



1. A bag contains a large number of counters. A third of the counters have a number 5 on them and the remainder have a number 1.

A random sample of 3 counters is selected.

(a) List all possible samples. (2)

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2. The continuous random variable  $Y$  has cumulative distribution function

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{4}(y^3 - 4y^2 + ky) & 0 \leq y \leq 2 \\ 1 & y > 2 \end{cases}$$

where  $k$  is a constant.

(a) Find the value of  $k$ . **(2)**

(b) Find the probability density function of  $Y$ , specifying it for all values of  $y$ . **(3)**

(c) Find  $P(Y > 1)$ . **(2)**

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3. The random variable  $X$  has a continuous uniform distribution on  $[a, b]$  where  $a$  and  $b$  are positive numbers.

Given that  $E(X) = 23$  and  $\text{Var}(X) = 75$

(a) find the value of  $a$  and the value of  $b$ .

(6)

Given that  $P(X > c) = 0.32$

(b) find  $P(23 < X < c)$ .

(2)

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4. The random variable  $X$  has probability density function  $f(x)$  given by

$$f(x) = \begin{cases} k(3 + 2x - x^2) & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

where  $k$  is a constant.

- (a) Show that  $k = \frac{1}{9}$  (3)

- (b) Find the mode of  $X$ . (2)

- (c) Use algebraic integration to find  $E(X)$ . (4)

By comparing your answers to parts (b) and (c),

- (d) describe the skewness of  $X$ , giving a reason for your answer. (2)

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5. In a village shop the customers must join a queue to pay. The number of customers joining the queue in a 10 minute interval is modelled by a Poisson distribution with mean 3

Find the probability that

(a) exactly 4 customers join the queue in the next 10 minutes, (2)

(b) more than 10 customers join the queue in the next 20 minutes. (3)

When a customer reaches the front of the queue the customer pays the assistant. The time each customer takes paying the assistant,  $T$  minutes, has a continuous uniform distribution over the interval  $[0, 5]$ . The random variable  $T$  is independent of the number of people joining the queue.

(c) Find  $P(T > 3.5)$  (1)

In a random sample of 5 customers, the random variable  $C$  represents the number of customers who took more than 3.5 minutes paying the assistant.

(d) Find  $P(C \geq 3)$  (3)

Bethan has just reached the front of the queue and starts paying the assistant.

(e) Find the probability that in the next 4 minutes Bethan finishes paying the assistant and no other customers join the queue. (4)

Horizontal lines for writing answers.







6. Frugal bakery claims that their packs of 10 muffins contain on average 80 raisins per pack. A Poisson distribution is used to describe the number of raisins per muffin.

A muffin is selected at random to test whether or not the mean number of raisins per muffin has changed.

- (a) Find the critical region for a two-tailed test using a 10% level of significance. The probability of rejection in each tail should be less than 0.05

(4)

- (b) Find the actual significance level of this test.

(2)

The bakery has a special promotion claiming that their muffins now contain even more raisins.

A random sample of 10 muffins is selected and is found to contain a total of 95 raisins.

- (c) Use a suitable approximation to test the bakery's claim. You should state your hypotheses clearly and use a 5% level of significance.

(8)

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7. As part of a selection procedure for a company, applicants have to answer all 20 questions of a multiple choice test. If an applicant chooses answers at random the probability of choosing a correct answer is 0.2 and the number of correct answers is represented by the random variable  $X$ .

(a) Suggest a suitable distribution for  $X$ . (2)

Each applicant gains 4 points for each correct answer but loses 1 point for each incorrect answer. The random variable  $S$  represents the final score, in points, for an applicant who chooses answers to this test at random.

(b) Show that  $S = 5X - 20$  (2)

(c) Find  $E(S)$  and  $\text{Var}(S)$ . (4)

An applicant who achieves a score of at least 20 points is invited to take part in the final stage of the selection process.

(d) Find  $P(S \geq 20)$  (4)

Cameron is taking the final stage of the selection process which is a multiple choice test consisting of 100 questions. He has been preparing for this test and believes that his chance of answering each question correctly is 0.4

(e) Using a suitable approximation, estimate the probability that Cameron answers more than half of the questions correctly. (5)

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