

Question	Answer	Marks	Guidance
1 (i)	<p><b>EITHER:</b></p> $S_{xy} = \sum xy - \frac{1}{n} \sum x \sum y = 40.66 - \frac{1}{60} \times 43.62 \times 55.15$ $= 0.56595$ $S_{xx} = \sum x^2 - \frac{1}{n} (\sum x)^2 = 32.68 - \frac{1}{60} \times 43.62^2$ $= 0.96826$ $S_{yy} = \sum y^2 - \frac{1}{n} (\sum y)^2 = 51.44 - \frac{1}{60} \times 55.15^2$ $= 0.74796$ $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{0.56595}{\sqrt{0.96826 \times 0.74796}} = 0.665$ <p><b>OR:</b></p> $\text{cov}(x,y) = \frac{\sum xy}{n} - \bar{x}\bar{y} = 40.66/60 - (43.62/60 \times 55.15/60)$ $= 0.0094325$ $\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(0.96826/60)} = \sqrt{0.016137\dots} = 0.1270$ $\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(0.74796/60)} = \sqrt{0.012466} = 0.1117$ $r = \frac{\text{cov}(x,y)}{\text{rmsd}(x)\text{rmsd}(y)} = \frac{0.0094325}{0.1270 \times 0.1117} = 0.665$	<p>M1*</p> <p>M1*</p> <p>A1</p> <p>M1 dep*</p> <p>A1</p> <p>[5]</p> <p>M1*</p> <p>M1*</p> <p>A1</p> <p>M1 dep*</p> <p>A1</p> <p>[5]</p>	<p>For method for <math>S_{xy}</math></p> <p>For method for at least one of <math>S_{xx}</math> or <math>S_{yy}</math></p> <p>For at least one of <math>S_{xy}</math>, <math>S_{xx}</math> or <math>S_{yy}</math> (to 2 sf) Note Allow 0.57322 for <math>S_{xy}</math> and 0.76634 for <math>S_{yy}</math> from rounding mean of <math>y</math> to 0.919.</p> <p>For structure of <math>r</math></p> <p>For answer rounding to 0.66 or 0.67</p> <p>[5]</p> <p>For method for cov (<math>x,y</math>)</p> <p>For method for at least one msd or rmsd</p> <p>For at least one of cov (<math>x,y</math>), msd or rmsd correct (to 2 sf)</p> <p>For structure of <math>r</math></p> <p>For answer rounding to 0.66 or 0.67</p> <p>Methods mixed – max M0M1A1M0A0</p> <p>[5]</p>

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1	(ii)	<p><math>H_0: \rho = 0</math>  <math>H_1: \rho &gt; 0</math> (one-tailed test)</p> <p>where <math>\rho</math> is the population correlation coefficient</p> <p>For <math>n = 60</math>, 5% critical value = 0.2144</p> <p>Since <math>0.665 &gt; 0.2144</math>, the result is significant.</p> <p>Thus we have sufficient evidence to reject <math>H_0</math></p> <p>There is sufficient evidence at the 5% level to <b>suggest</b> that there is <b>positive</b> correlation between FEV1 before and after the two-week course.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p><b>[6]</b></p>	<p>For <math>H_0, H_1</math> in symbols. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population correlation coefficient.</p> <p>For defining <math>\rho</math>. Condone omission of “population” if correct notation <math>\rho</math> is used, but if <math>\rho</math> is defined as the <b>sample</b> correlation coefficient then award <b>B0</b>. Allow “<math>\rho</math> is the pmcc”.</p> <p>For critical value</p> <p>For sensible comparison leading to a conclusion provided that <math> r  &lt; 1</math>.  The comparison can be in the form of a diagram as long as it is clear and unambiguous.  Sensible comparison: e.g. <math>0.665 &gt; 0.2144</math> is ‘sensible’ whereas <math>0.665 &gt; -0.2144</math> is ‘not sensible’.  Reversed inequality sign e.g. <math>0.665 &lt; 0.2144</math> etc. gets max M1 A0.</p> <p>For reject <math>H_0</math> o.e. FT their <math>r</math> and critical value from 5% 1-tail column.</p> <p>For correct, <b>non-assertive</b> conclusion in context (allow ‘<math>x</math> and <math>y</math>’ for context). E0 if <math>H_0</math> and <math>H_1</math> not stated, reversed or mention a value other than zero for <math>\rho</math> in <math>H_0</math>.</p>

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1	(iii)	The underlying population must have a bivariate Normal distribution.  Yes, since the scatter diagram appears to have a roughly elliptical shape.	B1  E1  [2]	Condone “bivariate Normal distribution”, “underlying bivariate Normal distribution”, but <b>do not allow</b> “the <b>data</b> have a bivariate Normal distribution”  Condone ‘oval’ or suitable diagram
1	(iv)	The significance level is the probability of rejecting the null hypothesis when in fact it is true.	E1*  E1dep*  [2]	For “probability of rejecting $H_0$ ” or “probability of a significant result”. For “when $H_0$ is true”
1	(v)	$\sum x = 43.62 + 0.45 = 44.07$ $\sum y = 55.15 - 0.45 = 54.70$ $\sum xy = 40.66$ $\sum x^2 = 32.68 + 1 - 0.55^2 = 33.3775$ $\sum y^2 = 51.44 - 1 + 0.55^2 = 50.7425$	B1  B1  B1  [3]	For $\sum x$ or $\sum y$ or $\sum xy$  For $\sum x^2$ or $\sum y^2$ (to 2 dp)  For all correct (ignore $n$ )
2	(i)	$P(\text{At least one has red hair}) = 1 - 0.97^{10}$ $= 0.263$	M1 A1  [2]	M1 for $1 - 0.97^{10}$ Allow 0.26
2	(ii)	(Because $X$ is binomially distributed), $n$ is large and $p$ is small.  Mean = 1.8	E1 E1  B1  [3]	Allow “sample is large” for $n$ is large Allow “ $np < 10$ ” or “mean $\approx$ variance” for “ $p$ is small” Do not allow “the probability is small”

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2	(iii) (A)	$P(X = 2) = e^{-1.8} \frac{1.8^2}{2!} = 0.2678$ OR $0.7306 - 0.4628 = 0.2678$	M1 A1 [2]	For calculation for $P(X = 2)$ FT their mean. Allow answer to 3sf.
2	(iii) (B)	$P(X > 2) = 1 - P(X \leq 2) = 1 - 0.7306$ $= 0.2694$	M1 A1 [2]	$1 - P(X \leq 2)$ used. e.g. $1 - P(X \leq 2) = 1 - 0.4628$ gets M0 CAO
2	(iv)	The mean ( $np = 1.8$ ) is too small It is not appropriate to use a Normal approximation	E1* E1dep* [2]	For “mean is too small” or “mean < 10” For “not appropriate”. Do not allow “ $p$ is too small”.
2	(v)	Binomial(5000, 0.03)	B1* B1dep* [2]	For binomial, or B( , ) For parameters
2	(vi)	Mean $5000 \times 0.03 = 150$ Variance $= 5000 \times 0.03 \times 0.97 = 145.5$ Using Normal approx. to the binomial, $X \sim N(150, 145.5)$ $P(X \geq 160) = P\left(Z \geq \frac{159.5 - 150}{\sqrt{145.5}}\right)$ $= P(Z > 0.7876) = 1 - \Phi(0.7876) = 1 - 0.7846$ $= 0.215 \text{ (to 3 sig.fig.)}$	B1 B1  B1 M1 A1 [5]	For mean (soi) For variance (soi)  For continuity corr.  For probability using correct tail and structure (condone omission of/incorrect c.c.) CAO, (Do not FT wrong or omitted CC) Allow 0.2155. Do not allow 0.216

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3	(i)	$P(Y = 76) = P\left(\frac{75.5 - 76}{12} \leq Z \leq \frac{76.5 - 76}{12}\right)$ $= P(-0.04166... < Z < 0.04166...)$ $= \Phi(0.04166...) - (1 - \Phi(0.04166...))$ $= 2 \times \Phi(0.04166...) - 1$ $= 2 \times 0.5167 - 1$ $= 0.0334$	B1 M1 M1 A1 <b>[4]</b>	For one correct continuity correction used For standardizing For correctly structured probability calculation. CAO inc use of diff tables. Allow 0.0330 – 0.0340 www.
3	(ii)	$P(Y \geq 80) = P\left(Z \geq \frac{79.5 - 76}{12}\right)$ $= P(Z > 0.2917) = 1 - \Phi(0.2917)$ $= 1 - 0.6148 = 0.3852 = 0.385 \text{ to 3 sig fig}$	B1 M1 A1 <b>[3]</b>	For correct cc used For correct structure CAO do not allow 0.386
3	(iii)	$3 \times 0.3852 \times 0.6148^2 = 0.4368$	M1 A1 <b>[2]</b>	$3 \times \text{their } p \times (1 - \text{their } p)^2$ FT their $p$ . Allow 2sf if working seen.

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3	(iv)	<p><b>EITHER:</b> <math>P(\text{Score} \geq k) = 0.1</math>  <math>\Phi^{-1}(0.9) = 1.282</math>  <math>\frac{k - 76}{12} = 1.282</math>  <math>k = 76 + (1.282 \times 12) = 91.38</math> or  <math>k = 76 + 0.5 + (1.282 \times 12) = 91.88</math>  <math>91.38 &gt; 90.5</math> or <math>91.88 &gt; 91</math>  so lowest reported mark = 92</p> <p><b>OR</b> Trial and improvement method  <math>P(\text{Mark} \geq 91) = P(\text{Score} \geq 90.5) = 0.1135</math>  <math>P(\text{Mark} \geq 92) = P(\text{Score} \geq 91.5) = 0.0982</math>  <math>P(\text{Mark} \geq 91) &gt; 10\%</math> and <math>P(\text{Mark} \geq 92) &lt; 10\%</math>  so lowest reported mark = 92</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For 1.282</p> <p>Allow <math>k - 0.5</math> used for <math>k</math>. Positive <math>z</math> used.</p> <p>For 91.38 or 91.88</p> <p>Relevant comparison (e.g. diagram)</p> <p>M1 for attempt to find <math>P(\text{Mark} \geq \text{integer})</math></p> <p>A1 for 0.1135</p> <p>A1 for 0.0982</p> <p>M1 for comparisons</p>	<p>www</p> <p>www</p>
3	(v)	<p><math>P(Y \leq 50) = 0.2</math>  <math>P(Z \leq \frac{50.5 - \mu}{12}) = 0.2</math>  <math>\frac{50.5 - \mu}{12} = \Phi^{-1}(0.2) = -0.8416</math>  <math>\mu = 50.5 + (12 \times 0.8416) = 60.6</math></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For 50.5 used</p> <p>For <math>-0.8416</math>. Condone <math>-0.842</math>  Condone 0.8416 if numerator reversed.</p> <p>For structure.</p> <p>CAO</p>	

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4 (i)	<p>H<sub>0</sub>: no association between sex and artist preferred H<sub>1</sub>: some association between sex and artist preferred</p> <table border="1" data-bbox="371 331 1070 459"> <thead> <tr> <th>EXPECTED</th> <th>Monet</th> <th>Renoir</th> <th>Degas</th> <th>Cézanne</th> </tr> </thead> <tbody> <tr> <td>Male</td> <td>12.13</td> <td>28</td> <td>13.07</td> <td>16.8</td> </tr> <tr> <td>Female</td> <td>13.87</td> <td>32</td> <td>14.93</td> <td>19.2</td> </tr> </tbody> </table> <table border="1" data-bbox="371 496 1070 624"> <thead> <tr> <th>CONTRIB'N</th> <th>Monet</th> <th>Renoir</th> <th>Degas</th> <th>Cézanne</th> </tr> </thead> <tbody> <tr> <td>Male</td> <td>1.4081</td> <td>0.3214</td> <td>1.8626</td> <td>0.2881</td> </tr> <tr> <td>Female</td> <td>1.2321</td> <td>0.2813</td> <td>1.6298</td> <td>0.2521</td> </tr> </tbody> </table> <p><math>X^2 = 7.28</math> Refer to <math>\chi_3^2</math></p> <p>Critical value at 10% level = 6.251</p> <p>Result is significant</p> <p>There is evidence to <b>suggest</b> that there is some association between sex and artist preferred</p> <p>NB if H<sub>0</sub> H<sub>1</sub> reversed, or 'correlation' mentioned, do not award first B1 or final E1</p>	EXPECTED	Monet	Renoir	Degas	Cézanne	Male	12.13	28	13.07	16.8	Female	13.87	32	14.93	19.2	CONTRIB'N	Monet	Renoir	Degas	Cézanne	Male	1.4081	0.3214	1.8626	0.2881	Female	1.2321	0.2813	1.6298	0.2521	<p>B1</p> <p>M1 A2</p> <p>M1 A2</p> <p>B1 B1</p> <p>B1 B1</p> <p>E1</p> <p>[12]</p>	<p>For both hypotheses in context</p> <p>For expected values (to 2 dp where appropriate) (allow A1 for at least one row or column correct)</p> <p>For valid attempt at <math>(O-E)^2/E</math> For all correct (to 2 dp) and presented in a table or clear list. (Allow A1 for at least one row or column correct)</p> <p>Allow 7.27 for 3 deg of f</p> <p>CAO for cv No FT from here if wrong or omitted, unless <math>p</math>-value used instead FT their <math>X^2</math></p> <p>For correct (FT their <math>X^2</math>), non-assertive conclusion, in context.</p> <p>NB: These three marks cannot be implied by a correct final value of <math>X^2</math></p> <p>www</p> <p>B1 for <math>p</math>-value = 0.0636</p>
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4	(ii)	<p>Monet: <b>More females and fewer males than expected</b> prefer Monet, as indicated by <b>large contribution(s)</b> (of 1.4081 and 1.2321).</p> <p>Renoir: Preferences are much <b>as expected</b>, as indicated by <b>small contributions</b>.</p> <p>Degas: <b>Fewer females and more males than expected</b> prefer Degas, as indicated by <b>large contribution(s)</b> (of 1.8626 and 1.6298).</p> <p>Cézanne: Preferences are much <b>as expected</b>, as indicated by <b>small contributions</b>.</p>	<p>E1* E1dep*</p> <p>E1</p> <p>E1* depE1*</p> <p>E1</p> <p>[6]</p>	FT their table of contributions	<p>NB MAX 3/6 for answers not referring to contributions (explicitly or implicitly).</p> <p>SC1 Renoir and Cézanne have correct comments for both but without referring to contributions</p>