Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE Mathematics/Further Mathematics

Statistics S2 (6684/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper or ag- answer given
- $\lfloor$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

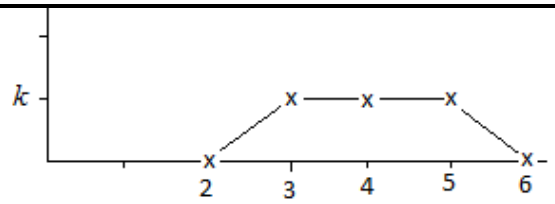
Question Number	Scheme		Marks
	Allow any letter instead of X or c for this question		
1 (a)	$X \sim \mathrm{B}(25, 0.2)$	**M1** Writing or using B(25,0.2)or B(25,1/5) [allow Po(5)] May be written in full or implied by a correct CR (allow written as a probability statement)	M1
	$[\mathrm{P}(X \geq 9) =] 0.0468$ $[\mathrm{P}(X \leq 1) =] 0.0274$	**1st A1** both awrt 0.0468 and awrt 0.0274 seen.	A1
	$X = [0 \leq] \ X \leq 1$	**2nd A1** $X \leq 1$ or $X < 2$ or $0 \leq X \leq 1$ or [0,1] or 0,1 or equivalent statements. $X \leq c$ and $c = 1$	A1
	$9 \leq X \ [\leq 25]$	**3rd A1d** dependent on seeing a probability from the B(25, 0.2) and $X \geq 9$ or $X > 8$ or $9 \leq X \leq 25$ or 9,10,11,12,13,14,15,16,17,18,19,20,21,22, 23,24,25 or [9,25] or equivalent statements. $X \geq c$ and $c = 9$	A1d
	NB These two final 2 A marks must be for statements with "X" only(or list) – not in probability statements **SC If** a probability from the B(25, 0.2) is seen **and** they either have both CR correct but written as probability statements **or** the CR is written as $1 \geq X \geq 9$ they get A1 A0 for final 2 marks		**(4)**
(b)	H_0: $p = 0.2$ H_1: $p < 0.2$	**B1** both hypotheses with p or π and clear which is H_0 and which is H_1	**B1**
	$\mathrm{P}(X \leq 6) = 0.1034$ or CR $X \leq 5$	**1st M1** writing or using B(50, 0.2) and writing or using P($X \leq 6$) or P($X \geq 7$) on its own. May be implied by a correct CR	**M1**
		1st A1 awrt 0.103. Allow CR $X \leq 5$ or $X < 6$. or if not using CR allow awrt 0.897.	**A1**
	Insufficient evidence to reject H_0, Accept H_0, Not significant. 6 does not lie in the Critical region.	**2nd M1** dependent on previous M being awarded. A correct statement (do not allow if there are contradicting non-contextual statements). ft their Prob/CR compared with 0.05/6/(0.95 if using 0.8979). Do not follow through their hypotheses	**M1d**
	No evidence that increasing the batch size has **reduced** the **percentage** of broken **pots (oe)** **or** evidence that there is **no change** in the **percentage** of broken **pots (oe)**	**2nd A1cso** Conclusion must contain the words **reduced/ no change/not affect oe number/percentage/proportion/ probability oe**, and **pots**. All previous marks must be awarded for this mark to be awarded. Do **not** allow the potters claim /belief is wrong/true **NB** Correct contextual statement on its own scores M1A1	**A1cso** **(5)**
			(Total 9)

2(a)(i)	$X \sim \text{Po}(2.5)$		
	$P(X \geq 4) = 1 - P(X \leq 3)$ $= 1 - 0.7576$	**M1** writing or using $1 - P(X \leq 3)$ implied by awrt 0.242	**M1**
	$= 0.2424$	**A1** awrt 0.242	**A1**
(ii)	$X \sim \text{Po}(0.625)$	**B1** Using Po(0.625)	**B1**
	$P(X = 3) = \dfrac{e^{-0.625}0.625^3}{3!}$	**M1** finding $P(X = 3)$ with any λ e.g $\dfrac{e^{-\lambda}\lambda^3}{3!}$ or $P(X \leq 3) - P(X \leq 2)$ – may be implied by awrt 0.0218	**M1**
	$= 0.02177...$	**A1** awrt 0.0218	**A1** **(5)**
(b)	$1 - P(X = 0) < 0.2$ $P(X = 0) > 0.8$	**1st M1** for writing or using $1 - P(X = 0) < 0.2$ or $P(X = 0) > 0.8$ **oe** allow use of = instead of > or <. May be implied by $e^{-l} = 0.8$ or $e^{-l} > 0.8$ or by awrt 5.36 or 0.089	**M1**
	$e^{-2.5t} > 0.8$ $t < 0.089\ldots$ hours = 5.36 mins	**2nd M1** writing an inequality of the form $e^{-l} > 0.8$ using any l . May be implied by or by awrt 5.36 or 0.089 Do not allow $e^{-l} = 0.8$	**M1**
	[$t <$] 5 mins	**A1cso** both the method marks must be awarded. Accept 5 or $t = 5$ or $t < 5$	**A1cso** **(3)**
(c)	$H_0: \lambda = 2.5\ (\lambda = 5)$ $H_1: \lambda > 2.5\ (\lambda > 5)$	**B1** both hypotheses using λ or μ - allow 5 or 2.5 and it must be clear which is H_0 and which is H_1	**B1**
	$P(X \geq 10) = 1 - P(X \leq 9)$ $= 1 - 0.9682$	**1st M1** writing or using Po(5) and $1 - P(X \leq 9)$ May be implied by a correct CR. Do not allow for writing $P(X \geq 10)$	**M1**
	$= 0.0318$	**1st A1** awrt 0.0318. Allow CR $X \geq 10$ or $X > 9$	**A1**
		NB allow M1A1 if not using CR route for $P(X \leq 9)$ = awrt 0.968	
	Sufficient evidence to reject H_0, Accept H_1, significant. 10 does lie in the Critical region.	**2nd M1** dependent on previous M being awarded. A correct statement (do not allow if there are contradicting non-contextual statements). ft their Prob/CR compared with 0.05/10 (0.95 if using 0.968)	**M1d**
	There is sufficient evidence that the mean rate of telephone **calls** has increased **(oe)**	**2nd A1** A correct contextual statement must include the word **calls** and the idea the rate has increased. (do not allow "it has changed" on its own **oe**). All previous marks must be awarded for this mark to be awarded. M1A1 is awarded for a correct contextual statement on its own provided previous marks have been awarded	**A1cso** **(5)**
			(Total 13)

3(a)	$E(X) = \frac{1}{9}\int_1^4 \left(4x^2 - x^3\right) dx$	**1st M1** Using $\int xf(x)\,dx$, multiplying out and at least one of $x^2 \to x^3$ or $x^3 \to x^4$ ignore limits	**M1**
	$= \frac{1}{9}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_1^4$	**1st A1** correct integration, ignore limits	**A1**
	$= \frac{1}{9}\left[\frac{4\times4^3}{3} - \frac{4^4}{4}\right] - \frac{1}{9}\left[\frac{4}{3} - \frac{1}{4}\right]$	**2nd M1d** subst in correct limits (allow 1 sign error)	**M1d**
	$= \frac{9}{4}$ or 2.25	**2nd A1** cao allow equivalent fractions	**A1**
			(4)
(b)	$P(X > 2.5) = \frac{1}{9}\int_{2.5}^4 x(4-x)\,dx$	**M1** for using $\frac{1}{9}\int_{2.5}^4 x(4-x)\,dx$ or $1 - \frac{1}{9}\int_1^{2.5} x(4-x)\,dx$ correct limits needed at some point Or $1 - \left[\frac{2}{9}x^2 - \frac{1}{27}x^3 - \frac{5}{27}\right]$ and attempt to subst 2.5	**M1**
	$= \frac{1}{9}\left[2x^2 - \frac{x^3}{3}\right]_{2.5}^4$	**1st A1** correct integration with correct limits at some point	**A1**
	$= \frac{3}{8}$ **oe** or 0.375	**2nd A1** allow equivalent fractions	**A1**
			(3)
(c)	P(both batteries working after 25 hours) $= (0.375)^2$	**M1** (their part(b))^2 or writing (P(X >2.5))^2	**M1**
	$= 0.140625$ or $\frac{9}{64}$	**A1** awrt 0.141	**A1**
			(2)
(d)	$P(X > 1.6) = \frac{1}{9}\int_{1.6}^4 x(4-x)dx$ $= \frac{96}{125}$ or 0.768	**B1** 0.768 or awrt 0.77 or 0.5898…or awrt 0.59. These may be seen in the conditional probability or implied by a correct final answer	**B1**
	P(works for 25 hours|worked for 16 hours) $= \frac{0.140625}{(0.768)^2}$	**M1** $\frac{\text{their part(c)}}{prob}$ or $\frac{(\text{their}(b))^2}{prob}$ and numerator < denominator	**M1**
	$= 0.2384\ldots$	**A1** awrt 0.238	**A1**
	NB if use one battery rather than 2 they could get B1 M0 A0		
			(3)
			(Total 12)

4.(a)	$\left[\mathrm{E}(X)=\frac{\alpha+\beta}{2}=3.5\right], \Rightarrow \alpha+\beta=7$	**B1** Correct equation. Need not be simplified	**B1**
	$\left[\mathrm{P}(X>5)=\frac{\beta-5}{\beta-\alpha}=\frac{2}{5}\right],$ $\Rightarrow 5(\beta-5)=2(\beta-\alpha)$	**M1** a second correct equation, Using simultaneous equations and eliminating α or β to gain a value of α and β.	**M1**
	$\alpha=-4$	**1st A1** for -4	**A1**
	$\beta=11$	**2nd A1** for 11	**A1**
		NB Award full marks for $\alpha=-4, \beta=11$	
			(4)
(b)(i)	$\frac{c+4}{15}=\frac{2}{3}$		
	$[c=]$ 6	**B1** for 6	**B1**
(ii)	$\mathrm{P}(6<X<9)=\frac{1}{15}\times(3)$	**M1** $\frac{1}{\beta-\alpha}\times(9-c)$ or $[\mathrm{F}(9)-\mathrm{F}(c)]=\frac{13}{15}-\frac{2}{3}$ **SC** if 9 > "their b" award for 1- $\frac{2}{3}$	**M1**
	$=0.2$	**A1cso** 0.2 **oe**	**A1cso**
			(3)
(c)	$[\mathrm{P}(S<45)]=\frac{3}{10}$	**B1** $\frac{3}{10}$ seen – it does not need to be associated with P ($S<45$)]	**B1**
	$[\mathrm{P}(S>55)]=\frac{1}{2}$	**B1** $\frac{1}{2}$ seen– it does not need to be associated with P ($S>55$)]	**B1**
	total $=\frac{3}{10}+\frac{1}{2}=\frac{4}{5}$	**M1** for adding their two areas and the total < 1. Do not allow 2′ a single area **A1** $\frac{4}{5}$ **oe** NB Award full marks for $\frac{4}{5}$	**M1A1**
			(4)
			(Total 11)

5(a)	$P(M < 10) = P\left(Z < \frac{12-14}{\sigma}\right) = 0.1$		
	$\Rightarrow \frac{12-14}{\sigma} =$, -1.2816	**M1** standardising ($\pm$) with 12, 14 and σ and setting equal to a z value where $\lvert z\rvert > 1$ **B1** ± 1.2816 or better	**M1** **B1**
	$\sigma = 1.5605.....$ =awrt 1.56 minutes	**A1** awrt 1.56 Do **not** allow answer written as an exact fraction.	**A1** **(3)**
(b)	T represents number less than 12 minutes. $T \sim B(15, 0.1)$	**B1** Writing or using B(15, 0.1) .	**B1**
	$P(T \leq 1)$	**M1** writing $P(T \leq 1)$ or $P(T < 2)$ any letter may be used.	**M1**
	$= 0.549$	**A1** awrt 0.549	**A1**
		NB 0.549 gets B1 M1 A1	**(3)**
(c)	[T ~ number of people who take less than 12 mins to complete the test] $T \sim B(n, 0.1)$		
	T can be approximated by $N(0.1n, 0.09n)$	**B1** mean = $0.1n$ and Var = $0.09n$ **oe** may be seen in an attempt at standardisation	**B1**
	$P\left(Z < \frac{8.5-0.1n}{\sqrt{0.09n}}\right) = 0.3085$	**M1** using a continuity correction either 8.5 or 7.5 in an attempt at standardised form. Allow 0.09 for sd.	**M1**
	$\frac{8.5-0.1n}{\sqrt{0.09n}} = -0.5$ or $\frac{8.5-0.1x^2}{0.3x} = -0.5$	**B1** a z value of awrt $\pm$ 0.5	**B1**
		M1 standardising using their mean and sd. (If these have not been given then they must be correct here) **and** one of 7.5, 8, 8.5, 9 or 9.5 **and** equal to a z value where $\lvert z\rvert > 0.4$. Allow any form	**M1**
		A1 A correct equation in **any form**. ISW. Do **not** allow if they have $0.3n$ rather than $0.3\sqrt{n}$	**A1**
	$0.1n - 0.15\sqrt{n} - 8.5 = 0$ $\sqrt{n} = 10$	**M1** using either the quadratic formula or completing the square or factorising or any correct method to solve **their 3 term quadratic**. If they write the quadratic formula down then allow one slip. If no formula written down then it must be correct for their equation. May be implied by seeing 10 or 8.5. They must show working if the equation used is not correct. **2nd A1** awrt 10.0 – do not need to see n or $\sqrt{n}$. Allow n = 10 May be implied by 100	**M1A1**
	$n = 100$	**3rd A1 cso** 100 If they have a second answer of 72.25 they must reject it to get this final mark.	**A1cso** **(8)**
			(Total 14)

6(a)		**B1** correct shape with the end points on the *x*-axis **B1** correct shape with *k*, 2,3,5,6 marked on in the correct places. Allow $^1/_3$ for *k*	**B1** **B1**
			(2)
(b)	$\frac{1}{2}\times k+2\times k+\frac{1}{2}\times k=1$	**M1** An attempt to find area using any correct method and putting equal to 1	**M1**
	$3k=1$		
	$k=\frac{1}{3}*$	**A1 cso**. AG Method must be shown and there must be no incorrect working. Need to have these 3 lines as a minimum.	**A1 cso**
	alternative		**(2)**
	$\int_2^3 k(x-2)\mathrm{d}x+\int_3^5 k\mathrm{d}x+\int_5^6 k(6-x)\mathrm{d}x=1$ $\left[\frac{kx^2}{2}-2kx\right]_2^3+[kx]_3^5+k\left[6x-\frac{x^2}{2}\right]_5^6=1$	**M1** Correct integration to find the whole area, put = 1 and an attempt to integrate, ignore limits for attempt $x^n \to x^{n+1}$	**M1**
	$\left(\frac{3}{2}k+2k\right)+(5k-3k)+\left(8k-\frac{35}{2}k\right)=1$		
	$3k=1$		
	$k=\frac{1}{3}$	**A1 cso** Method must be shown – at least one step between integration and $k = 1/3$ and there must be no incorrect working.	**A1 cso**
	SC For using verification they could get **M1 A0** if there are no errors		
(c)	$\mathrm{F}(x)=\begin{cases}0 & x<2\\ \frac{x^2}{6}-\frac{2x}{3}+\frac{2}{3} & 2\le x\le 3\\ \frac{x}{3}-\frac{5}{6} & 3<x<5\\ 2x-\frac{x^2}{6}-5 & 5\le x\le 6\\ 1 & x>6\end{cases}$	**Alternative** $\mathrm{F}(x)=\begin{cases}0 & x<2\\ \frac{1}{6}(x-2)^2 & 2\le x\le 3\\ \frac{x}{3}-\frac{5}{6} & 3<x<5\\ 1-\frac{1}{6}(6-x)^2 & 5\le x\le 6\\ 1 & x>6\end{cases}$	**M1A1** **M1A1** **M1A1** **B1**
			(7)

1st M1 For $2 \le x \le 3$, $\int_2^x \frac{1}{3}(t-2)\mathrm{d}t = \left[\frac{t^2}{6} - \frac{2t}{3}\right]_2^x$ and attempt to subst 2 and x

Or $F(x) = \frac{x^2}{6} - \frac{2x}{3} + C$ and using $F(2) = 0$

1st A1 for the second row in the above $F(x)$ **oe**. Condone < instead of $\le$ and vice versa

2nd M1 For $3 < x < 5$, $\int_3^x \frac{1}{3}\mathrm{d}t + "\frac{1}{6}" = \left[\frac{t}{3}\right]_3^x + "\frac{1}{6}"$ and attempt to subst 3 and x. Allow F(3) instead of "$\frac{1}{6}$"

or $F(x) = \frac{x}{3} + C$ and using $F(3) = \frac{1}{6}$ or $F(5) = \frac{5}{6}$

2nd A1 for the third row in the above $F(x)$ **oe**. Condone $\le$ instead of < and vice versa

3rd M1 For $5 \le x \le 6$, $\int_5^x 2 - \frac{t}{3}\mathrm{d}t + "\frac{5}{6}" = \left[2t - \frac{t^2}{6}\right]_5^x + "\frac{5}{6}"$ and subst 5 and x. Allow F(5) instead of "$\frac{5}{6}$"

or $F(x) = 2x - \frac{x^2}{6} + C$ and using $F(6) = 1$

3rd A1 for the fourth row in the above $F(x)$ **oe**. Condone < instead of and vice versa

B1 For both Top line of $F(x)$ ie $0 \quad x < 2$ and Bottom line of $F(x)$ ie $1 \quad x > 6$
Condone $\le$ instead of < and vice versa. Allow one of the lines to have otherwise as its range

(d)	$2x - \frac{x^2}{6} - 5 = 0.9$	**1st M1** using their cdf for $5 \le x \le 6 = 0.9$	**M1**
	$\frac{x^2}{6} - 2x + 5.9 = 0$ $x = \frac{2 \pm \sqrt{4 - 4 \times 1/6 \times 5.9}}{1/3}$	**2nd M1** using either the quadratic formula or completing the square or factorising or any correct method to solve their **3 term quadratic** which must have been correctly rearranged. If they write the formula down then allow a slip. If no formula written down then it must be correct for their equation. May be implied by awrt 5.23 or 6.77	**M1**
	x = awrt 5.23	**A1** awrt 5.23 – (allow $\frac{30-\sqrt{15}}{5}$). If they have 6.77… this must be eliminated	**A1**
			(3)
(e)	$E(X) = 4$		
	$F(5.5) - F(4) = \frac{11}{24}$	**M1** for writing or attempting to find $F(5.5) - F(4)$ or $P(X \le 5.5) - P(x \le 4)$ or $P(X < 5.5) - P(x < 4)$ or $F(5.5) - 0.5$ or $\int_4^5 k\,\mathrm{d}x + \int_5^{5.5} k(6 - x)\,\mathrm{d}x$ with correct limits and $x^n \to x^{n+1}$. May be implied by a correct answer.	**M1**
		A1 $\frac{11}{24}$ **oe** or awrt 0.458	**A1**
			(2)
			(Total 16)

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