# S2 June 2018 (MA)

Q1a i) $P(X > 5) = 1 - P(X \leq 5) = \boxed{0.8843}$

ii) $P(4 \leq X < 10) = P(4 \leq X \leq 9)$

$= P(X \leq 9) - P(X \leq 3) = 0.5874 - 0.0212$

$= \boxed{0.5662}$

b) For 7 hours, (420 min) $Y \sim Po(378)$

$\lambda$ is large $\therefore Y \simeq\sim N(378, 378)$

$P(Y < 370) = P(Y < 369.5)$ ← applying c.c

$= P\left(Z < \frac{369.5 - 378}{\sqrt{378}}\right) = P(Z < -0.44)$

$= 1 - P(Z < 0.44) = 1 - 0.67$

$= \boxed{0.330}$

c) let A = no. of days (/5) where there are fewer than 370 calls.

$A \sim B[5, 0.33]$

$P(\text{required}) = P(A \geq 4) = P(A = 4) + P(A = 5)$

$= \binom{5}{4}(0.33)^4(0.67)^1 + 0.33^5 = \boxed{0.0436}$

Q2a) - only 2 possible outcomes (heads & tails)
- no. of spins is fixed
- probability of a tail is fixed
- spins are independent of each other

(any 2)

b) $T \sim B[6, \frac{1}{2}]$

$P(T=5) = \binom{6}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 = \boxed{\frac{3}{32}}$

c) $P(\text{more tails than heads}) = P(T \geq 4)$

$= 1 - P(T \leq 3) = 1 - 0.6563$

$= \boxed{0.3437}$

d) $H \sim B[6, \frac{1}{4}]$ where $H =$ no. of heads for second coin.

$P(\text{required}) = P(H \geq 3) = 1 - P(H \leq 2)$

$= 1 - 0.8306 = \boxed{0.1694}$

Q3a) $E(T) = \frac{1}{2}\int_1^2 (t^2 - t)\,dt + \frac{1}{16}\int_2^4 (14t^2 - 3t^3 - 8t)\,dt$

$= \frac{1}{2}\left[\frac{t^3}{3} - \frac{t^2}{2}\right]_1^2 + \frac{1}{16}\left[\frac{14t^3}{3} - \frac{3t^4}{4} - 4t^2\right]_2^4$

$= \frac{1}{2}\left[\frac{8}{3} - 2\right] - \frac{1}{2}\left[\frac{1}{3} - \frac{1}{2}\right]$

$+ \frac{1}{16}\left[\frac{128}{3}\right] - \frac{1}{16}\left[\frac{28}{3}\right]$

$= \frac{5}{12} + \frac{25}{12} = \boxed{\frac{5}{2}}$

b) $E(T^2) - [E(t)]^2 = Var(T)$

$\therefore Var(T) = \frac{267}{40} - \left(\frac{5}{2}\right)^2 = \boxed{\frac{17}{40}}$

c) $\frac{1}{2}\int_1^t (t-1)\,dt = \frac{1}{2}\left[\frac{t^2}{2} - t\right]_1^t = \frac{1}{2}\left[\frac{t^2}{2} - t + \frac{1}{2}\right]$

$= \frac{1}{4}\left[t^2 - 2t + 1\right]$ //

$F(2) = \frac{1}{2}(2-1) = \frac{1}{2}$ //

interval $1 < t \leq 2$.

for $2 < t \leq 4$ :

$$F(2) = \frac{1}{4}(2^2 - 2(2) + 1) = \frac{1}{4}$$

$$\frac{1}{16}\int_2^t [14t - 3t^2 - 8]\,dt = \frac{1}{16}\left[7t^2 - t^3 - 8t\right]_2^t$$

$$= \frac{1}{16}\left[7t^2 - t^3 - 8t\right] - \frac{1}{16}\left[4\right]$$

$$\frac{1}{16}\left[7t^2 - t^3 - 8t\right] - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} = \frac{1}{16}\left[7t^2 - t^3 - 8t\right]$$

so . . .

$$F(t) = \begin{cases} 0, & t \leq 1 \\ \frac{1}{4}(t^2 - 2t + 1), & 1 < t \leq 2 \\ \frac{1}{16}(7t^2 - 8t - t^3), & 2 < t \leq 4 \\ 1, & t > 4 \end{cases}$$

d) $F(2) = 0.25$ so $20^{th}$ percentile lies in first interval.

$$\Rightarrow \frac{1}{4}(t^2 - 2t + 1) = \frac{(t-1)^2}{4} = 0.2$$

$$\Rightarrow (t-1)^2 = 0.8$$

$$\Rightarrow t - 1 = \pm\sqrt{0.8}$$

reject, $t > 1$

$$t = 1 \pm \sqrt{0.8} = \boxed{1.89} \text{ or } 0.106$$

e) $P(t > 1.5) = 1 - F(1.5)$

$$= 1 - \frac{(1.5-1)^2}{4} = \boxed{\frac{15}{16}}$$

f) $P(t > 3 \mid t > 1.5) = \dfrac{P(t>3)}{P(t>1.5)}$

$$= \frac{1 - F(3)}{1 - F(1.5)} = \frac{1 - \frac{3}{4}}{1 - \frac{1}{16}}$$

$$\left( F(3) = \frac{1}{16}\left(7(3^2) - (3)^3 - 8(3)\right) = \frac{3}{4} \right)$$

$$= \boxed{\frac{4}{15}}$$

Q4a) $X \sim U[\alpha, \beta]$

$E(X) = \dfrac{\alpha + \beta}{2} = 4$

$\therefore \alpha + \beta = 8 \sim (1)$

$Var(X) = \dfrac{(\beta - \alpha)^2}{12} = 12$

$\therefore (\beta - \alpha)^2 = 144 \sim (2)$

from (1), $\alpha = 8 - \beta$

(2) : $(\beta - 8 + \beta)^2 = 144$

$(2\beta - 8)^2 = 144$

$2\beta - 8 = \pm\sqrt{144} = \pm 12$

$\therefore 2\beta = 8 \pm 12$

as $\beta > \alpha$

$\beta > 0$ so $\beta = \frac{8+12}{2} = \boxed{10}$

and $\alpha = 8 - \beta = \boxed{-2}$

b) $P(\text{David is late}) = P(\text{oversleeps}) + P(\text{doesn't oversleep but } X > 5)$

$= (0.05) + (0.95)\left(\frac{10-5}{12}\right)$

$= \boxed{\frac{107}{240}}$

c) $P(\text{overslept} \mid \text{he is late}) = \frac{P(\text{oversleeps and late})}{P(\text{Late})}$

(if he oversleeps he will always be late)

$= \frac{0.05}{\frac{107}{240}} = \boxed{\frac{12}{107}}$

Q5a) $H_0: p = 0.35$
$H_1: p > 0.35$

$V \sim B(40, 0.35)$

$P(V \geq 18) = 1 - P(V \leq 17)$

$= 1 - 0.8761$

$= 0.1239$

$0.1239 > 0.05$

$\therefore$ Result is insignificant.

Accept $H_0$; insufficient evidence to suggest the proportion of customers buying organic vegetables

... has increased.



E = no. of customers buying organic eggs.

b) $E \sim B[50, 0.35]$

$P(E \leq 10) = 0.0160$
$P(E \leq 11) = 0.0342$ } 0.0160 is closer to 0.025

$P(E \geq 25) = 0.0207$
$P(E \geq 24) = 0.0396$ } 0.0207 is closer to 0.025

so C.R is $E \leq 10$ and $E \geq 25$

c) 8 is in the critical region. So the manager's claim is supported.

d) $0.0160 + 0.0207 = \boxed{0.0367}$
(3.67%)

e) $F \sim N(40, 32)$

$np = 200(0.2) = 40$

$np(1-p) = 200(0.2)(0.8) = 32$

$P(F < n) = 0.0465$

$P(F < n) = P(F < n - 0.5)$ ← applying c.c

$= P\left(Z < \frac{n - 0.5 - 40}{\sqrt{32}}\right) = P\left(Z < \frac{n - 40.5}{\sqrt{32}}\right)$

$\Rightarrow P\left(Z < \frac{n - 40.5}{\sqrt{32}}\right) = 0.0465$

$\Rightarrow P\left(Z < \frac{40.5 - n}{\sqrt{32}}\right) = 0.9535$

but $P(Z < 1.68) = 0.9535...$

$$\therefore 1.68 = \frac{40.5 - n}{\sqrt{32}}$$

$$\Rightarrow 40.5 - 1.68\sqrt{32} = \boxed{n = 31}$$

Q6a) $F'(x) = k(ax^2 - x^3) = f(x)$

for $2 < x \leq 4$
since $x = \frac{8}{3}$
at mode

$F''(x) = k(2ax - 3x^2) = f'(x)$.

(at the mode there is a turning point $\left(\frac{dy}{dx} = 0\right)$)

$$\text{so} \quad k(2ax - 3x^2) = 0$$

$$2ax - 3x^2 = 0$$

$$2a - 3x = 0$$

$$\frac{3x}{2} = a$$

at $x = \frac{8}{3}$ : $\quad a = \frac{3}{2} \times \frac{8}{3} = \boxed{4}$

b) $P(X < 2.5) = F(2.5) = k\left(\frac{4}{3}(2.5)^3 - \frac{(2.5)^4}{4}\right) + b$

So we need to find $k$ and $b$ ...

looking at first interval, $F(2) = \frac{4}{15}$

so substituting $x = 2$ into the 2nd interval should yield $\frac{4}{15}$.

$$F(2) = k\left(\frac{8}{3}(4) - 4\right) + b = \frac{4}{15}$$

$$\frac{20k}{3} + b = \frac{4}{15} \sim 1$$

and $F(4) = 1$

$$F(4) = k\left(\frac{64}{3}(4) - 64\right) + b = 1$$

$$\frac{64k}{3} + b = 1 \sim 2$$

solve 1 and 2 simultaneously...

2 − 1 : $\frac{44}{3}k = \frac{11}{15}$

$$k = \frac{3 \times \frac{11}{15}}{44} = \boxed{\frac{1}{20}}$$

$$\therefore b = 1 - \frac{64}{3}\left(\frac{1}{20}\right) = \boxed{-\frac{1}{15}}$$

so $F(2.5) = P(X < 2.5) = \frac{1}{20}\left(\frac{2125}{192}\right) - \frac{1}{15} = \boxed{0.487}$