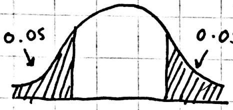
JUNE 2017 S2 MA

1a) $X \sim B(25, 0.2)$



$P(X \leq 1) = 0.0274$

$P(X \leq 8) = 0.9532$

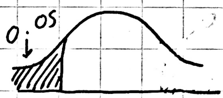
$1 - P(X \leq 8) = P(X \geq 9) = 0.04$

CR:

$\underline{X \leq 1}$ and $\underline{X \geq 9}$

b) $X \sim B(50, 0.2)$ $H_0: p = 0.2$

$H_1: p < 0.2$



$P(X \leq 5) = 0.0480$

CR: $X \leq 5$

6 is not within CR, $\therefore$ insufficient evidence to reject H_0, $\therefore$ accept H_0 $\therefore$ evidence that increasing number of pots does <u>NOT</u> reduce percentage of broken pots.

2 a i) $X \sim Po(2.5)$

$P(X \geq 4) = 1 - P(X \leq 3)$

$1 - 0.7576 = \underline{0.2424}$

ii) $Y \sim Po(0.625)$

$2.5 = 60$

$0.625 = 15$

$P(Y=3)$

$e^{-0.625} \times \frac{0.625^3}{3!} = \underline{0.0218}$ (3 s.f.)

b)

$P(X=0) > 0.8$

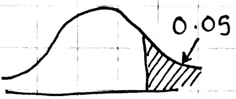
$e^{-2.5t} > 0.8$

$\log_{(e^{-2.5})}(0.8) = t = 0.08925\ldots$ hours

$60 \times 0.08925 = 5.36$ mins

to nearest minute, $\underline{t = 5}$

2 c) $2.5 \times 2 = 5$ $\qquad X \sim Po(5)$



$H_0 : \lambda = 5$

$H_1 : \lambda > 5$

$P(X \leq 9) = 0.9682$

$1 - P(X \leq 9) = 0.0318$

$1 - P(X \leq 9) = P(X \geq 10)$

CR: $X \geq 10$

$10 \geq 10$ $\therefore$ reject H_0 and accept H_1. There is sufficient evidence that the mean rate of telephone calls has increased.

3)

$$f(x) \begin{cases} \frac{1}{9}x(4-x) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

a) $E(X) = \frac{1}{9}\int_1^4 4x^2 - x^3 \, dx$

$\left(\int x f(x) \, dx\right)$

$$\frac{1}{9}\left[\frac{4}{3}x^3 - \frac{1}{4}x^4\right]_1^4$$

$$\frac{1}{9}\left(\frac{64}{3} - \frac{13}{12}\right)$$

$$= \frac{9}{4} = \underline{2.25}$$

b) $P(X > 2.5)$

$$\int_{2.5}^{4} \frac{1}{9}x(4-x)\,dx$$

$$\frac{1}{9}\int_{2.5}^{4} 4x - x^2\,dx$$

$$\frac{1}{9}\left[2x^2 - \frac{1}{3}x^3\right]_{2.5}^4$$

$$\frac{1}{9}\left(\frac{32}{3} - \frac{175}{24}\right)$$

$$\frac{1}{9} \times \frac{27}{8} = \frac{3}{8} = 0.375$$

c) life time X is in tens of hours

$\therefore$ 25 hours $= 2.5 \times$ ten

$= P(X > 2.5) = 0.375$

of 2 batteries $= (0.375)^2$

$= 0.141$

d) 16 hours $= 1.6 \times 10$

$P(X > 1.6)$

$$\frac{1}{9}\int_{1.6}^{4} 4x - x^2 \, dx$$

$$\frac{1}{9}\left[2x^2 - \frac{1}{3}x^3\right]_{1.6}^{4} = \frac{1}{9}\left(\frac{32}{3} - \frac{1408}{375}\right)$$

$$= \frac{96}{125} = 0.768$$

two batteries $\therefore = 0.788^2 = (0.768)^2$

$16 + 9 = 25$ hours

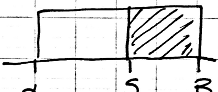
$$\therefore \frac{P(\text{working 25 hours})}{P(\text{working 16 hours})}$$

$$= \frac{0.141...}{(0.768)^2} = 0.2384...$$

4) $[\alpha, \beta]$

$E(X) = 3.5 \quad P(X>5) = \frac{2}{5}$

a) $E(X) = \frac{1}{2}(\alpha+\beta) = 3.5 \quad \ldots ①$



$P(X>5) = \frac{\beta-5}{\beta-\alpha} = \frac{2}{5} \quad \ldots ③$

from ① $\alpha+\beta = 7$

$\alpha = 7-\beta \quad \ldots ②$

② sub in ③

$$\frac{\beta-5}{\beta-(7-\beta)} = \frac{2}{5}$$

$$\frac{\beta-5}{2\beta-7} = \frac{2}{5}$$

$$\beta - 5 = \frac{2}{5}(2\beta-7)$$

$$\beta - \frac{4}{5}\beta = -\frac{14}{5} + 5$$

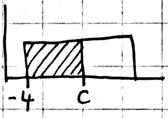
$$\frac{1}{5}\beta = \frac{11}{5}$$

$$\underline{\beta = 11}$$

$\alpha = 7-11$

$\underline{\alpha = -4}$

b) $P(X < c) = \frac{2}{3}$



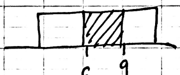
$$\frac{c - -4}{11 - -4} = \frac{2}{3}$$

$$\frac{c+4}{15} = \frac{2}{3}$$

$$c = \frac{2}{3} \times 15 - 4$$

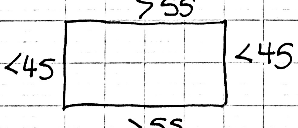
$$\underline{c = 6}$$

ii) $P(6 < X < 9)$



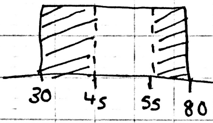
$$\frac{9-6}{15} = \frac{3}{15} = \frac{1}{5} = \underline{0.2}$$

c) $[30, 80]$ perimeter = 200



if one side is <45 the other must be >55 so that the perimeter = 200

$\therefore P(S<45) + P(S>55)$



$$\frac{45-30}{50} + \frac{80-55}{50} = \frac{3}{10} + \frac{1}{2} = \frac{4}{5}$$

5 a)

$P(M<12) = 0.1$

$P(z < \frac{12-14}{\sigma}) = 0.1$

0.1 not in tables $\therefore$ $1-0.1=0.9$

$1-P(z<\frac{12-14}{\sigma}) = 0.9$

$P(z<-\frac{12-14}{\sigma}) = 0.9$

closest to 0.9 in tables $= 0.8997$

$= 1.28$

$\therefore \frac{-12+14}{\sigma} = 1.28$

$\sigma = \frac{2}{1.28} = \frac{25}{16} = 1.5625$

b) $X \sim B(15, 0.1)$

$P(X<2) = P(X \leq 1) = 0.5490$

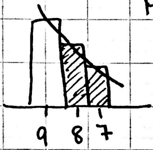
c $X \sim B(n, 0.1)$

$P(X < 9) = P(X \leq 8)$

$0.1 \times 0.9 = 0.09$

$X \sim N(0.1n, 0.09n)$

continuity correction



$P(X < 8.5) = 0.3085$

$P\left(Z < \dfrac{8.5 - 0.1n}{\sqrt{0.09n}}\right) = 0.3085$

$1 - P\left(Z < \dfrac{8.5 - 0.1n}{\sqrt{0.09n}}\right) = 1 - 0.3085$
$= 0.6915$

$P\left(Z - \dfrac{8.5 - 0.1n}{\sqrt{0.09n}}\right) = 0.6915$

$\therefore \dfrac{-8.5 + 0.1n}{\sqrt{0.09n}} = 0.5$

$-8.5 + 0.1n = 0.5 \times 0.3 \times n^{1/2}$

$\cancel{0.18}\ 0.1n - 0.15n^{1/2} - 8.5 = 0$

let $n^{1/2} = x$

$0.1x^2 - 0.15x - 8.5 = 0$

$x = 10$ or $x = -8.5$

continued c) accept $x = 10$ as time cannot be negative

$\therefore n^{1/2} = 10$

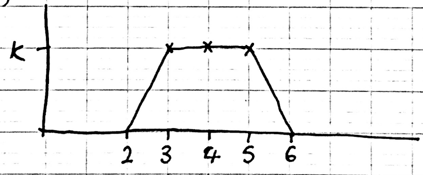
$n = 10^2 = 100$

$\underline{\underline{n = 100}}$

6)

$$f(x) \begin{cases} k(x-2) & 2 \leq x \leq 3 \\ k & 3 < x < 5 \\ k(6-x) & 5 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

a)



b) area under graph = 1

$\therefore \frac{1}{2}k + 2k + \frac{1}{2}k = 1$

$3k = 1$

$k = \frac{1}{3}$

c)

$$F(x)\begin{cases} 0 & x<2 \\ \frac{1}{6}x^2-\frac{2}{3}x+c_1 & 2\leq x\leq 3 \\ \frac{1}{3}x+c_2 & 3<x<5 \\ 2x-\frac{1}{6}x^2+c_3 & 5\leq x\leq 6 \\ 1 & x>6 \end{cases}$$

$$\frac{1}{6}x^2-\frac{2}{3}x+c_1 = 0 \quad \text{when } x=2$$

$$\therefore c_1 = \frac{2}{3}$$

$$\frac{1}{6}x^2-\frac{2}{3}x+\frac{2}{3} = \frac{1}{3}x+c_2 \text{ when } x=3$$

$$\therefore c_2 = -\frac{5}{6}$$

$$2x-\frac{1}{6}x^2+c_3 = 1 \quad \text{when } x=6$$

$$\therefore c_3 = -5$$

$$F(x)\begin{cases} 0 & x<2 \\ \frac{1}{6}x^2-\frac{2}{3}x+\frac{2}{3} & 2\leq x\leq 3 \\ \frac{1}{3}x-\frac{5}{6} & 3<x<5 \\ 2x-\frac{1}{6}x^2-5 & 5\leq x\leq 6 \\ 1 & x>6 \end{cases}$$

d) 90th percentile between
$5 \leq x \leq 6$

$\therefore 2x - \frac{1}{6}x^2 - 5 = 0.9$

$\frac{1}{6}x^2 - 2x + 5.9 = 0$

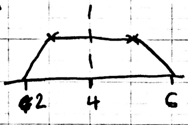
$$\frac{2 \pm \sqrt{2^2 - 4 \times \frac{1}{6} \times 5.9}}{\left(2 \times \frac{1}{6}\right)}$$

$x = \frac{30 \pm \sqrt{15}}{5} = 6.77$ or 5.23

$\underline{x = 5.23}$ as $6.77 > 6$ disgard

e) $P[E(X) < X < 5.5]$

$E(X) = 4$



$P(X < 5.5) - P(X < 4)$
$F(5.5) - F(4)$
$\left(2(5.5) - \frac{1}{6}(5.5)^2 - 5\right) - \left(\frac{1}{3}(4) - \frac{5}{6}\right)$
$= \frac{11}{24}$