

4767

**Mark Scheme
January 2006**

Question 1

(i)	Faults are detected randomly and independently Uniform (mean) rate of occurrence	B1 B1	2
(ii)	(A) $P(X = 0) = e^{-0.15} \frac{0.15^0}{0!} = 0.8607$ (B) $P(X \geq 2) = 1 - 0.8607 - e^{-0.15} \frac{0.15^1}{1!}$ $= 1 - 0.8607 - 0.1291 = 0.0102$	M1 for probability calc. M0 for tables unless interpolated A1 M1 A1	4
(iii)	$\lambda = 30 \times 0.15 = 4.5$ Using tables: $P(X \leq 3) = 0.3423$	B1 for mean (SOI) M1 attempt to find $P(X \leq 3)$ A1	3
(iv)	Poisson distribution with $\lambda = 10 \times (0.15 + 0.05) = 2$ $P(X = 5) = e^{-2} \frac{2^5}{5!} = 0.0361$ (3 s.f.) or from tables $= 0.9834 - 0.9473 = 0.0361$	B1 for Poisson stated B1 for $\lambda = 2$ M1 for calculation or use of tables A1 FT	4
(v)	Mean no. of items in 200 days $= 200 \times 0.2 = 40$ Using Normal approx. to the Poisson, $X \sim N(40, 40)$: $P(X \geq 50) = P\left(Z > \frac{49.5 - 40}{\sqrt{40}}\right)$ $= P(Z > 1.502) = 1 - \Phi(1.502) = 1 - 0.9334$ $= 0.0666$ (3 s.f.)	B1 for Normal approx. (SOI) B1 for both parameters B1 for continuity corr. M1 for probability using correct tail A1 cao , (but FT wrong or omitted CC)	5
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Question 2

(i) (A)	$X \sim N(42, 3^2)$ $P(X > 50.0) = P\left(Z > \frac{50.0 - 42.0}{3.0}\right)$ $= P(Z > 2.667)$ $= 1 - \Phi(2.667) = 1 - 0.9962$ $= 0.0038$	M1 for standardizing M1 for prob. calc. with correct tail A1 NB answer given	3
(i) (B)	$P(\text{not positive}) = 0.9962$ $P(\text{At least one is out of 7 is positive})$ $= 1 - 0.9962^7 = 1 - 0.9737$ $= 0.0263$	B1 for use of 0.9962 in binomial expression M1 for correct method A1 CAO	3
(i) (C)	If an innocent athlete is tested 7 times in a year there is a reasonable possibility (1 in 40 chance) of testing positive. Thus it is likely that a number of innocent athletes may come under suspicion and suffer a suspension so the penalty could be regarded as unfair. <i>Or</i> this is a necessary evil in the fight against performance enhancing drugs in sport.	E1 comment on their probability in (i) B E1 for sensible contextual conclusion consistent with first comment	2
(ii) (A)	$B(1000, 0.0038)$	B1 for $B(,)$ or Binomial B1 <i>dep</i> for both parameters	2
(ii) (B)	A suitable approximating distribution is Poisson(3.8) $P(\text{at least 10 positive tests})$ $= P(X \geq 10) = 1 - P(X \leq 9)$ $= 1 - 0.9942$ $= 0.0058$ <i>NB Do not allow use of Normal approximation.</i>	B1 for Poisson soi B1FT <i>dep</i> for $\lambda = 3.8$ M1 for attempt to use $1 - P(X \leq 9)$ A1 FT	4
(iii)	$P(\text{not testing positive}) = 0.995$ From tables $z = \Phi^{-1}(0.995) = 2.576$ $\frac{h - 48.0}{2.0} = 2.576$ $h = 48.0 + 2.576 \times 2.0 = 53.15$	B1 for 0.995 seen (or implied by 2.576) B1 for 2.576 (FT their 0.995) M1 for equation in h and positive z -value A1 CAO	4
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Question 3

(i)	<table border="1" style="width: 100%; text-align: center;"> <tbody> <tr> <td>Rank x</td> <td>1</td> <td>5</td> <td>4</td> <td>7</td> <td>6</td> <td>8</td> <td>10</td> <td>3</td> <td>9</td> <td>2</td> </tr> <tr> <td>Rank y</td> <td>2</td> <td>4</td> <td>5</td> <td>8</td> <td>9</td> <td>7</td> <td>10</td> <td>6</td> <td>3</td> <td>1</td> </tr> <tr> <td>d</td> <td>-1</td> <td>1</td> <td>-1</td> <td>-1</td> <td>-3</td> <td>1</td> <td>0</td> <td>-3</td> <td>6</td> <td>1</td> </tr> <tr> <td>d^2</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>9</td> <td>1</td> <td>0</td> <td>9</td> <td>36</td> <td>1</td> </tr> </tbody> </table> $r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 60}{10 \times 99}$ $= 0.636 \text{ (to 3 s.f.) [allow 0.64 to 2 s.f.]}$	Rank x	1	5	4	7	6	8	10	3	9	2	Rank y	2	4	5	8	9	7	10	6	3	1	d	-1	1	-1	-1	-3	1	0	-3	6	1	d^2	1	1	1	1	9	1	0	9	36	1	<p>M1 for ranking (allow all ranks reversed)</p> <p>M1 for d^2</p> <p>A1 CAO for $\sum d^2$</p> <p>M1 for structure of r_s using their $\sum d^2$</p> <p>A1 f.t. for $r_s < 1$ NB No ranking scores zero</p>	5
Rank x	1	5	4	7	6	8	10	3	9	2																																					
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d^2	1	1	1	1	9	1	0	9	36	1																																					
(ii)	<p>H_0: no association between x and y</p> <p>H_1: positive association between x and y</p> <p>Looking for positive association (one-tail test):</p> <p>Critical value at 5% level is 0.5636</p> <p>Since $0.636 > 0.5636$, there is sufficient evidence to reject H_0, i.e. conclude that there appears to be positive association between temperature and nitrous oxide level.</p>	<p>B1 for H_0</p> <p>B1 for H_1</p> <p>NB $H_0 H_1$ <u>not</u> ito rho</p> <p>B1 for ± 0.5636</p> <p>(FT their H_1)</p> <p>M1 for comparison with c.v., provided $r_s < 1$</p> <p>A1 for conclusion in words f.t. their r_s and sensible cv</p>	5																																												
(iii)	<p>Underlying distribution must be bivariate normal.</p> <p>If the distribution is bivariate normal then the scatter diagram will have an elliptical shape.</p> <p>This scatter diagram is not elliptical and so a PMCC test would not be valid.</p> <p>(Allow comment indicating that the sample is too small to draw a firm conclusion on ellipticity and so on validity)</p>	<p>B1 CAO for bivariate normal</p> <p>B1 indep for elliptical shape</p> <p>E1 dep for conclusion</p>	3																																												
(iv)	<p>$n=60$, PMCC critical value is $r = 0.2997$</p> <p>So the critical region is $r \geq 0.2997$</p>	<p>B1</p> <p>B1 FT their sensible c.v.</p>	2																																												
(v)	<p>Any three of the following:</p> <ul style="list-style-type: none"> • Correlation does not imply causation; • There could be a third factor (causing the correlation between temperature and ozone level); • the claim could be true; • increased ozone could cause higher temperatures. 	<p>E1</p> <p>E1</p> <p>E1</p>	3																																												
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Question 4

(i)	<p>H_0: no association between method of travel and type of school; H_1: some association between method of travel and type of school;..</p>	B1 for both	1
(ii)	<p>Expected frequency = $120/200 \times 70 = 42$ Contribution = $(21 - 42)^2 / 42$ = 10.5</p>	<p>M1 A1 M1 for valid attempt at $(O-E)^2/E$ A1 FT their 42 provided $O = 21$ (at least 1 dp)</p>	4
(iii)	<p>$X^2 = 42.64$ Refer to χ_2^2 Critical value at 5% level = 5.991 Result is significant There appears to be some association between method of travel and year group. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1 or final E1</p>	<p>B1 for 2 deg of f(seen) B1 CAO for cv B1 for significant (FT their c.v. provided consistent with their d.o.f. E1</p>	4
(iv)	<p>$H_0: \mu = 18.3$; $H_1: \mu \neq 18.3$ Where μ denotes the mean travel time by car for the whole population. Test statistic $z = \frac{22.4 - 18.3}{8.0/\sqrt{20}} = \frac{4.1}{1.789} = 2.292$ 10% level 2 tailed critical value of z is 1.645 $2.292 > 1.645$ so significant. There is evidence to reject H_0 It is reasonable to conclude that the mean travel time by car is different from that by bus.</p>	<p>B1 for both correct B1 for definition of μ M1 (standardizing sample mean) A1 for test statistic B1 for 1.645 M1 for comparison leading to a conclusion A1 for conclusion in words and context</p>	7
(v)	<p>The test suggests that students who travel by bus get to school more quickly. This may be due to their journeys being over a shorter distance. It may be due to bus lanes allowing buses to avoid congestion. It is possible that the test result was incorrect (ie implication of a Type I error). More investigation is needed before any firm conclusion can be reached.</p>	E1, E1 for any two valid comments	2
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Question 4 chi squared calculations

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