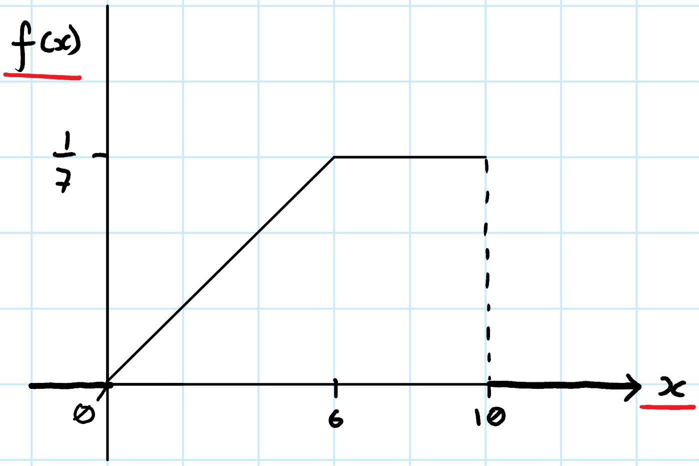
Mock MA - S2

1)

a)



b) $P(x \geqslant 5) = 1 - P(x < 5)$

Area under graph in domain 0<x<5

$$\frac{1}{2} \cdot \frac{5}{42} \cdot 5 = \frac{25}{84}$$

$$1 - \frac{25}{84} = \frac{59}{84}$$

c)

Probability that either of the 2 components break down in 50 hours is
1 - the probability that neither of them break down:

$$\frac{59}{84} \times \frac{59}{84} = \frac{3481}{7056}$$ Neither of them break down

$$1 - \frac{3481}{7056} = \frac{3575}{7056}$$ One or both break down

$$= 0.5067 \quad (4\text{dp})$$

2)

$$X \sim U[-5, 5]$$

a) $P(X < -4.2) = \frac{-4.2 - -5}{5 - -5}$

$= 0.08$

b) $P(|X| < 1.5) = P(X < 1.5) - P(X \leq -1.5)$

$= \frac{1.5 - -5}{5 - -5} - \frac{-1.5 - -5}{5 - -5}$

$= 0.65 - 0.35$

$= 0.3$

c) Let y be the number of lengths of piping, out of 10, cut within 1.5cm of the target length.

$$Y \sim B(10, 0.3)$$

$P(Y > \frac{10}{2}) = 1 - P(Y \leq 5)$

$= 1 - 0.9527$

$= 0.0473$

2)

d) $P(X < -4.2) = 0.08$

Let P be the number of lengths of piping (out of 60) that cannot be used
Let Q be a Poisson approximation of P

$$P \sim B(60, 0.08)$$

n is large, p is small therefore np ≈ npq

$$np = 4.8$$

$$Q \sim Po(4.8)$$

$$P(Q \leq 2) = P(Q=2) + P(Q=1) + P(Q=0)$$

$$= e^{-4.8}\left[\frac{4.8^2}{2!} + \frac{4.8^1}{1!} + \frac{4.8^0}{0!}\right]$$

$$= e^{-4.8}\left[\frac{288}{25} + 4.8 + 1\right]$$

$$= 0.1425 \quad (4dp)$$

3)

a) D is continuous (measure of length)

b) A suitable sampling frame would be a **list** of previous results
Randomly select 36 results from the list to use as your sample

c) Let X be the number of competitors, out of 36, that achieve their greatest distance on their second throw, assuming P(A = 2) = 1/3
Let Y be a normal approximation of X

$X \sim B(36, \frac{1}{3})$ p is close to 0.5, therefore normal aproximaion

$\mu = 12, \sigma^2 = 8$

$Y \sim N(12, \sqrt{8}^2)$

$Z \sim N(0, 1)$

$P(Y \geq 19.5)$

$z = \frac{19.5 - 12}{\sqrt{8}}$ $z = 2.65$

$P(Z \geq 2.$ $= 1 - P(Z < 2.65)$

$= 1 - 0.9965$

$= 0.0035$

d) Probability of 20 out of 36 results being 2 is very low,
so assumption that P(A = 2) = 1/3 is unlikely
P(A = 2) is moot likely greater than 1/3

4)

a)

Let X be the number of vases, out of 20, with defects.

$X \sim B(20, 0.15)$

No Value < 0

$P(X \leq 0) = 0.0388$

$X \leq 0$

$P(X \leq 6) = 0.9781$ ← closer to $1 - 0.025$

$P(X \leq 7) = 0.9941$

$X > 6$

Discrete distribution therefore:

$X \geq 7$

critical regions $X \leq 0$, $X \geq 7$

b)

$0.0388 + (1 - 0.9781) = 0.0607$

c)

Let Y be the number of number of vases sold in 4 weeks

$H_0: \lambda = 10 \qquad H_1: \lambda > 10$

$Y \sim Po(10)$

$P(Y \geq 15) = 1 - P(Y \leq 14)$

$= 1 - 0.9165$

$= 0.0835$

$0.0835 > 0.05$

More than 0.05 therefor not a significant result

Accept H_0, Reject H_1

Insufficient evidence to suggest that the **rate of sales por week** has increased in December

5) $F(1.5) = 1$

a) $\therefore k(2(1.5)^3 - 1.5^4) = 1$

$k\left(\frac{27}{16}\right) = 1$

$k = \frac{16}{27}$

b) $1 - F(1) = 1 - \frac{16}{27}(2-1)$

$= \frac{11}{27}$

c) $f(t) = F'(t)$

for $0 \leq t \leq 1.5$, $F'(t) = \frac{16}{27}(6t^2 - 4t^3)$

$$f(t) = \begin{cases} \frac{32}{27}(3t^2 - 2t^3) & 0 \leq t \leq 1.5 \\ 0 & \text{otherwise} \end{cases}$$

d) $E(T) = \int_0^a t f(t)\,dt$

$= \frac{32}{27}\int_0^{1.5}(3t^3 - 2t^4)\,dt$

$= \frac{32}{27}\left[\frac{3}{4}t^4 - \frac{2}{5}t^5\right]_0^{1.5}$

$= \frac{32}{27}\left[\frac{243}{64} - \frac{243}{80} - 0 + 0\right]$

$\frac{32}{27} \times \frac{243}{320} = 0.9$

e)

$$F(0.9) = \frac{16}{27}\left(2(0.9)^3 - 0.9^4\right)$$

$$= \frac{16}{27}(1.458 - 0.6561)$$

$$= \frac{16}{27} \times \frac{8019}{10{,}000}$$

$$= 0.4752$$

f)

Probability of spending more than an hour on homework:

$$P(T>1) = \frac{11}{27}$$

Probability of spending more than the mean time:

$$P(T > E(T)) = 1 - 0.4752$$

$$= \frac{328}{625}$$

$$P(T>1 \mid T>E(T)) = \frac{11}{27} \div \frac{328}{625}$$

$$= \frac{6875}{8856}$$

$$= 0.7763 \text{ (4 dp)}$$

6)

Let X be the number of customers to the post office in a given 10 minute period

a) $X \sim Po(3)$

$P(X \geqslant 4) = 1 - P(X \leqslant 3)$

$1 - 0.6472 = 0.3528$

b) Let Y be the number of customers to the post office in a given 30 minute period

$Y \sim Po(9)$

$P(Y \leqslant 7) = 0.3239$

c) Let P be the number of customers to the post office in the 5 minute period

$P \sim Po(1.5)$

$P(P = 0) = 0.2231$

$= e^{-1.5}$

Let Q be the number 5 minute periods, out of 6, that have no customers

$Q \sim B(6, e^{-1.5})$

$P(Q \leqslant 1) = \binom{6}{1} e^{-1.5} (1 - e^{-1.5})^{6-1} + \binom{6}{0} e^{0} (1 - e^{-1.5})^{6-0}$

$= 6 \times 0.2231 \times 0.2830 + 1 \times 1 \times 0.2198$

$= 0.5987$ (4 dp)

6)

d) $\frac{7}{2} \times 6 \times 3 = 63$

Let W be the number of customers to the post office in a given 210 minute period
Let N be the Normal approximation of W

$W \sim Po(63)$

$N \sim N(63, \sqrt{63}^2)$

$Z \sim N(0, 1)$

$P(W > 49) \approx P(N > 49.5)$

$\frac{49.5 - 63}{\sqrt{63}} = -1.70$

$P(Z > -1.70) = P(Z < 1.70)$

$= 0.9554$ 4dp