. (List can be implied). *Not* random numbering unless correct subsequent method (e.g. sort them numerically) Mention random numbers. *Not* "select numbers randomly". Must be random method. NB: Using 263 × calculator Rand # is biased: B0. But "Ran#(263)" is unbiased. Deal with problem of > 263, *or* repeats. "Select 20 random numbers between 1 and 263": B1B0 *[If this, need to mention repeats to get last B1]* *Example:* "put numbers/house names (etc) into hat and select": B1B0B0
2	α	$\mu = \frac{48+57}{2}$ = 52.5 $\Phi^{-1}(0.9332) = 1.5$ $4.5 \div 1.5$ [$\sigma = 3$]	M1 A1 B1 M1	Use symmetry to find μ Obtain μ = 52.5 1.5 seen, e.g. in 4.5 ÷ 1.5 4.5 ÷ their Φ^{-1}, or 1.645 ÷ their Φ^{-1}, must be +ve, allow cc
	β	$\frac{57-\mu}{\sigma} = 1.5, \frac{48-\mu}{\sigma} = -1.5$ Solve simultaneously: μ = 52.5 [$\sigma = 3$]	M1 A1 B1 A1	$\frac{57-\mu}{\sigma} = z$, $\frac{48-\mu}{\sigma} = -z$ M1 for one, ignoring cc, σ^2, sign or "1 –" errors, RHS must be Φ^{-1} (*not* Φ [e.g. 0.8246 or 0.5267] or 0.0668 or 0.9332); A1 for both completely correct except for value of *z*. *z* = 1.5 or –1.5 in at least one equation Solve without obvious errors, get μ = 52.5, OK from wrong *z* [NB: 52.5 from both signs wrong: A0]
		$\mu + \frac{4.5}{1.5} \times 1.645$ = **57.4**(35)	M1 B1 A1 **7**	$\mu + z\sigma$ *[Their* μ *and* σ*, anything recognisable as z]* [expect to see 52.5 + 3×1.645] *z* = 1.645 seen Answer in range [57.4, 57.45], cwo
3		CV $20 - \frac{5}{\sqrt{16}} \times 2.326$ = 17.0925 P(X > 17.0925) = $\Phi\left(\frac{17.0925-15}{5/\sqrt{16}}\right)$ = Φ(1.674) Answer **0.0471**	M1 B1 A1 M1* A1 dep M1 A1 **7**	Attempt 20 – 5*z*/√16, allow SD ↔ var errors, allow 20 ± 5*z*/√16, *not* 20 + 5*z*/√16, allow cc 2.326 seen CV a.r.t. 17.1 *[NB: not 17.9075]* Standardise any attempt at a CV (from μ = 20) with 15 and any SD that would have got first M1, allow cc *z* = 1.674 seen or implied, e.g. by *p* = 0.047 or 0.953 or 0.9535, allow anything in range [1.67, 1.68] Probability < 0.5, or > 0.5 if their CV is < 15 Answer, a.r.t. 0.047 [including 0.0465 from CV 17.1] *Notes:* 16 missing: can get M0B1A0M1A0M1A0, or even last two A1's if 16 used then

4	(i)		M1 A1 **2**	Positive parabola, all above axis. [Don't worry about being pointed unless extreme.] Correct place, touches x-axis, not beyond the limits suggested by their axes, symmetric ends, not too straight
	(ii)	$\frac{3}{16}\int_0^4 x^2(x-2)^2\,dx$ $= \frac{3}{16}\left[\frac{x^5}{5} - x^4 + 4\frac{x^3}{3}\right]_0^4$ $[= 6\frac{2}{5}]$ $\sigma^2 = 6\frac{2}{5} - 2^2$ $= 2\frac{2}{5}$	M1 M1 B1 B1 A1 **5**	Attempt $\int x^2 f(x)dx$, limits 0 and 4 Method for integration, e.g. multiply out *[indept]* [Or use $\sigma^2 = \frac{3}{16}\int_0^4 (x-2)^4\,dx$] Correct indefinite integral, limits not needed, e.g. parts: $\frac{3}{16}\left[\frac{x^2(x-2)^3}{3} - \frac{x(x-2)^4}{6} + \frac{(x-2)^5}{30}\right]$ Subtract 2^2 Final answer 2.4, any equivalent exact form, cwo
	(iii)	No because x represents a value taken by the random variable *[not an event that "occurs"]*	B1 **1**	Show clear understanding that x is a value of X. Usual misunderstanding is "X is an event that may or may not occur, depending on x". However: SR: Allow B1 for answer clearly indicating that probabilities higher where curve higher, *or* clearly stating that all probabilities are effectively zero. E.g.: "Agree as area under graph [or "f(x)"] increases", or "minimum at 2" B1 "True only between 0 and 4": B0 unless explanation Mention of variance etc: 0. "Agree because the graph shows this": B0
5	(i)	H_0: $p = 0.4$; H_1: $p < 0.4$ B(10, 0.4)	B1B1 M1	Both: B2. Allow π. One error, B1, but x or r: 0. ***SEE NOTES AT START AND END*** B(10, 0.4) stated or implied, e.g. N(4, 2.4) [*P(=1) [=0.0404] or P(≥ 1) [=0.9940] or P(<1) [=0.0060] or Poisson or normal, or RH tail for CR, gets no more marks in (i)*]
	α	$P(\leq 1) = 0.0464$ < 0.05 so reject H_0	A1 A1	This probability or 0.9536 only Explicit comparison with 0.05, or 0.9536 with 0.95
	β	CR is ≤ 1 and compare 1 Probability of this is 0.0464	A1 A1	Comparison needn't be explicit in this method This probability needs to be seen
		Reject H_0. Significant evidence that % who book with travel agents reduced	M1 A1✓ **7**	Correct method, ✓, comparison and first conclusion Interpreted in context, "evidence that" or equiv needed, ✓ on numbers
	(ii)	Can't deduce cause-and-effect	B1 **1**	Equivalent comment, regardless of answer to (i). Ignore wrong answer if right answer seen "Other factors haven't been considered" B1 "Sample is small" , or "test may be wrong" B0

6	(i)	H_0: $\mu = 24.3$; H_1: $\mu \neq 24.3$	B1B1	Both: B2. 1 error, B1, but *t*, *x* etc: B0 ***SEE NOTES AT START AND END***
		$\bar{t} = 26.28$	B1	26.28 seen or implied
		$\hat{\sigma}^2 = \frac{50}{49}\left[\frac{36602.17}{50} - 26.28^2\right]$	M1	Correct formula for biased estimate [= 41.405]
			M1	Multiply by 50/49
				[Single formula: M2, or give M1 if wrong but 49 divisor seen]
		= 42.25	A1	42.25 or 6.5 seen or implied
	α	$z = \frac{26.28 - 24.3}{\sqrt{42.25/50}} = 2.154$	M1	Standardise their $\bar{t}$ with 24.3, √50, allow sign/√/cc errors, their variance
			A1	2.15(4) *or p* in range [0.0153, 0.0158], *not* –2.154 unless 0.015(6) subsequently used, *not* 1-tail
		< 2.576	A1	Compare *z* with ±2.576, or $p > 0.005$, or 2*p* with 0.01, *not* from $\mu = 26.28$
	β	CV $24.3 + 2.576 \times \sqrt{\frac{42.25}{50}}$	M1	24.3 + *zs*/√50, allow cc, √ errors, allow ± but not – only. Not 26.28 – *zs*/√50
			A1	*z* = 2.576, *not* from $\mu = 26.28$ or 50 omitted, *not* from 1-tail
		= 26.67 and 26.28 < 26.67	A1	Correct CV, ✓ on *z*, and compare sample mean
		Do not reject H_0. Insufficient evidence of a change in maximum daily temperature.	M1	Conclusion, ✓, needs method, like-with-like, 50, *not* from $\mu = 26.28$, *doesn't* need correct *z*
			A1✓ **11**	Contextualised, recognise uncertainty, ✓ on numbers
				NB: Clear evidence of $\mu = 26.28$: can't get last 4 marks. *See exemplars γ and δ*
	(ii)	*n* is large	B1 **1**	This answer ***only*** or "*n* > number" where number ≥ 29, *not* both this and "distribution unknown". But "*n* is large so we can approximate even though we don't know the distribution" is B1 "Possible as *n* = 50" B0.
7	(i)	Po(11)	M1	Po(11) stated or clearly implied
		$1 - P(\leq r) = 0.854$ gives $r = 14$	M1	Find 1 – 0.146 in tables, e.g. answer 14 [RH tail, e.g. "7", or single value only: max M1M0A0]
		so $n = 15$	A1 **3**	$n = 15$ only, allow "≥ 15"
	(ii)	Po(44) ≈ N(44, 44)	M1	Normal, mean attempted 2.2×20
			A1	Both parameters 44, allow var = √44 or 44^2
		$\Phi\left(\frac{37.5 - 44}{\sqrt{44}}\right) = \Phi(-0.980)$	M1	Standardise, their 44, allow cc, √ errors, e.g. ans 0.283 or 0.2036 or 0.4411, *not* ÷20
			A1	√ and cc both correct
		= **0.1635**	A1 **5**	Answer in range [0.163, 0.164]
	(iii)	B(40, 0.146)	M1	B(40, 0.146) stated or implied, e.g. by Po(5.84)
		≈ N(5.84, 4.98736)	M1	Normal, attempt at mean = *np* [Poisson etc, or exact binomial (0.22132): no more marks]
			A1	Both parameters correct [Poisson(5.84) → N(5.84, 5.84): M0A0]
		$1 - \Phi\left(\frac{7.5 - 5.84}{\sqrt{4.98736}}\right) = 1 - \Phi(0.7433)$	M1	Standardise with their *np* and *npq*, allow cc, √ errors, e.g. ans 0.3838 or 0.302 or 0.370
			A1	cc and √ both correct
		= **0.2286**	A1 **6**	Answer in range [0.228, 0.229]
				SC: B(40, 0.854) ≈ N(34.16, 4.98736): can get full marks, but if *R* > 7 used, max 3

8	(a) (i)	Several calls may all refer to the same incident	B1	**1**	Any reason showing correct understanding of "independent", but not just "singly" or equivalent. Ignore extra condition(s) unless clearly wrong in which case B0. Not "*fires*" independent. "Fires might spread" B0
	(ii)	Calls occur at constant average rate	B1		This condition only, allow "average" omitted, *not* "constant *probability*", *not* "random" unless clearly correct interpretation follows. No third condition unless fully justified by subsequent answer. Need contextualising *somewhere* in this part.
		E.g. No, because incidents are less/more common at night	B1	**2**	Any comment (with either yes or no) showing correct understanding, but "Fires might not occur at constant average rate" is not enough (gets B1 B0) "Different rates at different times of year": B0
	(b) (i)	$1-\left(1+2.74+\frac{2.74^2}{2!}\right)e^{-2.74}$	M1		Formula for any one correct Poisson probability for $r \geq 1$ [1 – (0.06457 + 0.17692 + 0.24238)]
			M1		Correct overall formula, allow 1 error (e.g. 1 term extra or missing or no "1 –")
		= 0.516(1)	A1	**3**	Answer, a.r.t. 0.516 [Interpolation (0.51604) or no working: B0 or B3]
	(ii)	$(e^{-2}\times1)(e^{-3}\times3) + (e^{-2}\times2)(e^{-3}\times1)$	M1		Two correct terms multiplied, or all 4 bits seen, e.g. .1353×.1494 + .2707×.4979 = 0.0202 + 0.0135
			A1		Correct expression
		= **0.0337**	A1	**3**	Answer, a.r.t. 0.0337
	(iii)	$(e^{-\lambda}\times1)(e^{-\mu}\times\mu) + (e^{-\lambda}\times\lambda)(e^{-\mu}\times1)$	M1		Correct algebraic expression *[Ignore 1! throughout]*
		$= e^{-\lambda}\times e^{-\mu}(\lambda + \mu)$	M1		Take out factor of $e^{-\lambda}\times e^{-\mu}$ or equivalent essential step
		$= e^{-(\lambda+\mu)}(\lambda + \mu)$	A1		Correctly obtain exact answer [allow $e^{-\lambda-\mu}(\lambda + \mu)$]
		$= P(T = 1)$	A1	**4**	All correct, and write down correct formula for $P(T = 1)$ [NB: T needed] Allow working towards middle SR: $\lambda = 2$, $\mu = 3$: Can get M1M1A1A0 if e^{-2} and e^{-3} retained. As soon as decimal approximations seen, no more marks.

Specific examples for question 5(i)

α	H_0: $p = 0.4$; H_1: $p < 0.4$ N(4, 2.4) P(≤ 1) = 0.0533 > 0.05 So do not reject H_0. Insufficient evidence that % who book with travel agents reduced	B1B1 M1 A0 M0 **3**	δ	H_0: $p = 0.4$; H_1: $p < 0.4$ B(10, 0.4) P(≥ 1) = 0.9939 > 0.95 So reject H_0 Insufficient evidence that % who book with travel agents reduced	B1B1 M1 A0 A0 M0 A0 **3**
β	H_0: $p = 0.4$; H_1: $p < 0.4$ B(10, 0.4) "P(= 1) = 0.0464" *[allow this]* < 0.05 So reject H_0 Insufficient evidence that % who book with travel agents reduced	B1B1 M1 A1 A1 M1 A0 **6**	ε	H_0: $p = 0.4$; H_1: $p \neq 0.4$ *[two-tailed]* B(10, 0.4) "P(= 1) = 0.0464" > 0.025 So do not reject H_0 Insufficient evidence that % who book with travel agents reduced	B1B0 M1 A1 A0 M1 A1 **5**
γ	H_0: $p = 0.4$; H_1: $p < 0.4$ B(10, 0.4) P(= 1) = 0.0404 *[look out for this]* < 0.05 so reject H_0 Significant evidence that % who book with travel agents reduced	B1B1 M1 A0 A0 M0 A0 **3**	ζ	H_0: $p = 0.4$; H_1: $p < 0.4$ B(10, 0.4) P(= 1) = 0.0464 *[no explicit comparison]* So reject H_0. Significant evidence that % who book with travel agents reduced	B1B1 M1 A1 A0 M1 A1 **6**

Specific examples for question 6(i)

α	H_0: $\bar{t} = 24.3$; H_1: $\bar{t} \neq 24.3$ *[wrong symbol]* $\bar{t}$ not seen explicitly *[implied by …]* $\hat{\sigma}^2 = \left[\frac{36602.17}{50} - 26.28^2\right] = 41.405$ *[biased est]* $z = \frac{26.28 - 24.3}{\sqrt{41.405/50}} = 2.1758$ < 2.576 Accept H_0, maximum temp unchanged *[over-assertive, otherwise A1]*	B0B0 B1 M1 M0 A0 M1 A0 A1 M1A0 **5**	δ	$H_0 = 24.3$; $H_1 \neq 24.3$ *[missing symbol]* $\bar{t} = 26.28$ $\hat{\sigma}^2 = ... = 42.25$ $z = \frac{24.3 - 26.28}{\sqrt{42.25/50}} = -2.154$ *[loses 1]* > -2.576 Insufficient evidence to reject H_0. No change in maximum daily temperature. *[OK]*	B1 only B1 M1M1 A1 M1 A0 A1 M1 A1 **9**
β	H_0: $\mu = 26.28$; H_1: $\mu \neq 26.28$ *[WRONG]* $\bar{t} = 24.3$ *[explicitly]* $\hat{\sigma}^2 = ... = 42.25$ $z = \frac{26.28 - 24.3}{\sqrt{42.25/50}} = 2.154$ *[allow this – BOD]* < 2.576 Accept H_0. Insufficient evidence of a change in maximum daily temperature.	B0B0 B0 M1M1 A1 M1 A1 A1 M1 A1 **8**	ε	H_0: $\mu = 24.3$; H_1: $\mu > 24.3$ *[one-tail]* $\bar{t} = 26.28$ $\hat{\sigma}^2 = ... = 42.25$ $z = \frac{26.28 - 24.3}{\sqrt{42.25/50}} = 2.154$ < 2.326 Accept H_0. Insufficient evidence of a change in maximum daily temperature.	B1B0 B1 M1M1 A1 M1 A1 A0 M1 A1 **9**
γ	H_0: $\mu = 26.28$; H_1: $\mu \neq 26.28$ *[WRONG]* $\bar{t}$ not seen separately *[implied]* $\hat{\sigma}^2 = ... = 42.25$ $z = \frac{24.3 - 26.28}{\sqrt{42.25/50}} = -2.154$ *[DON'T allow this]* > -2.576 Accept H_0. Insufficient evidence of a change in maximum daily temperature.	B0B0 B1 M1M1 A1 M1 A0 A0 M0 A0 **5**	ζ	$z = \frac{24.3 - 26.28}{\sqrt{42.25/50}} = -2.154$ *but then…* So $p = 0.0156 > 0.005$ *[OK here]* Accept H_0. Insufficient evidence of a change in maximum daily temperature.	M1 A1 A1 M1 A1 **(11)**
			η	$z = \frac{26.28 - 24.3}{\sqrt{42.25}} = 0.3046$ *[no √50]* < 2.576 Accept H_0. Insufficient evidence of a change in maximum daily temperature.	M0 A0 A0 M0 A0 **(6)**

The following guidance notes are provided.

1 Standardisation using the normal distribution.

(a) In *stating* parameters of normal distributions, don't worry about the difference between σ and σ^2, so allow N(9, 16) or N(9, 4^2) or N(9, 4). When *calculating* $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$, the following mistakes are accuracy mistakes and not method mistakes so can generally score M1A0:
confusion of σ with σ^2 or √σ ; *n* versus √*n* ; wrong or no continuity corrections.

(b) Use of $\frac{\mu-\bar{x}}{\sigma}$ instead of $\frac{\bar{x}-\mu}{\sigma}$ is not penalised if it leads to a correct probability, but if the candidate is using a *z*-value in a hypothesis test, an answer of *z* = –2.15 when it ought to be 2.15 is an accuracy error and loses the relevant A1. When finding μ or σ^2 from probabilities, some candidates are taught to use $\frac{\mu-\bar{x}}{\sigma}$ whenever $\mu > \bar{x}$; provided the signs are consistent this gains full marks.

(c) Some candidates are taught to calculate, for example, P(*X* > 5) from N(9, 16) by calculating instead P(*X* < 13). This is a correct method, though it looks very strange the first time you see it.

(d) When calculating normal approximations to binomial or Poisson, use of the wrong, or no, continuity correction generally loses the last two marks: A0 A0.

2 Conclusions to hypothesis tests. There are generally 2 marks for these.

(a) In order to gain M1, candidates must not only say the correct "Reject/do not reject H_0" but have done the whole test in essence correctly apart from numerical errors. In other words, they must have compared their *p* value with a critical *p* value or other "like-with-like" (e.g. *not* say 0.0234 with 1.96), using the correct tail (e.g. *not* –2.61 with +2.576), and the working should in general have accuracy errors only. Thus miscalculation of *z*, comparison with 1.645 instead of 1.96, or using *n* instead of √*n*, or omission of a continuity correction when it is necessary, are all accuracy errors and the candidate can still gain the last M1 A1. Omission of √*n* where it is necessary is a method mistake and so gets M0. In hypothesis tests using discrete distributions, use of P(≤ 12) or P(> 12) or P(= 12) when it should be P(≥ 12) is a method mistake and usually loses all the final marks in a question.

(b) The A1 mark is for interpreting the answer *in the context of the question*, and *without over-assertiveness*. Thus "The mean number of applicants has increased" is over-assertive and gets A0 (although we allow "There is sufficient evidence to reject H_0. The mean number of applicants has increased", A1), and "There is sufficient evidence that the mean has increased" is not contextualised, so that too is A0.

(c) A wrong statement such as –2.61 > –2.576 generally gets B0 for comparison but can get the subsequent M1A1. Otherwise:

(d) If there is a self-contradiction, award M1 only if "Reject/Accept H_0" is consistent with their comparison. Thus if, say, we had $z = 2.61 > z_{crit} = 2.576$:
"Reject H_0, there is insufficient evidence that the mean number of … has changed" is M1A0.
but "Do not reject H_0, there is evidence that the mean number of … has changed" is M0A0.

(e) We don't usually worry about differences between "Reject H_0" and "Accept H_1" etc.

Question			Answer	Marks	Guidance
1			$\hat{\mu} = \bar{x} = 3.65$	B1	3.65 stated explicitly, *not* isw
			$S^2 = \frac{739.625}{50} - 3.65^2$ [= 1.47]	M1	Correct formula for biased estimate used, award if 1.47 seen
			$\hat{\sigma}^2 = \frac{50}{49}S^2$	M1	$n/(n-1)$ factor used, or if wrong single formula, M1 if $n-1$ divisor anywhere. Correct single formula: M2
			= 1.5	A1 **[4]**	Answer 1.5 or exact equivalent only
2			Po(4.2)	M1	Po(np) stated or implied
			$e^{-4.2}\frac{4.2^5}{5!}$ **= 0.1633**	M1 A1	Poisson formula or tables, allow for .1944, .1144, .16(0), .1663; Answer, a.r.t. 0.163
			n large, p small	B1	One condition ⎫ Needs Poisson. If inequalities
			or $n > 50$, $np < 5$	B1	The other condition ⎭ used, must be these, but allow $p < 0.1$ if $n > 50$ already stated
			and nothing extraneous	**[5]**	
3			μ = 60	B1	μ = 60 stated or implied, can be written down
			$\frac{63.8-\mu}{\sigma} = \Phi^{-1}(0.9) = 1.282$	M1 B1	Standardise 63.8 or 56.2 with σ, allow √ or cc errors, equate to Φ^{-1} 1.282 (or 1.281 or 1.28) seen
			σ = 2.96(4)	A1	σ, in range [2.96, 2.97], can be implied by what follows, *not* σ^2
			$1-\Phi\left(\frac{65-60}{2.964}\right)$ = 1 – Φ(1.687)	M1	Standardise 65 with their μ and σ, allow √ or cc errors
			= 0.0458	A1 **[6]**	Final answer, a.r.t. 0.046, c.w.o.
4			N(2.5, 0.025)	M1	Normal (any – can be implied by standardisation)
			$\Phi\left(\frac{2.59-2.5}{\sqrt{0.025}}\right)$ = Φ(0.5692)	A1 A1 A1	Mean 2.5 Variance or SD 1.25 ÷ 50 stated or used Standardise 2.59 or 2.61, with √(1.25/50)
			= **0.7154**	A1 **[5]**	Answer in range [0.715, 0.716] *or* [0.736, 0.737] from 0.632

5	(i)	(a)	${}^6C_4\ 0.6^4\ 0.4^2$ = **0.311**[04] [= 972/3125]	M1 A1 **[2]**	This formula, allow $0.6 \leftrightarrow 0.4$, or tables used correctly Final answer, exact fraction or a.r.t. 0.311
5	(i)	(b)	${}^{12}C_4 \times {}^8C_2 \div {}^{20}C_6$ [= 495×28 ÷ 38760] **= 0.3576** [= 231/646]	M1 A1 **[2]**	Product of two ${}^nC_r$ divided by ${}^nC_r$, *or* ${}^6C_2\left(\frac{12}{20}\times\frac{11}{19}\times\frac{10}{18}\times\frac{9}{17}\times\frac{8}{16}\times\frac{7}{15}\right)$ Final answer, exact fraction or a.r.t. 0.358
5	(ii)		B(60, 0.6) ≈ N(36, 14.4) $1-\Phi\left(\frac{29.5-36}{\sqrt{14.4}}\right) = 1-\Phi(-1.713)$ **= 0.9567** No effect as population is large *or* yes but not by much	B1 B1 M1 A1 A1 B1 **[6]**	N(36, …) or N(24, …); 14.4 or √14.4, both from B(60, 0.6) Standardise with their *np* and √*npq* (or *npq*) Both their √*npq* and cc correct [30.5 if using 24] Answer in range [0.956, 0.957] Need all of one of these [*not* "sample"], or equiv, nothing wrong
6			$H_0: \lambda = 6.3$ [*or* μ] $H_1: \lambda < 6.3$ [*or* μ] $P(\leq 2) = e^{-6.3}(1 + 6.3 + 19.845)$ **= 0.0498** < 0.05 Therefore reject H_0. Significant evidence that average number of pips has been reduced.	B2 M1 A1 A1 B1 M1 A1√ **[8]**	Both: B2. One error e.g. "$H_0 = 6.3$", or "$H_1: \lambda \neq 6.3$", B1, but *x*, *r* etc: 0 Correct formula for at least 2 terms, can be implied by 0.0134 Fully correct formula for ≤ 2, can be implied by answer Answer, a.r.t. 0.0498 SR tables: B2 if a.r.t. 0.0506, else 0 [then can get B1M1A1] Explicitly state < 0.05, *not* from $H_1: \lambda \neq 6.3$, *or* CR ≤ 2 and explicitly state 2 in CR, needs essentially correct distribution *Not needed for final M1A1* Correct method, comparison and first conclusion Interpreted in context acknowledging uncertainty somewhere, √ on *p* etc SR: P(< 2) [0.0134] or Po(= 2) [0.0364]: B2 M1 A0 B0 M0 but *allow* "Po(= 2) = 0.0498" etc SR: Normal: B2 M1 A0 B0

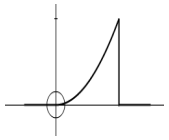
7	(i)	(a)	$\int_1^4 \frac{1}{2\sqrt{x}} x dx = \left[\frac{1}{3}x^{\frac{3}{2}}\right]_1^4$ = 7/3 or 2.333…	M1 B1 A1 **[3]**	Attempt to integrate $xf(x)$, correct limits Correct indefinite integral, a.e.f. Final answer 7/3 or equiv or a.r.t. 2.33
7	(i)	(b)	$\int_1^m \frac{1}{2\sqrt{x}} dx = 0.5$ $\sqrt{m} - 1 = 0.5$ $\boldsymbol{m = 2.25}$	M1 A1 A1 **[3]**	This or complementary integral, limits needed [not "$-\infty$"], equated to 0.5, needn't attempt to evaluate This equation, any equivalent simplified form Answer 9/4 or exact equivalent only
7	(ii)		$1.5\int_1^\infty y^{-2.5} y^2 dx = 1.5\left[\frac{y^{0.5}}{0.5}\right]_1^\infty$ Upper limit gives infinite answer	M1 B1 A1 **[3]**	Attempt to integrate $y^2f(y)$, limits 1 and ∞, allow any letter Correct indefinite integral [$=3\sqrt{y}$], ignore μ [= 3] Give correct reason, c.w.o. apart from constant, allow "$= \infty$"
8	(i)		Location of bacteria must be independent – the position of one does not affect that of another ***Examples*** α Number of bacteria occurring in a particular volume is independent of the number in another interval of the same volume. Number in one volume occurs randomly. M1A0 β Bacteria are distributed independently from one another. This means that they cannot be in groups. M1A0 γ Position of each bacterium must be independent of the position of other bacteria. Not well modelled by Poisson if they tended to form groups, they must not be influenced by the surrounding bacteria or certain conditions (e,g, heat). M1A0 δ Bacteria need to be independent. The results of one cannot influence the result of another. M1A0 ε Bacteria must occur independently, so the state of one bacterium has no effect on any other bacteria. M1A0 ς Probability of bacteria must be independent, they cannot affect the probability of another bacterium occurring. M1A1 η Bacteria must occur independently, so if one occurs it can't cause more to appear. M1A1	M1 A1 **[2]**	"Found independently": M1. Allow "are independent", "singly". Context needed somewhere in answer. Correct explanation, not just of "singly", e.g. not "must not group together". No extra or wrong conditions, but allow both "singly" and "independently". Right explanation, not "independent": M1A0

8	(ii)		$1 - P(\leq 4)$ [= 1 – 0.8912] **= 0.1088**	M1 A1 **[2]**	Allow M1 for 1 – .9580 [= 0.042] or wrong λ. 0.8912 etc: M0 0.109 or 0.1088 or better
8	(iii)		Po(0.925) $e^{-0.925}\frac{0.925^2}{2!}$ **= 0.169**(64)	M1 M1 A1 **[3]**	Po(0.925) stated or implied [37/40] Correct Po formula for $r = 2$, any λ, can be implied by: Answer 0.17(0) or 0.1696 or better
8	(iv)		Po(250) $\lambda > 15$ or λ large [or μ] N(250, 250) $\Phi\left(\frac{239.5-250}{\sqrt{250}}\right) = 1-\Phi(0.664)$ = **0.2533**	B1 B1 M1* A1√ Dep*M1 A1√ A1 **[7]**	Po(250) stated or implied Either of these N, mean their 100 × 2.5 … … variance (or SD) their mean Standardise, allow wrong or no cc and/or no √ or σ^2, needs A1 Continuity correction and √ correct Final answer a.r.t. 0.253, c.w.o.
9	(i)		H_0: $\mu = 8$; H_1: $\mu \neq 8$ where μ is the population mean amount of sleep obtained by Year 11 pupils	B2 B1 **[3]**	Both, B2. One error, B1, allow *x*/*r*/*t* here, but not $\overline{H}$ Need "population" or equivalent, but allow "average amount of sleep obtained by Year 11 pupils". Allow "μ is population mean".
9	(ii)		$\Phi\left(\frac{0.28}{0.87/\sqrt{64}}\right) = \Phi(2.575)$ 2 × (1 – above) = **0.01** or **1%**	M1 A1 M1 A1 **[4]**	Standardise, with √*n* or *n*, allow cc, √ errors *z* = 2.575 or 2.57 or 2.58, can be implied by, e.g., 0.005 or 0.995 Correct handling of tails Answer 0.01 or 1% correct to 2 SF, c.w.o.
9	(iii)		Rejecting H_0 when $\mu = 8$	B1 **[1]**	Or equivalent, some mention of context, *not* "probability of …"
9	(iv)		$\Phi\left(\frac{8.28-7.9}{0.87/\sqrt{64}}\right) - \Phi\left(\frac{7.72-7.9}{0.87/\sqrt{64}}\right)$ = Φ(3.494) – Φ(–1.655) [= 0.99976 – (1 – 0.951) or 1 – …] **= 0.951**	M1 M1 A1 **[3]**	Find P(between 7.72 and 8.28 | $\mu = 7.9$), allow 1 – 2×P(1 tail) (need attempt to find correct region, *not* isw – i.e., *not* ans 0.049) Correct handling of tails, needn't attempt to evaluate, needs 64 Final answer, a.r.t. 0.951. SR: One tail only used: M1M0A0. 0.951 from no working: B2

Question			Answer	Marks	Guidance	
1			Number CDs (sequentially) Select using random numbers	B1 B1 **[2]**	List needn't be stated, but must mention CDs. *Not* "select numbers randomly". Hat, etc: B1B0 Systematic: 66 or 67 B1, random start B1	Assume sequential unless stated otherwise. If "number CDs randomly", B1 max unless "sort by number". Stratified: apply scheme
2	(i)		$\left(\frac{71.2-72.0}{\sigma/\sqrt{40}}\right) = -0.3853$ [σ = 13.13,] Var(V) = **172.4**	M1 A1 B1 A1 **[4]**	Standardise with Φ^{-1} & $\sqrt{40}$, allow cc, $\sqrt{}$ errors eg σ^2 Square roots and sign correct, no cc, no "1 –" error z in range (±) [0.385, 0.386] seen *Final* answer in range [172, 173], or 13.1^2 cwo	RHS must be Φ^{-1}, i.e. *not* 0.7411 or 0.2589 or 0.6368 or 0.35. "1 –" error or ×40/39: M1A0 [0.674 may be from "1 – 0.35 = 0.75"] Needs variance, not SD NB: Look out for –13.1 → 172, M1A0B1A0
2	(ii)		Parent distribution not known n is large	B1 B1 **[2]**	Or clear equivalent. Not "*sample* not normal" Or clear equiv, e.g. sample size > 30. Extras: max 1 "n large, $n > n_0$": B1 if $n_0 \geq 30$.	Don't bother about order of these statements. If numerical must be 30. Ignore "continuous".
3			H_0: $p = \frac{1}{3}$ [or 0.33 or better] H_1: $p \neq \frac{1}{3}$ [or 0.33 or better]	B2	Allow π, but $\mu = \frac{1}{3}$ etc B1. Any other letter, B0 One-tailed, or no symbol, B1 max	Not $\mu = 4$ *(if in doubt, consult)*
		α:	B(12, ⅓) stated or implied P(≥ 7) = 1 – 0.9336 = 0.0664 > 0.025	M1 A1 A1	B(12, ⅓) stated or implied, allow for N(4,8/3), Po(4) Probability in range [0.066, 0.067] Explicit comparison with 0.025, or 2p with 0.05	*If N used, or P(≤ 7) or P(= 7), no more marks* 1-tailed: A0 here regardless of value
		β:	CR is ≥ 8, 7 not in CR Probability is 0.0188	A1 A1	Needs explicit comparison of 7 with CV Must be ≥ 7, 0.019 or 0.0188 or better, allow 0.9812	Need to be clear that CR is being used – look for comparison with 7. See also ζ.
			Do not reject H_0. Insufficient evidence that statement is false.	M1 A1√ **[7]**	Needs correct method, including like-with-like, correct tail, ≥ 7 (or ≤ 6). If CV, needs right tail A1 needs "evidence" or equivalent. "Statement" is enough context here	Allow from 1-tail. 0.9812 or 0.0188 or 0.0476: M0 unless "≥ 7" stated or clearly using β. √ on their p/CR. Withhold if answer refers only to p.

Question			Answer	Marks	Guidance	
4	(i)		Crystals must occur independently of one another	B1 **[1]**	Allow interpreted, or "randomly" but nothing else. Must be contextualised; no other answers included.	Ignore "singly" (meaningless in this context). But allow "probability… is independent"
4	(ii)		$e^{-3.2}\frac{3.2^5}{5!}$ **= 0.114(0)**	M1 A1 **[2]**	Formula, or .0608 or .1781 or .1075 or .1203 (tables) Answer a.r.t. 0.114, implies both marks	
4	(iii)		Po(2.368) $1-e^{-2.368}(1+2.368+\frac{2.368^2}{2})$ **= 0.4219**	M1 M1 A1 **[3]**	Po(0.74 × 3.2) stated or implied 1 – correct Poisson terms, their λ, allow ± 1 term Answer, a.r.t. 0.422, implies all 3 marks	Allow for 0.75 × 3.2 etc, e.g. Po(2.4) Don't allow second M1 from λ in tables, e.g. if MR, treat as E–1. If no working: don't give M1A0
4	(iv)		Po(32) ≈ N(32, 32) $1-\Phi\left(\frac{35.5-32}{\sqrt{32}}\right)$ = 1 – Φ(.619) = **0.2681**	M1 A1 M1 A1 A1 **[5]**	N(λ, λ) stated or implied, allow $\sqrt{\lambda}$ or λ^2 for var N(32, 32), allow $\sqrt{32}$ or 32^2 for var Standardise with λ and $\sqrt{\lambda}$ or λ, allow cc errors but not $\sqrt{n}$; both cc and $\sqrt{}$ correct Final answer, a.r.t. 0.268	Needs $\lambda \geq 15$ Can get (M0A0) M1A1 from $\lambda < 15$ Typically, no cc → 0.2203, or 32 → 0.4565, 3/5 (but needs evidence, not just answer)

Question			Answer	Marks	Guidance	
5			H_0: $\mu = 6.1$ H_1: $\mu \neq 6.1$	B2	Both: B2. One error, B1, but $\bar{x}$, x, r etc: 0. 6.2: B0	
			$\hat{\mu} = \bar{x} = 6.2$	B1	6.2 [31/5] seen somewhere (other than hypotheses)	
			$\hat{\sigma}^2 = \frac{80}{79}\left(\frac{3126}{80} - 6.2^2\right) = 0.643$	M1	Correct formula for biased estimate [0.635 or 127/200]	If single formula used, M2 or, if wrong, allow M1 for divisor 79 anywhere
				M1	Divide by 79 somewhere	
		α:		A1	Variance estimate, a.r.t. 0.643, can be implied	[254/395 leading to 127/15800]
			$z = \frac{6.2 - 6.1}{\sqrt{0.643/80}} = 1.115$	M1	Standardise their 6.2 with reasonable variance attempt, needs 80, allow cc	*80 needed, otherwise M0 and no more marks* *If clearly $\mu = 6.2$ used, no more marks*
			[1 – Φ(1.115) = 0.1325>0.05]	A1	z ∈ [1.11, 1.12] (not –) or $p \in$ [0.1323, 0.1333]	A1 uses number used for comparison
			1.115 < 1.645	A1	Compare z with 1.645 (allow –1.645 if $z < 0$) or p *(< 0.5)* with 0.05	Withhold if inequality incorrect or if 1-tailed Must be consistent signs/tails and like-with-like
		β:	CV $6.1 + 1.645 \times \sqrt{\frac{0.643}{80}}$ = 6.247 and 6.2 < 6.247	M1 A1 A1√	6.1 + $z\sqrt{(\sigma^2/80)}$, allow ±, √ errors CV, a.r.t. 6.25, needs z = 1.645, allow biased $\hat{\sigma}^2$ Compare 6.2 with CV from + sign, √ on z (but not σ)	Allow 6.2 – (or ±) but no more marks afterwards If no 79 earlier but used here, recovers M1A1 E.g. 1.96 → 6.276 or 1.282 → 6.215 [gets M1A0A1
			Do not reject H_0. Insufficient evidence that pH value is not 6.1	M1 A1√ **[11]**	Needs essentially correct method and comparison, needs 80 but no need for correct variance Needs context and "evidence" or equivalent, ft on their z/p/CV	First conclusion wrong: M0A0 even if second correct. "1.115 > 1.645 so do not reject H_0" etc: (A0)M1A1
Notes:			Biased estimate used : typically gets B2B1 M1M0A0 M1A0A1 M1A1 [total 8]		$\bar{x}$ and μ interchanged: allow final M1A1 if *anywhere* right, but if always wrong (in hypotheses and z) M0A0. This would typically get B0B0B1 M1M1A1 M1A0A0 M0A0 [total 5]	

Question			Answer	Marks	Guidance	
6	(i)		B(32, 0.4) ≈ N(12.8, 7.68) Valid as 12.8 and 19.2 > 5 $1 - \Phi\left(\frac{17.5 - 12.8}{\sqrt{7.68}}\right)$ [= 1 – Φ(1.696)] **= 0.0449**	B1 M1A1 B1 M1 A1 A1 **[7]**	B(32, 0.4) stated or implied, e.g. by Po(12.8) N(their attempt at *np*, *npq*); N(12.8, 7.68) Or "*n* large and *p* close to 0.5". Not *npq* or 7.68 > 5. Standardise, their *np*,*npq*, allow wrong/no cc or no √ 17.5 and √*npq* correct Answer, a.r.t. 0.045	Poisson [0.09888], or exact [0.046269]:B1max SC: B(12.8, 7.68/32): M1A0 Allow *np* and *nq* both asserted > 5 ÷32: M0
6	(ii)		B(90, 0.01) ≈ Po(0.9) $e^{-0.9}\frac{0.9^3}{3!}$ **= 0.0494**	B1 M1 M1 A1 **[4]**	B(90, 0.01) stated or implied. Po(their attempt at *np*) Correct formula or use of tables, e.g. 0.1646 or 0.0112 Final answer in range [0.049, 0.05) [i.e., *not* 0.05]	Exact [0.049003]: B1 max. Don't treat *p* = 0.1 as MR. If *np* > 5, M0M0 No working, wrong answer ⇒ M0A0, but right answer ⇒ M1A1 provided clearly Po SC: B(90, 0.1), N(9, 8.1), [0.015, 0.016] cwo B2
7	(i)		 Values of *X* close to *a* are more likely than those close to 0	M1 A1 B1 **[3]**	Positive parabola (only), through 0, nothing below *x*-axis Clear truncation at ends Withhold if concept misunderstood. Need to have probability of *values* (not of *occurring*); not just shape. Allow for U-shape but nothing else	*k* < 0: M0 even if *k* > 0 as well. Don't need any scales, vertical line at *a* etc. Can be vertical at *A*, needn't be horizontal at *O*. E.g.: "More likely to *occur* for *x* close to *a*": B0. Ignore extra comments like "exponential"
7	(ii)	(a)	$\int_0^a kx^2\,dx = 1 \Rightarrow k = \frac{3}{a^3}$ $\int_0^a \frac{3}{a^3}x^3\,dx = \frac{9}{2} \Rightarrow a = 6$	M1 A1 M1 A1 A1 A1 **[6]**	Attempt to integrate kx^2, ignore limits Correct limits and equate to 1 Attempt to integrate kx^3, ignore limits Correct limits and equate to 4.5 One correct equation connecting *k* and *a*, can be implied Correctly obtain *a* = 6 only	Must attempt integration Must attempt integration Don't need *k* in terms of *a* here $ka^3 = 3$ or $ka^4 = 18$, a.e. simplified form No marks explicitly for *k* [= 1/72 or 0.01388…]

Question			Answer	Marks	Guidance	
7	(ii)	(b)	$\int_0^6 \frac{1}{72}x^4 \mathrm{d}x$ $[= \frac{108}{5}]$ $21.6 - 4.5^2$ **= 1.35**	M1 A1 A1 **[3]**	Attempt to integrate kx^4, their a, k, can be algebraic Subtract 4.5^2 (given in question) 1.35 or exact equivalent only	Must attempt integration; limits 0, a√ Somewhere [=27/20]
8	(i)		$30 + 1.645 \times \frac{8}{\sqrt{18}} = 33.102$ so CR is $\bar{X} > 33.1$	M1 A1 A1 A1√ **[4]**	30 + z×8/√18, allow √ errors, cc 1.645, requires + only 33.1 a.r.t. 33.10 ≥ their RH CV√, allow ≤ their LH CV *as well*, allow >, allow no letter or X but no other letter	Allow ± but not – only. No 18: 0 in this part. Don't allow "accept if ≤ 33.1, reject if > 33.1" Inequality required in final line
	(ii)		Type I [error]	B1 **[1]**	Nothing else unless it's just an amplification. Allow "Type 1"	
	(iii)		B(20, 0.05): P(≥ 4) = 0.0159 so unlikely that $\mu = 30$	M1 A1 A1√ **[3]**	B(20, 0.05) stated or implied. Not B(20, 1/5) Probability, a.r.t. 0.016 Justified conclusion, e.g. "I think $\mu = 30$ as not less than 0.01". FT on their p.	No reason: A0. Not over-assertive. But "I think $\mu = 30$ as probability is small" is A0.
	(iv)		$\frac{33.1 - \mu}{8/\sqrt{18}} = -0.253$ μ = **33.58**	M1 A1 A1 A1 **[4]**	Needs Φ^{-1}, their CV, SD right or same as in (i), allow cc Signs correct, can be implied by answer > their CV z in range (±)[0.25, 0.26] Final answer 33.55 ≤ μ ≤ 33.60, 4 SF needed.	*Not* 30. Allow omission of √18 only if omitted in (i). "1 –" errors: can get M1A0A1 Typically 32.62 probably gets 2/4.

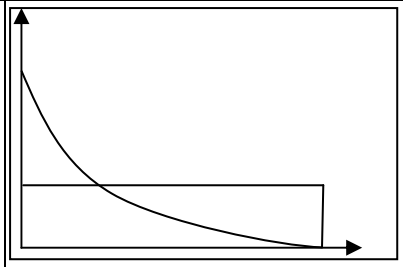
Question			Answer	Marks	Guidance	
1			$n = 9$ CR is ≤ 2 **0.0083**	B1 M1A1 A1 **[4]**	Stated explicitly 2 seen but not ≤: M1A0. Allow "P(≤ 2)" Or more SF. "$n = 9$, CR ≥ 3", 0.0083 seen: B1M1A0A1	CR must be stated explicitly for A1 SR: ≤ 3 with 0.0424: (B1)M1A0 SR: If 0, give B1 for at least 3 of 0.0083, 0.0113, 0.0026, 0.0197, 0.0034 seen
2	(i)		$\hat{\mu} = \bar{x} = 38$ $\frac{\Sigma x^2}{10} - 38^2 \quad [= 16.2]$ ×10/9 to get **18**	B1 M1 M1 A1 **[4]**	38 stated separately Use of $\Sigma x^2/n - \bar{x}^2$ Multiply by 10/9 18 or a.r.t. 18.0 only	Correct single formula: M2 If single formula, divisor of 9 seen anywhere gets second M1
2	(ii)		$\Phi\left(\frac{40 - 38}{\sqrt{18}}\right) = \Phi(0.4714) =$ **0.3187**	M1 A1 **[2]**	Standardise with their μ and σ, allow cc, √ errors Answer, a.r.t. 0.319	√10 used: M0. Allow a.r.t. 0.311 [0.3106] from 16.2
3	(i)		Allocate 4-digit number to each DVD; Select using random numbers Ignore random numbers outside range	B1 B1 B1 **[3]**	"DVD" & "4 digits/1 to 9000/sequentially" etc must be mentioned *somewhere* Mention random numbers Unbiased method, mention of "outside range" or "repeats"	*Not* allocate "random" numbers, unless subsequently sorted If "pick random numbers in range 1 to 9000", must mention repeats
3	(ii)		B(100, 0.24) ≈ N(24, 18.24) $\Phi\left(\frac{19.5 - 24}{\sqrt{18.24}}\right) = \Phi(-1.0537)$ = **0.1461**	M1 A1 M1 A1 A1 **[5]**	N(attempt at *np*) Both parameters correct Standardise with their *np* and √*npq* or *npq* Both cc correct and √*npq* used Answer, a.r.t. 0.146	Allow 18.24/100 A1 but then M0A0 Allow cc/√ errors.

Question			Answer	Marks	Guidance	
4	(i)		Values taken by X	B1 **[1]**	This answer only	Not "values taken by f"
4	(ii)		$\int_0^a kx\,dx = 1 \Rightarrow k = \frac{2}{a^2}$	M1 A1 **[2]**	Use definite integral and equate to 1, Correctly obtain $2/a^2$	*Or* clear argument from triangle area
4	(iii)		$\int_0^a kx^2\,dx = \left[k\frac{x^3}{3}\right]_0^a = \frac{2}{3}a$ $\int_0^a kx^3\,dx = \left[k\frac{x^4}{4}\right]_0^a = \frac{a^2}{2}$ $\frac{a^2}{2} - \left(\frac{2}{3}a\right)^2 = \frac{1}{18}a^2$	M1 B1 A1√ M1* depM1 A1 **[6]**	Attempt to integrate xf(x), limits 0 and a Correct indefinite integral seen Correct mean *or* correct E(X^2) [= $a^2/2$], √ on k Attempt to integrate x^2f(x), limits 0, a Subtract their μ^2 Correct final answer, ae exact f, no k now	either here or for x^2f(x) Can be in terms of k Or decimal, $0.056a^2$ or better
5	(i)		Po(4200) ≈ N(4200, 4200) $1 - \Phi\left(\frac{4350.5 - 4200}{\sqrt{4200}}\right)$ = 1 – Φ(2.322) = **0.010**(1)	M1 M1 M1 A1 A1 **[5]**	Po(60λ) stated or implied N(60λ, 60λ) Standardise with their 60λ and $\sqrt{60\lambda}$ or 60λ 4350.5 explicitly seen and $\sqrt{60\lambda}$ not wrong Answer, allow a.r.t. 0.010	 Allow wrong or no cc, or no √ $\sqrt{60\lambda}$ needn't be explicit Allow [0.0103, 0.0106] from no CC, but *not* 0.0105 from wrong CC
5	(ii)		B(30, 0.010(1)) ≈ Po(0.30(3)) 1 – 0.9997 = **0.0003** *or:* $1 - (q^{30} + 30q^{29}p + 435q^{28}p^2 + 4060q^{27}q^3)$ = 1 – (.7397 + .2242 + .0328 + .0031) = 1 – .999777 = 0.0002226	M1 A1 A1 **[3]**	B(30, their (i)) stated or implied Po(0.3) or 0.303 etc Final answer a.r.t. 0.0003 Exact binomial: 1 – (3,4 or 5 terms) (M1)M1 Answer a.r.t. 0.0002: A1 Normal (0.3, 0.297) (M1)M1 Answer 0 (4 dp) (z = 5.87) A1	 [0.30→0.000266. 0.303→0.000276. 0.309→0.000297] Needs clear ${}^nC_r$ or right answer No mention of dist: assume exact

Question			Answer	Marks	Guidance	
6	(i)		H_0: $\mu = 28.0$ H_1: $\mu > 28.0$ α: $\frac{28.98-28}{12/\sqrt{30}} = 0.4473$ [$p = 0.3274$] $z < 1.645$, or $p > 0.05$ OR: CC: $28.98 - \frac{1}{60} \rightarrow 0.4397$, $p = 0.33$	B2 M1 A1 A1	One error, e.g. p, or μ_0, μ_1, or 2-tail: B1. Standardise with √30, allow √ errors, cc Correct value of z or p: z = art 0.447 or p in range [0.327, 0.328] Compare z (incl 30) with 1.645, or p with 0.05, or with 0.95 if correct tail	But $\bar{x}$ etc: B0 CC is CORRECT here *Not* –0.447 but can be recovered if 0.327 used. Not 0.455/0.3246 Needs μ and $\bar{x}$ right way round
			β: $28 + 1.645 \times 12/\sqrt{30}$ $= 31.6$ $28.98 < 31.6$	M1 A1√ A1	$28 + z \times 12/\sqrt{30}$, allow √ errors, cc Correct CV, √ on z (only) Explicitly compare 28.98	Ignore 28 – …, do not allow 28.98 – ...
			γ: Totals used: $\frac{869.4-840}{12\sqrt{30}} = 0.4473$		Same scheme	NB: If totals used, allow ANY plausible CC or none
			Do not reject H_0. Insufficient evidence of an increase in mean score SD unchanged, *or* random sample/indept	M1 A1 B1 **[8]**	Consistent first conclusion Contextualised, "evidence" or exact equivalent somewhere One of these seen, nothing irrelevant	Needs correct method & comparison, 30 used, μ and $\bar{x}$ right way round "Evidence" in either part of conclusion
6	(ii)		Yes because population not stated to be normal	B2 **[2]**	Partial answer: B1 "Yes as parent distribution not normal" (i.e., "stated to be" omitted): B2 SR: "No as assumed normal" if in (i): B1	"Yes, because n large": B1 "Yes, as not normal and n large": B1 "Yes as not normal, but can be used as n large": B2
7	(i)		$\frac{\mu-20}{\sigma/\sqrt{n}} = 1.0$; $\frac{35-\mu}{\sigma/\sqrt{n}} = 2.0$ Solve to get $\sigma = 5\sqrt{n}$	M1 A1 B1 M1 A1 **[5]**	Standardise either 20 or 35, equate to Φ^{-1} Both equations completely correct Both correct z-values seen (to 3 SF at least) Correctly obtain $\sigma = k\sqrt{n}$ or $\sigma^2 = kn$ $\sigma = 5\sqrt{n}$ or $\sqrt{25n}$ only.	With $\sqrt{n}$ or n and z, allow "1 –", cc Including signs, but can have wrong z Independent of previous marks Allow √ errors, ALLOW from not Φ^{-1} [only mark from 0.7998 & 0.8358]
7	(ii)		Binya is right $\mu = 25$ $1 - \Phi\left(\frac{32-\mu}{5}\right) = 1 - \Phi(1.4)$ $= 1 - 0.9192$ = **0.0808**	B1 B1 M1 A1 **[4]**	Binya stated $\mu = 25$ following no wrong working Standardise with their $\sigma/\sqrt{n}$ and their numerical μ Answer, a.r.t. 0.081, CWO.	"Aidan" used: max B0B1M0 But allow if $\sqrt{n}$ omitted or wrong NB: use of 1.282 probably implies "Aidan"

Question			Answer	Marks	Guidance	
8	(i)		Failures do not occur at regular or predictable intervals	B1 **[1]**	*Not* equivalent of "independent". *Not* "equally likely at any moment"	Both right and wrong: B0
8	(ii)		Failures occur independently; Might not happen if a power cut … and at constant average rate; Might not happen if manipulated to change more rapidly at peak times	B1 B1 B1 B1 **[4]**	"Failures" needed in one reason, else B0(B3) Plausible reason Exact equivalents only Must be during one day and not week/year Allow any answers that show correct statistical understanding, however implausible	*Not* "randomly", allow "singly" only if also "independent" in this part Not "equal probability", *not* "constant rate", but allow second mark if OK. Extra wrong reason loses explanation mark
8	(iii)		$e^{-\lambda}\frac{\lambda^7}{7!} = e^{-\lambda}\frac{\lambda^8}{8!} \Rightarrow \lambda = 8$ **0.1396**	M1 A1 M1 A1 B1√ **[5]**	At least one correct formula Both sides correct Cancel exp and some λ Obtain $\lambda = 8$ only, CWO Answer in range [0.139, 0.14], √ on their λ	 [before rounding]
9	(i)		4.81% or 0.0481	B1 **[1]**	One of these only, or more SF	N(18, 7.2) → 0.0468: B1
9	(ii)		P(≥ 14) = 0.7077	M1 A1 **[2]**	Allow M1 for answer 0.5722 or 0.8192 0.708 or 0.7077 or more SF	0.2923: 0 N(15, 7.5) → 0.78: M1A1; 0.8194 or 0.7674: M1A0
9	(iii)		Only way that $p = 0.5$ for second test is if Type II error on first, where 0.2 × 0.7077 = 0.14154. Therefore 0.14154 × 0.2923 + 0.85846 × 0.0481 = **0.0827**	M1 M1 M2 A1 **[5]**	0.2 × 0.7077 × 0.2923 [= 0.04137] Consider 1 – 0.14154 0.2 × (ii) × (1 – (ii)) + (1 – [0.2 × (ii)]) × (i) [= 0.04137 + 0.04127] Answer, a.r.t. 0.083 OR: 0.8 × 0.0481 × 0.0481 [0.00185] + 0.8 × 0.9519 × 0.0481 [0.03663] M1 + 0.2 × 0.2923 × 0.0481 [0.00281] M1 + 0.2 × 0.7077 × 0.2923 [0.04137] M1 Add up 4 terms of 3 multiplications M1 Answer 0.0827 A1	Normal: 0.1416×0.292 + 0.8584×0.0468 or 0.00175+0.03569+0.00273+0.04135 = 0.0815: full marks Any two of these three M1 Third of these three M1 This one M1 SR: No 0.8 or 0.2 but 2 products: M1 4 products: M2

Question			Answer	Marks	Guidance	
1	(i)		89, 90, 91, 91, 92	B2 **2**	All correct; B2; one error (e.g. all –1), B1	Allow 088, etc
	(ii)		Not all equally likely (91 more than 90 etc) Multiply by 1000 and ignore if > 853	B1 B1 **2**	Imply different likelihood/probability Or equivalent method. Not "ignore repeats". Ignore extras.	*Not* "same pupil *is* selected twice" Number students, use random numbers *and ignore outside range*: B1
2			Po(2×10^6) $\approx$ N(2×10^6, 2×10^6) $\Phi\left(\frac{1998999.5 - 2\times10^6}{\sqrt{2\times10^6}}\right) = \Phi(-0.70746)$ **= 0.2396**	M1 A1 A1 A1 **4**	N(their 40λ) Both parameters correct, allow $\sqrt{}$ here Standardise, mean 40λ, sd $\sqrt{40\lambda}$ (*not* 40λ) Answer, a.r.t. 0.240 (no cc: M1A1A0A1)	*Correct cc must be seen for this A1* NB: no cc gives $\Phi(-0.7071)$, 0.23975, wrong cc gives $\Phi(-0.70675)$, 0.23986
3	(i)		$\frac{\mu - 157.18}{\sigma/\sqrt{80}} = 1.282$; $\frac{\mu - 164.76}{\sigma/\sqrt{80}} = 0.5244$ Solve simultaneously: μ **= 170** σ **= 89.44**	M1 A1 B1 B1 A1 A1 **6**	Standardise once with $\sqrt{80}$ or 80 and z, signs may be wrong, allow "1–" errors Both correct ***including signs***, no cc 1.28(155) seen anywhere, correct to 3 SF [0.524, 0.525] seen anywhere μ, a.r.t. 170 to 3 SF (169.98) σ, in range [89, 90], *not* isw *Don't* allow surds, e.g. $40\sqrt{5}$	Allow cc, but *not* 0.1, 0.7, 0.9, 0.3 or Φ(these) [= .5398, .758, .8159, .6179] z may be wrong (provided it *is* z) Ignore signs Ignore signs CWO×2 but allow from inaccurate z if answer(s) within limits. Look out for –89.44: A0A0
	(ii)	(a)	In using normal tables	B1	Or equiv, e.g. "standardising", "dist of $\overline{Y}$ "	Any reference to $\sigma/\sqrt{80}$: B0
		(b)	Parent distribution not known	B1	Allow "it is not normal", etc	No extras
		(c)	n large, nothing wrong seen [must be in correct order, no repeats]	B1 **3**	If numerical, must be of the form "$n > n_0$" or "$n \geq n_0$" with $30 \leq n_0 \leq 60$	*Not* "≥ 80".

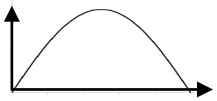
Question			Answer	Marks	Guidance	
4			$H_0: \lambda = 3.2$ (or 0.32) [Allow μ] $H_1: \lambda > 3.2$ (or 0.32) [Allow μ] $R \sim$ Po(3.2) α: P($R \geq 6$) = 0.1054 > 0.01 β: CR ≥ 9 and 6 < 9, with probability 0.0057 Do not reject H_0. Insufficient evidence of an increase in the number of floods.	B2 M1 A1 A1 A1 A1 M1 A1 ft **7**	Both correct, B2. One error, e.g. wrong/ no/different symbols, or two-tail, B1 Stated or implied, e.g. N(3.2, 3.2) [0.105, 0.106] before rounding Explicit comparison with 0.01 CR ≥ 9 stated; allow CV = 9 if comparison ft 0.0057 or 0.9943 seen, and 6 compared Consistent first conclusion Conclusion, mentions "floods", "evidence" *Not* "evidence of no increase" P($R \leq 6$) = 0.9554; P($R > 6$) = 0.0446; P($R = 6$) = 0.0608: max B2 M1 P($R < 6$) = 0.8946 *and* compare 0.99 etc: can get full marks. Else A0A0M0A0	But x, $\bar{x}$, r, t etc: B0. E(X), words: B1 E.g. H_0: $\lambda_0 = 3.2$, H_1: $\lambda_1 > 3.2$: B1 P(= 6) or (≤ 6) or > 6 or normal: no more marks, maximum B2M1. needs correct method and like-with-like comparison, but 0.01 needn't be explicit
5	(i)			M1 A1 B1 **3**	Upwards parabola, not below x-axis Correct place, not extending beyond limits, ignore pointed at a Horizontal straight line, not beyond limits, y-intercept below curve (unless curve makes this meaningless)	*[scales/annotations not needed]* Touching axes (not asymptotic) Don't need vertical lines i.e., 3/3 only if wholly right
	(ii)		$\int_0^a \frac{3}{a^3} x(x-a)^2 \,\mathrm{d}x$ $= \int_0^a \frac{3}{a^3}(x^3 - 2ax^2 + a^2x)\mathrm{d}x$ $= \left[\frac{3}{a^3}\left(\frac{x^4}{4} - \frac{2ax^3}{3} + \frac{a^2x^2}{2}\right)\right]_0^a$ $= \frac{a}{4}$	M1 M1 A1 B1 A1 **5**	Attempt this integral, correct limits seen somewhere Method for $\int xf(x)$, e.g. multiply out or parts, independent of first M1 Correct form for integration, e.g. multiplied out correctly, or correct first stage of parts Correct indefinite integral $\frac{a}{4}$ or exact equivalent (e.g. 0.25a) only	Multiplication: needs 3 terms E.g. $\frac{3}{a^3}x\frac{(x-a)^3}{3} - \int \frac{3}{a^3}\frac{(x-a)^3}{3}\mathrm{d}x$ E.g. $\frac{3}{a^3}x\frac{(x-a)^3}{3} - \frac{3}{a^3}\frac{(x-a)^4}{12}$ Limits not seen anywhere: can get M0M1A0B1A0

Question			Answer	Marks	Guidance	
5	(iii)		*S* is concentrated more towards 0 Therefore *T* has bigger variance	M1 A1 **2**	Reason that shows understanding of PDF Correct conclusion	*Not*, e.g., "*T* is constant"
6			$H_0 : \mu = 38.4$ [Allow E(*X*) both times] $H_1 : \mu \neq 38.4$	B2	Both correct: B2. One error e.g. no or different symbols, one-tail etc, B1	But $\bar{x}$, *x*, *t* etc B0. E.g. H_0: $\mu_0 = 38.4$, H_1: $\mu_1 \neq 38.4$: B1
			$\hat{\mu} = \bar{x} = 36.68$	B1	36.68 seen anywhere	H_0: $\mu = 36.68$, H_1: $\mu \neq 36.68$: B0B0B1 *See below and exemplars*
			$\hat{\sigma}^2 = \frac{50}{49}\left(\frac{70027.37}{50} - 36.68^2\right) = 56.25$	M1 M1 A1	Use biased variance formula [55.125] Multiply by 50/49 56.25	Single formula: M2 or M0. If M0, a divisor of 49 seen anywhere gets M1 Allow rounded if clearly correct
			α: $z = \frac{36.68 - 38.4}{\sqrt{56.25/50}} = -1.62$ > -2.576 [*or* $0.0525 > .005$]	M1 A1 A1ft	Standardise using $\sqrt{50}$ or 50 *z*, a.r.t. –1.62 or *p* = 0.0525 Compare $-z$ with -2.576 or $+z$ with 2.576	If 50 missing, no more marks *p* in range [0.052, 0.053] Ft on *z*. Or *p* explicitly with 0.005
			β: CV is $38.4 - 2.576\sqrt{\frac{56.25}{50}} = 35.6677$ $36.68 > 35.6677$	M1 A1 A1ft	CV $38.4 - z\sigma/\sqrt{50}$, ignore 38.4 + anything A.r.t 35.7 CV ft and correct comparison	$36.68 + z\sigma/\sqrt{50}$: M1A0A0, M0A0 Ft on wrong *z* or on $\sqrt{}$ only
			Do not reject H_0. Insufficient evidence of a change in crop yield	M1 A1ft	Correct first conclusion, needs correct method & comparison if seen Contextualised, "evidence" somewhere *Not* "evidence of no change"	Like-with-like, needs μ and $\bar{x}$ right way round, needs 50 Ft on wrong TS and/or CV
					Biased variance [55.125; –1.638 or 0.0508] can get B2B1 M1M0A0 M1A0A1M1A1 (max 8) σ^2 used [–1.529 or 0.0632, or –0.l2162 or 0.4144]: B2B1 M1M1A1 M1A0A1M1A1 (max 10) No $\sqrt{50}$ [–0.2293 or 0.4092]: B2B1 M1M1A1M0 (max 6) H_0/H_1 in terms of 36.68: can get last 4 marks *only* if (36.68 – 38.4) seen, and not (38.4 – 36.68)	
				11		

Question			Answer	Marks	Guidance	
7			H_0: $p = 0.35$	B2	One error (e.g. μ, no symbol, 2-tailed) B1, but $\bar{x}$, t etc: B0. Allow π	H_0: $\mu = 42$, H_1: $\mu > 42$: B1 only
			H_1: $p > 0.35$			
			B(120, 0.35)	M1	B(120, 0.35) stated or implied	
			≈ N(42, 27.3)	M1	N(*np*, *npq*), their attempt at 120 × 0.35	120 × 0.35 × 0.65 *Not* N(*np*, *nq*).
			α: $z = \frac{49.5 - 42}{\sqrt{27.3}}$	A1ft	Standardise, with their *np* and √*npq*, right cc Allow both 49.5 and 50.5 and both in CR	√50 or √120: M1M1A0A0A1M0A0
			= 1.435	A1	*z* in range [1.43, 1.44] before rounding	Or *p* in range [0.075, 0.0764]
			> 1.282 [or 0.0757 < 0.1]	A1ft	Comparison with 1.282, ft on *z*/*p* or √120	Or *p* explicit comparison with 0.1
			β: CV = 42.5 + 1.282 × √27.3 [= 49.198]	A1ft	CV 42.5 + *z*×√27.3, ignore LH, ft on *np*, *npq*	No cc: 48.618, can get A0A1A0
			z = 1.282 and compare 50	A1	*z* = 1.282 used in RH CV and compare 50	
			CR ≥ 50 or ≥ 49.2	A1ft	CV correct ft on *z*, but don't worry about ≥	Must round up. 49 from 49.2: A1A1A0
			Reject H_0.	M1	Consistent first conclusion, needs correct method and comparison	Can give M1A1 even if comparison not explicit. Allow from exact binomial
			Significant evidence that proportion who know regulations has increased	A1ft	Contextualised, needs "who know regulations" or "pupils", and "evidence"	Ft on TS & CV Or exact equivalent somewhere
			np > 5 [= 42] from normal attempted	M1	From *p* = 0.35 or 5/12, don't need 42	*or n* large or *p* close to 0.5 asserted
			nq = 78 > 5 and no others apart from *n* large	A1	Need 78, or 70 from 5/12, *not npq*	*and* the other qualitative reason asserted
			SC: If B0, B(120, 5/12):		Wrong or no cc [1.627, 0.0519 or 1.5311, 0.0629]: loses (α) first two A1A1 only	
			N(50, 29.17) M1M1		Exact B(120, 0.35): P(≥ 50) = 0.076824, CR ≥ 50. B2M1, M0A0A0A0, M1A1M0A0	
			np > 5, *nq* = 70 > 5: M1A1 Max 4		*NB: If S3 difference of proportions test used, consult PE*	
			SC: P(≥ 42): B2 M1M1A0A0A1M0A0			
				11		
8	**(i)**		B(14, 0.25): Critical region ≥ 7	M1	Use B(14, 0.25) and find *r* for an upper tail *All marks need upper tail*	e.g. CV 5 or 6 or 7, or .1117, .0383, .0103, 0.8883, 0.9617, .9897
				A1	CR ≥ 7 or AR ≤ 6 stated or clearly implied	Not just "CV = 7"
			B(14, 0.4): P(≤ 6)	M1	Find P(in AR when *p* = 0.4) *[indept of M1]*	*Not* P(≥ *r*), e.g. final answer 0.3075
			= 0.6925	A1	Answer 0.692 or 0.693 or a.r.t. 0.6925 or 0.6924 only, *not* isw [0.692452]	NB: expect CV 8 or 9 and answer 0.9825 or 0.9417: M0M0
				4		

Question			Answer	Marks	Guidance	
	(ii)		(a) Decreases	B1	One correct answer & one correct reason *or* two correct answers	Allow from numerical calculation
			(b) Decreases; increased prob (Type I) ⇔ decreased prob (Type II)	B1 **2**	Two correct answers and one correct reason, e.g. "CR becomes larger", etc	Allow equivalent or similar reason Allow from numerical calculation
9	**(i)**		Constant *average* rate; *or* [*] same statement *plus* "breakdowns independent"	B1	State "average" or equiv, "random" or "uniform".	No extras apart from independence (ignore "singly")
			Otherwise it means that they occur at exactly regular intervals	B1 **2**	Correct explanation	Can't get from [*]
	(ii)		No because breakdowns more likely in rush hours, etc	B1 **1**	Any plausible reason for either "yes" or "no" that shows understanding of what the *statistical* concept means	Not "equally likely". *Not* reason for (in)dependence, unless [*], which needs *both* conditions if affirmed
	(iii)		13	B1		
			0.0739	B1 **2**	0.074 or a.r.t. 0.0739. Marks independent	
	(iv)		$e^{-\lambda}\frac{\lambda^2}{2!} = 0.0072$	M1*	Correct formula = their 0.0072 seen	
			$\lambda = \sqrt{(0.0144e^{\lambda})}$	M1dep	Rearrange $e^{-\lambda}$ and square root, to get $\lambda = f(\lambda)$	Allow even if left with e^{λ} or $e^{-\lambda}$ or exact equivalent
			$= 0.12e^{\lambda/2}$	A1	Correctly obtain AG, with $k = 0.5$	
			$8.5 \rightarrow 8.4126$; $8.6 \rightarrow 8.8440$	A1	Two correct evaluations to 4 dp at least	4 dp explicitly required
			Therefore solution between 8.5 and 8.6	A1 **5**	All completely correct and deduction stated	CWO, except allow if only 3 SF

Question		Answer/Indicative content	Marks	Guidance
1		N(35, 10.5) $1-\Phi\left(\frac{40.5-35}{\sqrt{10.5}}\right) = 1-\Phi(1.697)$ $= 1 - 0.9552$ $= \mathbf{0.0448}$	M1 A1 M1 A1 A1 **[5]**	Normal, mean 35 Both parameters correct, allow $\sqrt{10.5}$ or 10.5^2 Standardise, their *np*, *npq*, allow no $\sqrt{}$ or 10.5^2, allow wrong or no cc Both 40.5 and $\sqrt{npq}$ [Ans 0.0448 or 0.9552 can imply first 4 marks] Answer, a.r.t. 0.045. [Exact binomial (0.040232): 0/5]
2	(i)	 *np* = 147 > 5 so not Poisson *nq* = 3 < 5 so not normal	M1 A1 A1 **[3]**	Consider any two conditions, out of *np*, *nq* (allow *npq*), size of *n*, size of *p* 147 stated, or "*p* not small", no wrong conditions for Poisson seen 3 [not *just* 2.94] stated, or "*p* not close to ½", no wrong conditions for normal seen (apart from *npq*) *If spurious extra reasons seen ("not independent" etc), max 2/3*
2	(ii)	$A \sim B(150, 0.98)$ so $150 - A \sim B(150, 0.02)$ $\approx$ Po(3) $P(A < 146) = P(150 - A > 4) = 1 - 0.8153$ $= \mathbf{0.1847}$	M1 A1 M1 A1 **[4]**	Clearly consider complement, with $p = 0.02$ Po(3) stated or implied 1 – Po(3) probability, e.g. 0.3528 or 0.0839 0.185 or better [Exact binomial (0.1830): 0/4. N(3, 2.94): M1A0M0A0]
3	(i)	$\frac{\mu - 40}{\sigma} = 0.9544$	M1 B1 **[2]**	Standardise with μ and σ and equate to Φ^{-1}, allow σ^2 but not $\sqrt{n}$, allow 1–, cc, wrong signs. P(…): M0 here. But can recover both marks from part (ii). [0.954, 0.955] seen
3	(ii)	$\frac{60 - \mu}{\sigma} = 0.674(5)$ Solve to get $\sigma = 12.3$ [12.278] $\mu = 51.7(18)$	M1 B1 A1 A1 **[4]**	Standardise as in (i) but do not give if "1 –" or wrong signs in *either* equation [0.674, 0.675] seen. (Other errors lead to loss of A marks.) σ, a.r.t. 12.3, cwo μ, a.r.t. 51.7, cwo [NB: ***CARE!*** either or both can be obtained from wrong equns.] {note for scoris zoning – (i) to be visible in marking (ii)}
3	(iii)	Based on a sample/small sample, etc	B1 **[1]**	Any similar comment, e.g. "frequencies not probabilities" (but not *just* "*n* is small") *and* no wrong comments. Not "because data is grouped". No scattergun.

4	(i)	Snakes must occur independently of one another	B1 **[1]**	Contextualised ("snakes" must be mentioned); not *just* "singly" but allow both independent and singly. Allow explanation, e.g. "Occurrence of one snake doesn't affect occurrences of others". Allow "snakes must occur randomly". Otherwise, more than one condition, "e.g. "randomly, independently, singly and at constant rate": 0.
4	(ii)	$1 - P(\leq 5)$ $= 1 - 0.7851$ $= \mathbf{0.2149}$	M1 A1 **[2]**	Give M1 for 0.3712, 0.1107 or 0.2307. Answer 0.7851 is M0. Answer, a.r.t. 0.215
4	(iii)	Po(3.08) $e^{-3.08}\left(\frac{3.08^2}{2!}+\frac{3.08^3}{3!}\right)$ [= 0.2180 + 0.2238] $= \mathbf{0.4418}$	M1 M1 A1ft A1 **[4]**	Po(3.08) stated or implied. [Just $\lambda = 3.08$ is M0 unless Poisson later.] Correct formula for Po ($r > 0$) used at least once, can be implied Completely correct formula for their λ (not 4), can be implied Final answer, a.r.t. 0.442 No working: last 3 marks either 0 or 3, no "nearly right".
5	(i)	$\int_0^1 \frac{\pi}{2}\sin(\pi x)\,dx = \left[-\frac{1}{2}\cos(\pi x)\right]_0^1 = \frac{1}{2}-\left(-\frac{1}{2}\right)=1$ and function non-negative for all x in range	M1 B1 A1 B1 **[4]**	Attempt to integrate f(x), limits (0, 1) somewhere, evidence e.g. "from calculator" Correctly integrate $\sin(\pi x)$ to $-\frac{1}{2}\cos(\pi x)$ Fully correct, need to see $-\frac{1}{2}\cos(\pi x)$ and final 1, no wrong working seen Non-negative asserted explicitly, allow positive or equivalent. Not just graph drawn. *(Most will not get this mark!)*
5	(ii)	 E(X) = ½	M1 A1 B1 **[3]**	Correct shape, through 0, allow below axis outside range. Allow partial curve if clearly part of sine curve. Fully correct including no extension beyond [0, 1]. Don't worry about grads at ends. Ignore labelling of axes ½ or 0.5, needs to be simplified, no working needed, *no* ft
5	(iii)	$\int_q^1 \frac{1}{2}\pi\sin(\pi x)dx = 0.75$; $\{\left[-\frac{1}{2}\cos(\pi x)\right]_q^1 = 0.75\}$ $\cos(\pi q) = 0.5$ Solve to get $q = \frac{1}{3}$	M1 A1 A1 **[3]**	Equate integral to correct probability, correct limits *somewhere* allow complementary probability (= 0.25) only if limits (0, q) $\cos(\pi q) = 0.5$ or exact equivalent $q = \frac{1}{3}$ or a.r.t. 0.333. SR: Numerical (no working needed): 0.333 B3, 0.33 B2
5	(iv)	$\int_0^1 \frac{\pi}{2}x^2\sin(\pi x)\,dx - \left(\frac{1}{2}\right)^2$	M1 A1ft **[2]**	Integral part correct, allow limits omitted, ignore dx Subtract their [E(X)]^2, allow μ in form of integral, correct limits needed, not just "μ^2" {note for scoris zoning – (ii) needs to be visible here}
5	(v)	Values of x in range close to E(X) are more likely than those further away	B1 **[1]**	Need to see "values of x" or equivalent, and probably not "occur" *Not* "the probability of x is greater when x is close to E(X)" etc. *Not* "PDF greater …"

6	(i)	Sample is random	B1 **[1]**	Indicate random sample. Allow "unbiased sample" or "randomly selected" or "all equally likely". Allow "representative" provided it's clearly "of company" (not city) Not just "independent". Withhold if extra wrong bits.
6	(ii)	List population, number sequentially Select using random numbers	B1 B1 **[2]**	List can be implied; must imply employees or people. "Sequential" can be assumed. Not "select numbers randomly", Don't need "ignore outside range" etc. Number randomly *and* select randomly, B1, but "assign random nos & arrange", B2 SC: Put names into hat/lottery machine and take them out: B2 SC: Systematic: B1 for list, can get second B1 if starting-point random
6	(iii)	H_0: $p = 0.4$; H_1: $p < 0.4$ B(12, 0.4) α: P(≤ 2) = **0.0834** > 0.05	B2 M1 A1 A1	Both correct, B2. Allow π. One error, e.g, μ or no symbol, B1, but $\bar{x}$, z etc: B0. B(12, 0.4) stated or implied. Can be implied by N(4.8, 2.88) but no further marks. 0.1673, 0.0398, 0.1513, 0.0421: M1A0(A1M1A1) P(≤ 2) = 0.0834, or P(> 2) = 0.9166. Compare numerical P(≤ 2) with 0.05, or P(> 2) with 0.95
		β: CR is ≤ 1 0.0196 seen and compare 2 with ≤ 1	A1 A1	CR is ≤ 1 stated. Explicitly compare 2 with CR, probability 0.0196 must be seen
		Do not reject H_0. Insufficient evidence that proportion of employees from group *Z* is less.	M1 A1ft **[7]**	Correct first conclusion, needs P(≤ 2 | $p = 0.4$) or fully consistent equivalent In context (mention "employees", "city" etc), acknowledge uncertainty ("evidence") *Not* "there is evidence that the proportion of employees is 0.4" FT on wrong *p*-value or wrong critical value if previous mark gained SC: Normal: B2 M1 max SC: P(= 2) or P(≥ 2) or P(< 2): B2 M1 max SC: two-tailed: can get B1B0 M1A1A0 M1A1 (don't give second A1 for 0.05)
6	(iv)	Yes as H_0 is rejected	M1 A1 **[2]**	Realise this changes conclusion (FT!), or "more likely to reject H_0", "larger CR" More supportive [just "more supportive" without evidence is M0A0]
7	(i)	$\hat{\mu} = \bar{x} = 81$ $\frac{329800}{50} - 81^2$ [= 35] $\times \frac{50}{49}$; = 35.71 $1 - \Phi\left(\frac{90-81}{\sqrt{35.71}}\right) = 1 - \Phi(1.506) = 1 - 0.9339$ **= 6.61%** or **0.0661**	B1 M1 M1 A1 M1 A1 **[6]**	81 only, can be implied Correct formula for biased estimate, their "81", can be implied Multiply by 50/49. SC: single formula: M2, or M1 if wrong but divisor 49 anywhere [can be recovered if correctly done in part (ii)] A.r.t. 35.7 – can't be recovered from part (ii). Can be implied Standardise with their μ and σ, allow σ^2, cc but not $\sqrt{50}$ Answer, a.r.t. 6.6% or 0.066

7	(ii)	H_0: $\mu = 80$ H_1: $\mu \neq 80$	B2	Correct, B2. One error, e.g. wrong or no symbol, >, B1, but x or $\bar{x}$ or t etc, or 81, B0. NB: If both hypotheses involve 81, *can't* get final M1
		α: $z = \dfrac{81-80}{\sqrt{35.71/50}} = 1.183$ [*or* $p = 0.1183$]	M1 A1	Standardise, with $\sqrt{50}$, allow $\sqrt{}$, sign or cc errors, allow from biased variance z, a.r.t. 1.18, or p, a.r.t. 0.118. Allow –1.18.
		< 1.645	B1	Their $z < 1.645$ or $p > 0.05$, *not* if one-tail. Allow –1.18 > –1.645. *Not* just 1.645 seen.
		β: CV $80 + 1.645\sqrt{\dfrac{35.71}{50}} = 81.39$	M1 B1	$80 + zs/\sqrt{50}$, allow $\sqrt{}$ or cc errors, ignore – (no marks for – alone); $z = 1.645$ used in this expression (not just seen), *not* from one-tail
		$81 < 81.39$	A1	Compare CV with 81, allow 81.08 from one-tailed ($z = 1.282$) (but not on their σ) SC: $81 - 1.645\sqrt{\dfrac{35.71}{50}}$: If H_0: $\mu = 80$: (B2) M1B1A0M0A0. If H_0: $\mu = 81$: (B0) M1B1A1 (79.61) M0A0
		Do not reject H_0.	M1	Correct first conclusion, needs $\sqrt{50}$, correct comparison type, μ and $\bar{x}$ not consistently wrong way round (thus H_0: $\mu = 81$ can get B0 M1A1A1 M0A0, max 3/7) In method β, it needs to be clear that comparison involves $\bar{x}$.
		Insufficient evidence that the mean time is not 80 minutes.	A1ft **[7]**	Contextualised (mention "time"), acknowledge uncertainty ("evidence that…") *Not* "significant evidence that mean time is 80" FT on wrong z-value or wrong critical value if previous mark gained SC: One-tailed: can get B1B0 M1A1B0 M1A1, max 5/7 No $\sqrt{50}$: can get B2 M0 B1 M0, max 3/7
7	(iii)	(a) Yes (single observation only) (b) No, CLT applies to large sample	B1 B1 **[2]**	No reason needed, but withhold if wrong reason seen. Allow "yes, no dist^n given" "No" *and* refer to central limit theorem or "large sample" {note for scoris zoning – (a) and (b) to be in single zone}
8	(i)	P($W = 0 \mid \lambda = 3.6$) = **0.0273** or 2.73%	M1 A1 **[2]**	Use this conditional probability. *Not* 0.9727, not *just* 2.5% etc Answer a.r.t. 0.0273 or 2.73%. ISW if appropriate (e.g. "0.0273, $\therefore$2.5%")
8	(ii)	$1 - e^{-\lambda_0} = 0.8$ $e^{-\lambda_0} = 0.2$ $\lambda_0 = -\ln(0.2)$ = **1.609**	M1 A1 M1 A1 **[4]**	Use P($W > 0 \mid \lambda = \lambda_0$), formula needed but allow if wrong This exact equation, or $e^{\lambda_0} = 5$, or exact equivalent RHS Solve using ln or otherwise [independent of first M1, e.g. –ln(0.8) = 0.223 is M1 here] Final answer, exact or a.r.t. 1.61, cwo SC: No working: 1.60 (tables etc): B0. 1.61 (T&I): SC B4.